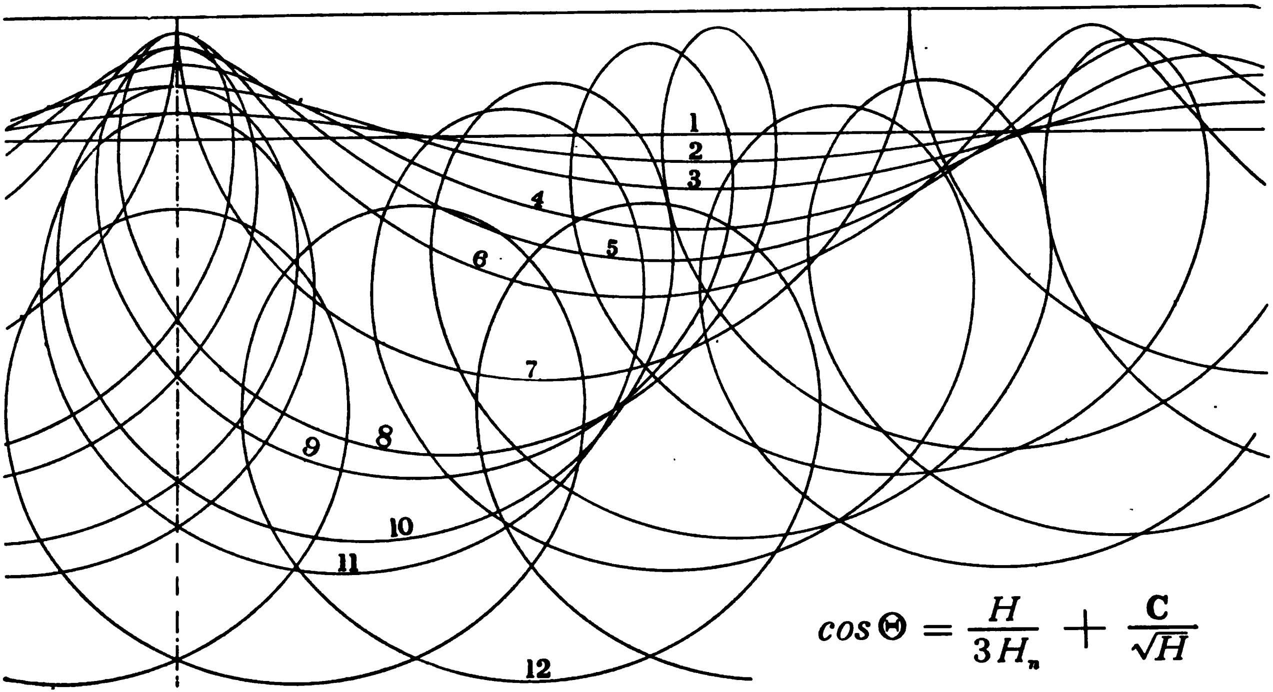
AERODONETICS



$$\cos\Theta = \frac{H}{3H_n} + \frac{C}{\sqrt{H}}$$

THE PHUGOID CHART. THE FLIGHT PATH PLOTTED FROM THE EQUATION.

[*Frontispiece.*

AERODONETICS

CONSTITUTING THE SECOND VOLUME OF A COMPLETE WORK ON AERIAL FLIGHT

BY

F. W. LANCHESTER

With Appendices on the Theory and Application of the Gyroscope, on the Flight of Projectiles, etc.

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PREFACE

THE object and scope of the present work have been stated at length in the preface to Vol. I., which appeared as recently as December, 1907; there is but little to add to this statement.

In conducting the investigations and otherwise in collecting and arranging the matter included in the two volumes now completed, the author has made an effort to do, in as thorough a manner as possible, that preparatory work which, in his opinion, should properly precede the serious and hitherto hazardous undertaking of full-scale experiment. It is in other words the author's intention to provide such foundation theory and data as to bring the problem of mechanical flight into the legitimate domain of the engineer, and to obviate in the future all need for empiricism.

The author is aware that considerable work remains to be done in the way of extension, both in the more complete elucidation of many questions relating to the flight of birds, and in the direction of the application of theory in the design of the flying-machine for aerial navigation. It has not been found practicable to deal specifically with these extensions in the present volumes, but much that is suggestive will be found embodied appropriately in the text. The *passive mode* of bird flight, as involved in gliding and soaring, has however been discussed at length, for it is in the study of this *mode* of flight that the greater part of that which is essential may be learned.[1]

[1] There is a false impression—one that is far too prevalent—that the *essence* of flight consists in the flapping of wings. Nothing is further from the truth.

So far is this error current that attempts have been sometimes made to

The present time is one of considerable importance in the history of aerial flight: the motive power engine has, during the last few years, reached a stage in its development at which its weight has become sufficiently reduced to render mechanical flight possible, and already several partially successful machines have been built, and flights of several miles have been made. Some of these machines are deficient in many important respects; the propellers for example are of relatively small diameter and are commonly of too quick a pitch and too high a revolution speed for best efficiency. The longitudinal stability also, instead of being automatic, is only maintained by the watchful attention of the aeronaut and the dexterous manipulation of a horizontal rudder. Further than this the cooling of the motor cylinders is not altogether effective, and the duration and range of flight is largely a matter of how long elapses before the water is boiled away. The question of weight makes it difficult to employ a thoroughly efficient "radiator" as used on road vehicles, and without doubt direct air cooling will sooner or later come into vogue. For the above reasons the flights so far made have been of comparatively brief duration.

In the future it is unlikely that the flying-machine will be

limit the term *flying-machine* to appliances of the wing-flapping kind. As illustrative of this vein of thought the statement is sometimes made that an aerodrome is not a flying-machine at all, that it does not *fly*, and that it is only sustained (presumably like a satellite) by the speed at which it travels!

Critics who talk thus know little or nothing of the subject of flight as taught by nature. It is true that many of the smaller birds and most insects are capable of stationary flight, but the heavier birds, as for example the eagle, the vulture, the stork, the albatros, the larger gulls, and nearer home the swan, the goose, and even the wild duck, cannot fly without considerable horizontal velocity. When leaving the ground (or water) these birds require a run of many yards in order to attain the necessary velocity. According to Mouillard, who was an accurate observer, most birds of more than a pound or two in weight may be effectively "caged" if confined in a simple fenced enclosure (open to the sky) of appropriate size; they are unable to acquire the velocity necessary to their flight.

Beyond the above very many of the larger birds rarely use the active (wing-flapping) mode of flight at all, but nevertheless it is not said that they no longer fly on that account.

limited in its performance to short flights over prepared ground at a few metres height, ready to come to earth at a moment's notice; it will rather seek safety in altitude, probably flying in most part at a height of at least two or three thousand feet, where in the event of any minor mechanical failure or other hitch the time of gliding descent will be at the disposal of the aeronaut for adjustment, or, if it is found necessary to land, for the selection of a suitable site. Thus, if an aerodrome be flying one mile above the surface of the earth, there will be a circle of some ten or twelve miles diameter available within which to choose a place to alight; or, if we assume the velocity of flight to be forty miles per hour, the aeronaut will have at his disposal a period of some eight or ten minutes before he need finally come to earth.

The manner in which the industries connected with locomotion have proved mutually helpful is very striking; the dependence of the march of progress on such mutual assistance, nearly always traceable in the evolution of mechanism, is perhaps nowhere more apparent. Thus the modern automobile may almost be said to owe its existence to the pneumatic tyre, an invention whose utility was first established by the bicycle; and, in turn, the flying-machine has only become possible through the development of the internal combustion engine in the hands of the automobile engineer. It is evident that the market for the light weight petrol motor has grown up as a direct consequence of the demand for high speed, a demand that could not have arisen in the absence of the pneumatic tyre; hence even the old "hobby horse" may be regarded as one of the stepping stones to the conquest of the air, and we are led to regard the flying-machine as marking a great step in a vast evolutionary movement in locomotion, rather than as constituting in itself an independent invention.

The title of the present volume, "Aerodonetics," is one of two alternatives suggested in the preface to Vol. I., chosen as being from its derivation the more appropriate. The arrangement is as follows:—

Chapter I. is an introductory account of the general pirnciples concerned in the equilibrium and stability of an aerodone in flight, illustrated by actual examples, including an account of the author's earlier experiments.

Chapters II. and III. consist of an analytical investigation of the flight path on a basis restricted by a hypothesis excluding the influence of the size and moment of inertia of the aerodone, and assuming resistance to flight to be either absent or accounted for by an equal and opposite force of propulsion. This investigation culminates with the plotting of the flight path from the equation (as given in the frontispiece), and includes a discussion of certain special cases, notably that of the flight path or *phugoid*[1] of small amplitude. This investigation is the foundation of the greater part of the subsequent work, where it is frequently referred to under the title of the Phugoid Theory; it constitutes the key to the quantitative study of longitudinal stability, and to the solution of many kindred problems in free flight.

Chapter IV. is devoted to the discussion of some of the more immediate and self-evident consequences of the Phugoid Theory, including the preliminary consideration of the influence of wind fluctuations both as isolated gusts and as possessing a definite periodicity.

Chapter V. deals with an important extension of the Phugoid Theory, by which account is taken of influences excluded by the initial hypothesis, *i.e.*, resistance and moment of inertia. In this chapter the theory is brought to a point at which it becomes of very great value as bearing on the correct proportioning of an aerodone or aerodrome; the investigation culminates in an equation, the *equation of stability*, by which the conditions of the permanence of the flight path are clearly defined.

Chapter VI. is an account of the experimental verification of the previous work (Chapters II., III., and V.). This account comprises a brief *résumé* of observations by earlier writers as confirmatory of the Phugoid Theory, a series of experiments

[1] From the Greek φυγη and ειδος (lit. *flight-like*); compare Glossary.

devised and carried out by the author as a direct test, and the application of the theory to the investigation of the stability of birds in flight, and to pre-existing flight models, including the gliding machine of the late Herr Lilienthal. The results are conclusive.

Chapter VII. consists of a series of investigations on lateral and directional stability, and may be looked upon as an independent section of the work. The method of treatment adopted is the initial discussion of these two kinds of stability separately, and their subsequent treatment in combination under the title *rotative stability;* this investigation also culminates in an equation which defines the limiting conditions of the stability of the kind under consideration.

Chapter VIII. constitutes in part a *résumé* and in part an extension of that which has gone before. It comprises a review of the groundwork of the theoretical investigations, with some notes and discussion of the limitations and existing deficiencies of the work; also an extension of the investigation of Chapter V. to cover the conditions that arise in the case of an aerodrome propelled by prime mover and screw propeller, with a further investigation on the rate of damping of the phugoid oscillation. The chapter concludes with a discussion of the theory of corresponding speed and its application to scale model experiment, and with some remarks on forms of aerodone that may be considered as departures from elementary type.

Chapter IX. is a distinct branch of the work and deals with the phenomenon of soaring, both from the point of view of observation and in the light of theory, much of the preceding work being brought to bear on this complex subject with considerable effect. The first portion of the work includes quotations from observations by Darwin, J. A. Froude, Mouillard, Langley, and others, and the theoretical investigations have for their starting point the well-known dictum of Rayleigh, that when soaring is possible the motion of the wind is either not horizontal or not uniform.

Chapter X. is principally an account of experimental method, and includes many notes, observations, and hints that will be found of value to those who are tempted to take up the experimental study of aerial flight.

The terminological innovations adopted in the present work are given in the glossary following Chapter X., in which are included new words or words bearing a special or restricted meaning, as employed in both Vols. I. and II. The present glossary is thus, in great part, a repetition of that previously given.

Numerical work has been done by the aid of an ordinary 25 c.m. slide rule, with a liability to error of about one-fifth of one per cent., an amount which is quite unimportant.

It is again with very great pleasure that the author tenders his thanks to Mr. P. L. Gray for his assistance in the reading and correction of the proof sheets.

May, 1908.

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VOLUME I.—ERRATA.

P. 4, line 14 from foot, *for* "plane the" *read* "the plane."

P. 10, line 6 from foot, *for* "icthyoid" *read* "ichthyoid."

P. 53, line 9 from foot, *for* "Poissuille" *read* "Poiseuille."

P. 85, line 8 from top, the cavity of the labyrinth of the ear is given as an example of a triply connected region; this is in error, *delete*.

P. 113, line 4 below Fig. 50, *for* "were" *read* "was."

P. 199, line 7 from foot, *for* "$4 \times I$" *read* "4×1."

P. 231, line 4 from foot (also in footnote), *for* "Moulliard" *read* "Mouillard."

P. 269, line 12 from foot, *for* "k" *read* "κ."

P. 300, line 6 from top, *for* "$\tan(\theta - \gamma)$" *read* "$\tan(\theta + \gamma)$," comp. p. 299, line 3.

P. 431, line 3 from foot should read, "angle of the course relatively to the *apparent direction of the* wind would be the sum of."

P. 432, line 7 from top, *delete words*,—"about twice that stated even in the most carefully designed craft."

AERODONETICS

CHAPTER I

FREE FLIGHT, GENERAL PRINCIPLES AND PHENOMENA

§ 1. Introductory.—The equilibrium and stability of a bird in flight, or of an aerodrome or flying machine, has in the past been the subject of considerable speculation, and no adequate explanation of the principles involved has hitherto been given.

The question of stability is studied to the greatest advantage in the case of gliding flight, for the problem is then presented in its simplest form; it is probable that the underlying principles are identical, whether a bird is in active flight, or whether it be poised on rigid pinions, as when soaring, or when merely gliding from a point of greater to one of less altitude.

There are two methods employed by nature to secure stability where animal life is concerned, *i.e.*, *mechanical means* and *nervous control*. As an illustration of the first may be cited the case of a quadruped, whose equilibrium when standing is of the mechanical order; and of the second the biped, whose equilibrium is maintained by the action of the nervous system.[1]

Mechanical stability does not necessarily involve an obvious apparatus like the four legs of a table, as might be supposed from the above illustration; the stability of a spinning top for instance, or of a common hoop, is entirely mechanical, but these

[1] The equilibrium of a man standing erect is only maintained by a series of reflex adjustments, although a rigid image or statue may be made mechanically stable.

are cases of *dynamic* stability, in contradistinction to the static stability of a table or a three-legged stool.

On the other hand, nervous control in the matter of stability does not of the least necessity involve any conscious action of the brain; there are special nerve centres to which the function of maintaining equilibrium is deputed.

It is always difficult in the case of any animal or bird to determine to what extent equilibrium is maintained by nervous agency and to what extent it is automatic. In the case of a bicycle rider we know that the equilibrium is to some extent automatic,[1] but that it is not so entirely may be reasonably inferred from the fact that it is necessary to *learn to ride*. In the case of a bird in flight, we have no means of ascertaining directly to what extent equilibrium is maintained by a continual series of reflex adjustments, and to what extent by purely dynamic action consequent upon its geometrical form considered as an inanimate body.

It is almost certain that, for a bird in flight, in any sudden evolution necessary to regain equilibrium after an unexpected gust of wind, the nervous system is called into play, and perhaps at times the whole conscious brain of the bird is involved in the effort to restore the balance that has been momentarily lost. These happenings may be said to be obvious to anyone who has watched the flight of birds in stormy weather. It would seem to follow as a matter of inference that a considerable number of minor reflex adjustments are made and perhaps even a continuous reflex adaptation may take place, invisible to an observer, but the extent to which this is the case can only at the moment be a matter of conjecture.

It is evident that the most promising way to examine the subject, both theoretically and experimentally is by an investigation of the inanimate model or *aerodone*, and having ascertained all

[1] It is well known that a cyclist can frequently ride his machine perfectly, when either from cerebral injury or inebriation he is quite unable to walk without assistance.

that can be done and accounted for on purely dynamical principles, to revert to the subject of bird flight, equipped with the knowledge so obtained.

§ 2. Introductory Remarks (continued).—There is an initial difficulty associated with the study of equilibrium that perhaps has been felt by others who have in the past attempted to investigate the subject; this difficulty is the *indefiniteness* of that which is to be investigated.

Whatever kind of thing an aerodrome or aerodone may be, it is evident that when not in flight it is absolutely unconstrained in any one of its *degrees of freedom*; it is, in fact, free to move bodily in any of the three co-ordinate directions of space, or to rotate about any of the three co-ordinate axes. In any other locomotive appliance we are accustomed to start with some definite limitations; thus a road vehicle is essentially constrained to motions in one plane, and therefore has only three degrees of freedom (*i.e.*, motion in two co-ordinate directions and rotation about one of the co-ordinate axes); a railway locomotive is deprived of all but one of its degrees of freedom, longitudinal motion alone being permitted.

When the aerodrome or aerodone is in free flight, its stability depends upon the limitation that is imposed on its freedom by its functional organs; it must either be deprived of its undesirable degrees of freedom altogether, or its motions permitted in these undesirable degrees must continually react one on another so as to in effect impose a safe limitation in every case; the only kind of continuous motion that is permissible being translation in the line of flight. It is the study of these actions and reactions between the forces and couples in and about the co-ordinate axes that forms the basis of the author's present investigations.

The subject may be conveniently approached by a preliminary discussion of the *ballasted aeroplane* and other simple forms of aerodone; the present chapter is principally devoted to this

discussion, including an account of the author's earlier (1894) experiments.

§ 3. The Ballasted Aeroplane. A Simple Case.—The simplest construction of flight model known is that described in §§ 162, 241, *et seq.*, of the author's "Aerodynamics," under the title of the *Ballasted Aeroplane.* If an aeroplane of about the proportions shown in Fig. 1 be loaded by appropriately disposed ballast, to bring its centre of gravity to a point somewhere about one-quarter of its width from the leading edge, it is found to be capable of gliding flight and is stable both laterally and longitudinally within certain limits.

For experimental purposes the author has found a mica plate about 2 inches by 8 inches by about three to four thousandths of

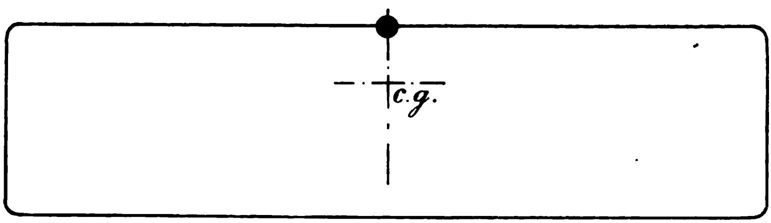


Fig. 1.

an inch thick, gives the most satisfactory results, the ballast taking the form of a split lead shot secured by closing on to the leading edge of the mica plate at a point midway between its ends.

The ballasted aeroplane constructed as described is easily upset by any sudden puff of wind, but in still air, as when experimenting inside a building, it may be employed to demonstrate many important points relating to the equilibrium question. A ballasted aeroplane may be launched directly by hand, the mica plate being taken between the finger and thumb and projected forward with as nearly a parallel motion as possible. A better way is to employ a launching staff, Fig. 2, which consists of a straight rod a few feet in length, capped with a small rectangular platen on which the aeroplane is carried; the

staff is held in an approximately vertical position with the lower end about shoulder high, and the launching is effected by giving a swaying motion to the body.

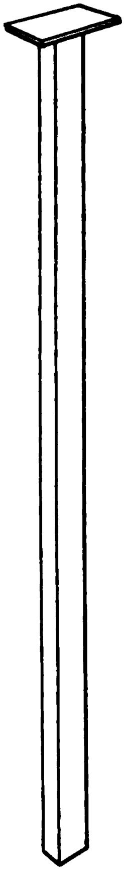


FIG. 2.

By experimenting with a model constructed as above described the following facts may be demonstrated :—

(1) There is a critical velocity and angle at which if the aeroplane be launched it will continue to glide indefinitely. These may be termed the *natural velocity* and *natural gliding angle.*

(2) If the aeroplane be launched at other than its natural velocity and gliding angle, it will perform a wave-like trajectory, oscillating about the gliding path of natural velocity, the oscillations gradually diminishing in amplitude, and the path of the aeroplane approximating more and more closely to a uniform glide (Fig. 3).

(3) The natural velocity of a ballasted aeroplane of given dimensions will depend upon its weight and upon the position of its centre of gravity, being greater when the weight is greater and when the centre of gravity is nearer the front edge.

(4) That there is a particular position of the centre of gravity that results in a least value of natural gliding angle, and for planes of the form stated the least value of the gliding angle is about 8 or 9 degrees.

§ 4. The Ballasted Aeroplane. Longitudinal Stability.—The subject of the longitudinal stability of the ballasted aeroplane is dealt with briefly in the author's "Aerodynamics," § 162, from which the following passage is quoted :—

"Let us suppose that the position of the centre of gravity be such as will coincide with the centre of pressure when the plane makes an angle $= \beta_1$ with its direction of motion. Now we

know (§ 184) that the position of the centre of pressure varies as a function of β and that its distance from the front edge of the plane diminishes the less the angle; if then the angle β from any accidental cause become less than β_1, the centre of pressure will move forward in advance of the centre of gravity, so that the forces acting on the plane will form a couple tending to increase the angle, and so restore the condition of equilibrium. Likewise if the angle become too great the centre of pressure will recede, and the resulting couple will tend to diminish the angle, and again the equilibrium is restored; thus the conditions are those of stable equilibrium, the plane tends to maintain its proper inclination to its line of flight."

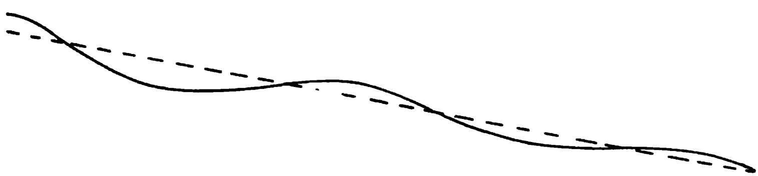


FIG. 3.

"There is not only equilibrium between the *angle of the plane* and its *direction of motion* as above demonstrated, but also between the *gliding angle* and the *velocity of flight*; thus if the velocity is deficient, so that the weight is insufficiently sustained, the gliding angle and the component of gravity in the line of flight automatically increase and the aerodrome (or aerodone) undergoes acceleration. Conversely, if the velocity is excessive the gliding angle (and so the propulsive component) diminishes, and the velocity is thereby reduced." [1]

It would thus appear that there are two distinct kinds of equilibrium involved in the longitudinal stability of the ballasted

[1] The above explanation of the automatic stability of an aerodone or aerodrome is, in a condensed form, that given by the author in his paper to the Birmingham Natural History and Philosophical Society in 1894. A more complete exposition is to be found in the author's specification of patent 3608 of 1897, p. 6, the text of which is given in Appendix II. of the present volume.

aeroplane—(1) an equilibrium between the *attitude*[1] of the plane and the direction of flight, that is to say, the plane tends to preserve a predetermined angle to its line of travel; (2) equilibrium of a dynamical kind between the kinetic and potential energy of the aeroplane, by which constancy of gliding angle and velocity is secured.

It may be stated at once that the same principles apply quite generally to aerodones of more fully developed type, in fact, so far as the author is aware, there is no other way by which longitudinal stability can be automatically secured, and any aerodone to fly successfully must comply with the necessary conditions.

§ 5. The Ballasted Aeroplane. Lateral Stability.—The lateral stability of the ballasted aeroplane is closely connected with the

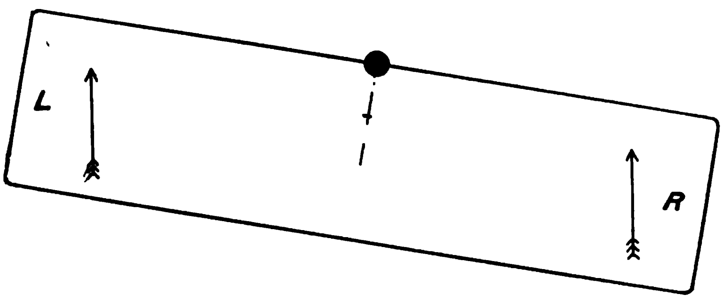


FIG. 4.

square-cut form of its extremities; it is at least certain that an oval or an elliptical plan form is relatively deficient in this respect.

It is evident that so long as a rectangular plane is travelling in a direction parallel to its end edges, the sustaining forces are symmetrical, and there is no turning moment about the axis of flight. If from any accidental cause, such as a slight motion of the air, the plane finds itself moving so that its ends make an angle with the line of flight (Fig. 4), the leading end L will, so

[1] The word *attitude* is here used to denote the position of the aerodone about a horizontal transverse axis; it is the analogue of *aspect* as defining the position of the plane about the vertical axis.

to speak, be eating its own up-current, and consequently it will experience an excess of pressure, and a turning moment will arise tending to elevate the end L of the plane. When the plane acting under the influence of this couple has assumed a laterally inclined position (Fig. 5), the forces W (the weight of the plane) and F (its pressure reaction) will have a tangential resultant, and the plane will proceed to slide downhill, that is in the direction indicated by the arrow; this motion will result in an alteration of *course* which will continue until the direction of flight is again parallel to the end edges of the plane, when the turning couple about the axis of flight will vanish.

If the initial disturbing cause be some want of symmetry of the plane, such as a slight twist or "wind" (a defect to which

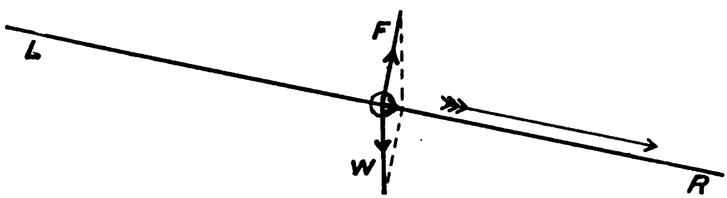


FIG. 5.

mica is extremely liable), then the final position of equilibrium is such that the edges of the plane are slightly diagonal to its line of flight, the couple due to the want of symmetry being then balanced by an equal and opposite couple due to the added reaction on the one or the other end of the plane. It is obvious that the same causes that result in the plane setting itself "square" to the line of flight in the case of a symmetrical plane result in it automatically setting itself at the appropriate angle in the case of a plane of defective symmetry.

It is evident that the motions of the plane after a disturbance will not be *dead-beat* as has been suggested, but that owing to the moment of inertia about the axis of flight, there will be a period of oscillation while the plane is settling down to its position of equilibrium. In practice the moment of inertia is

diminished as much as possible by concentrating the ballast in the middle of the front edge of the plane, when it is found that the oscillations die out with great rapidity. If the ballast is distributed along the front edge, or if the plane itself be of too great weight in proportion to the total weight of the plane and ballast, the oscillations do not die away, but tend to increase in amplitude, so that instability results.

§ 6. Ballasted Aeroplane. Directional Stability.—The directional stability of an aerodone may be defined as its stability about a vertical axis. It is evident that if an aerodone were liable to rotational changes of position about a vertical axis, its stability in other respects would be impaired; if, for example, in an extreme case it were liable to slue completely round in a manner analogous to the "side slip" of a motor car, it might at any moment find itself travelling stern foremost, and its longitudinal stability would have vanished.

Furthermore, we have already seen that lateral stability is dependent upon freedom of lateral motion, and to some extent the constancy of angular position (about a vertical axis) has been assumed in the explanation given (§ 5).

In the ballasted aeroplane the directional stability is primarily due to the influence of *skin friction*. It is manifest that if there were no viscous connection between the plane and the air, if skin friction were absent in fact, there would be nothing whatever to restrain the plane from rotation about an axis at right angles to its surfaces, and any accidental irregularity in its form, such as a slight "wind" or twist, would result in it receiving a spin of continually increasing angular velocity. The influence of skin friction is to damp out any rotary motion that may be accidentally acquired. This influence is an appreciable factor when a plane is translationally at rest, but it is far more potent when the plane is in motion. The reason for this is that the skin friction varies approximately as the square of the velocity, and the difference between this quantity for the right and left hand "wings" of

the aeroplane, for a given rate of spin, is greater the greater the velocity of flight.

If, as suggested in § 5, the mica plate have a slight twist or *wind*, the pressure reaction on either the right or left hand "wing" will be in excess, and, as we have already seen, this results in the aeroplane setting itself diagonally to the line of flight, so that the torque about the axis of flight due to the defect of symmetry is balanced by that due to the end effect.

Under these conditions the permanence or otherwise of *direction* depends upon the alignment of the resistance resultant, and

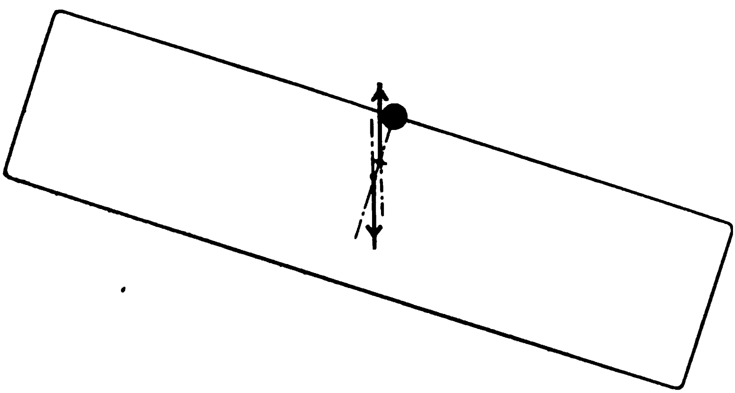


FIG. 6.

the gravity component which forms the propulsive force. Thus if (Fig. 6) these two forces are not in the same straight line they constitute a couple, and give rise to a torque about an axis normal to the aeroplane, and the flight path will not be straight. The same considerations apply if the initial want of symmetry is due to other causes, such as an unsymmetrical distribution of ballast, an unsymmetrical variation of the smoothness, etc., etc., the most frequent cause of trouble, however, when mica is employed, is that stated.

So long as the plane is carefully constructed from a plate of mica that is a sufficient approximation to a true plane, the turning moment that arises about a vertical axis is never of

sufficient magnitude to be of consequence; if, however, the mica plate is defective, the path of flight will be of spiral form, and if the defect is serious the spiral may become a curve of decreasing radius and instability may thus result.

The *sense of direction* of a ballasted aeroplane may be sometimes improved by the addition of a small fin near the front edge, conveniently attached to the ballast (Fig. 7). With the same end in view, the front corners of the plane may be "dog-eared" slightly (Fig. 8), a procedure that is otherwise advantageous. The simple ballasted plane has no "right side up," if properly

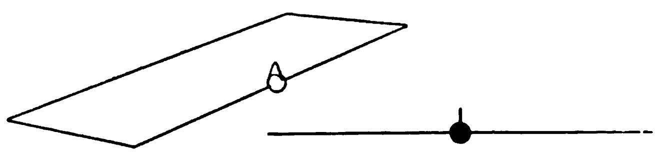


FIG. 7.

FIG. 8.

launched it will fly with either face topmost, but there is some risk that in the act of launching the reaction will be downwards instead of upwards, in which case the path of flight is entirely modified, in a manner that will be explained later in the work. In the case of the plane with the corners dog-eared (upwards), this does not apply to the same extent, and the launching may be effected with greater ease and certainty. Although the above expedients may sometimes be found advantageous it is the better plan to reject at the outset mica that is perceptibly on the twist or otherwise contorted.

§ 7. Ballasted Aeroplane. Interaction of Motions in and about the co-ordinate Axes.—It is of interest to summarise the inter-

action of motions in the different *degrees of freedom* (compare § 2) involved in the maintenance of equilibrium as above expounded.

In the longitudinal stability three kinds of motion are involved and interact: (1) rotation about a horizontal axis transverse to the line of flight; (2) motion of translation in a vertical direction, and (3) motion or change of motion in the direction of flight (taken as horizontal for the present purpose). Employing the usual axis symbols, and calling the direction of (mean) flight x, and the axis at right angles thereto in a vertical plane y, the transverse axis will be z; and the three kinds of motion become (1) rotation about z; (2) translation along y, and (3) translation along x. Thus if we were to take away the other three degrees of freedom by arranging the flight model to slide between two parallel vertical planes or walls, we should have the problem of longitudinal stability left in its entirety and nothing else. This is always a convenient supposition to make when the question of longitudinal stability is under discussion.

In the maintenance of lateral stability two degrees of freedom are primarily involved: (4) rotation about the axis of x, *i.e.*, the axis of flight, and (5) translation along the axis of z, *i.e.*, the transverse axis.

The sixth degree of freedom, *i.e.*, rotation about a vertical axis, primarily concerned in direction maintenance, is also closely associated with the question of lateral stability, and it will be subsequently shown that motion of this kind requires to be taken into account when dealing with that subject. There is a peculiar kind of skew instability that is prone to arise if this point is neglected.

§ 8. Other Forms of Aerodone.—The ballasted aeroplane has been chosen for the purpose of initial discussion as being the most elementary form of aerodone known, and as presenting an object lesson in automatic stability of the simplest possible kind. Several experimenters have developed gliding models or aerodones of more elaborate form; these, though generally inferior to the

simple mica aeroplanes above described, are worthy of mention not only as constituting part of the history of the subject, but also as forming a fitting introduction to the study of a more highly developed type employed by the author for the purpose of experimental study, and subsequently made the object of a mathematical analysis.

In general the early experimenters appear to have taken their inspiration from the models provided by nature, the aerodones employed bearing, in most cases, a strong resemblance to a bird in gliding flight.

In addition to the examples described in the following section, a number of partially successful flying models or *aerodromes* have been produced, notably by Hargraves (1885), an account of whose experiments will be found in *Engineering*[1]; the author (1894), whose apparatus is described later in the present work, and Langley (1896), of which particulars are given in a recent publication.[2]

There have also been a number of successful captive machines and also a few effective attempts at actual flight; the former are of but little interest to us from the present point of view, and the latter (as in the gliding machine of Lilienthal) have been reported as relying upon the skill of the aeronaut for their equilibrium, either in whole or in part, and may therefore be dismissed from the present stage of the discussion.

§ 9. Some Successful Gliding and Flying Models.—Some of the earliest successful model experiments were made by Penaud about the year 1870, an account of these is given in *L'Aeronaut*.[3] Penaud experimented with small paper-winged models, to some of which he applied twisted rubber and a screw propeller as a means of propulsion; a model so fitted (Figs. 9 and 10) is stated to have crossed a pond 40 or 50 yards wide, and to have shown

[1] *Engineering*, vol. xlix., p. 687; and vol. l., p. 769.

[2] "Report Smithsonian Institution," 1900.

[3] *L'Aeronaut*, tome v., p. 2, 1872.

complete stability. It would thus appear, unless some previous records exist, that to M. Penaud belongs the credit of producing the first stable flight model of a type that may be regarded as an embryo flying machine. It is unfortunate that the particulars given are not sufficient to permit of the exact reproduction of the model and repetition of the experiments; there is, however, no reason to question the accuracy of the facts stated. Penaud also calls attention to the flight path

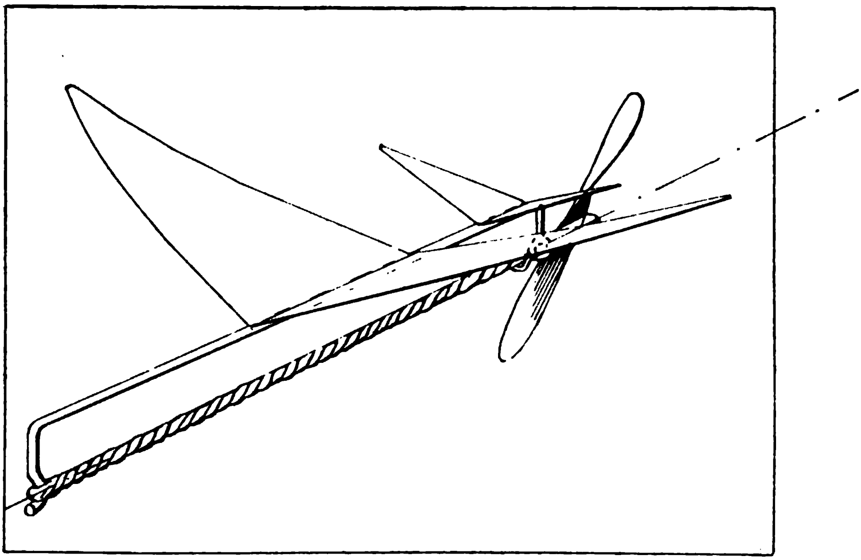
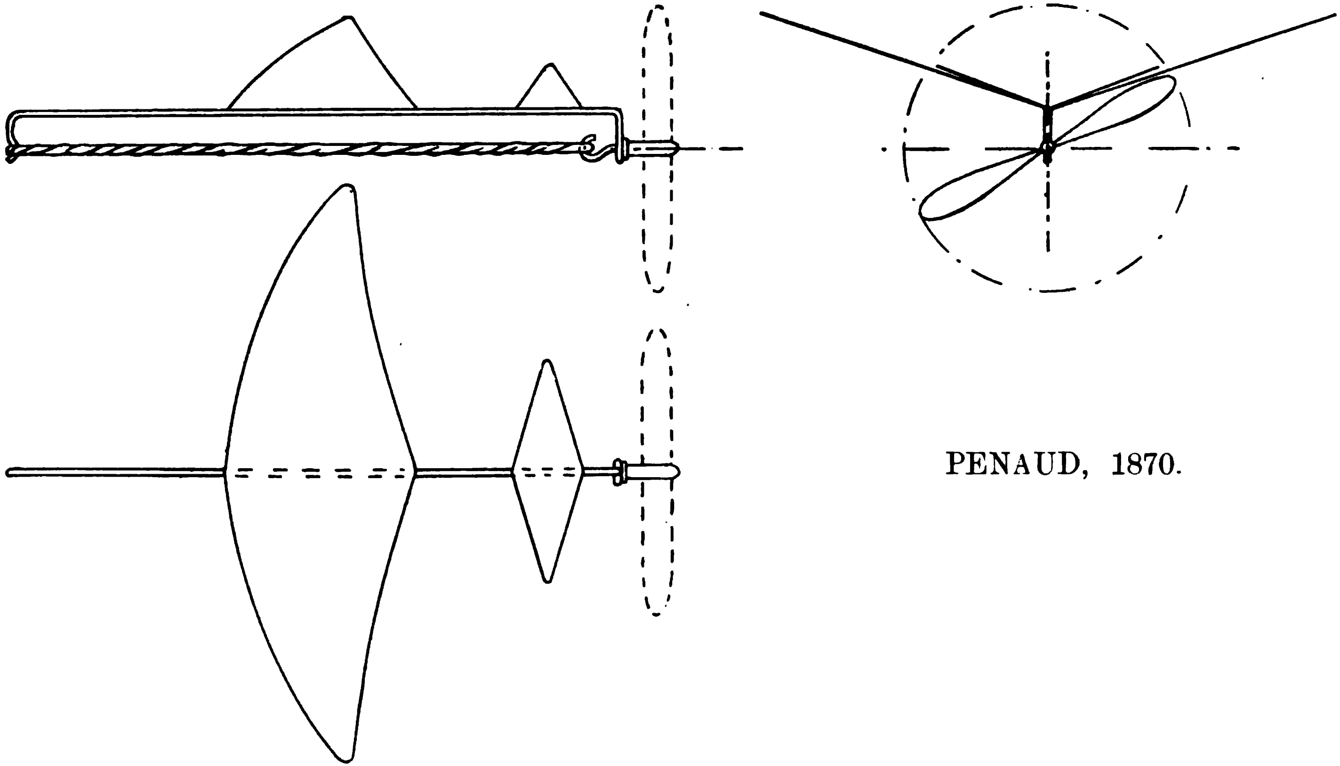


FIG. 9.

oscillation already referred to in connection with the ballasted aeroplane.[1]

Fig. 11 is a representation of a form of gliding model devised by Mons. J. Pline, and figured in Marey's "Vol des Oiseaux"; this model appears to have been stable in flight, but from an aerodynamic standpoint it is not a form of high efficiency, the

[1] Compare § 65. Penaud says: . . . "et l'on observe alors assez souvent des oscillations dans le vol, comme nous en voyons decrire aux passereaux et principalement au pic-vert" (green woodpecker).



PENAUD, 1870.

FIG. 10.

gliding angle appears to have been excessive. According to the particulars given the model consists of a folded sheet of paper ballasted by a needle that may be adjusted as required, but the one view given by Marey leaves the exact form in doubt.[1]

Fig. 12 is a form due to Weiss, and derived from experiments with models shaped like a bird; it shows complete stability, and is prone to oscillate in its gliding path just as in the

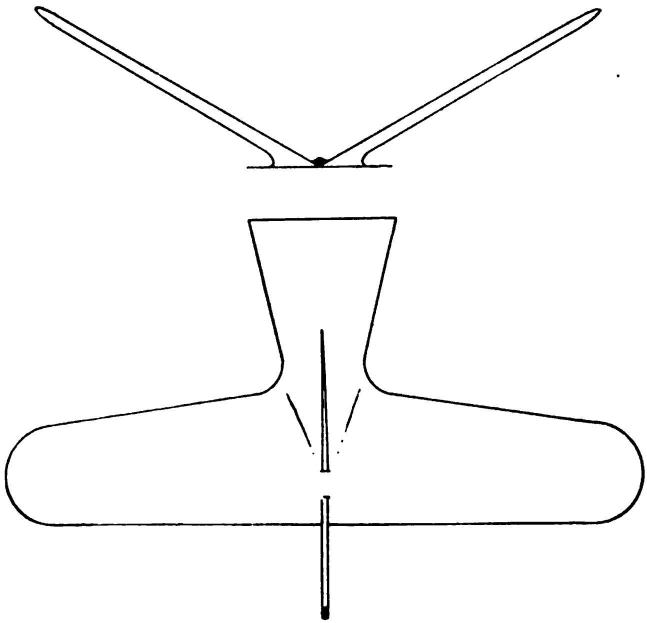


FIG. 11.[1]

case of the ballasted aeroplane. In construction it consists of a sheet of paper cut to the form shown, the front being folded to give some stiffness to the leading edge, and ballasted by a small piece of lead glued in position. The tips of the "wings" are turned up slightly in the manner shown in the

[1] Fig. 11 is from a model made by the author from Professor Marey's description; a measurement of the gliding angle gave approximately 20 degrees.

line-of-flight elevation. The gliding angle of this model is good, being approximately 9°.

Fig. 13 is a form due to Hele Shaw, and exhibited by him to the Royal Society in May, 1907; it also obviously derives its inspiration from nature, being a close representation of the plan-form of a swallow; it is quite stable, and also shows the flight path oscillation to a pronounced degree; this glider is simply cut from a sheet of paper and ballasted by means of split shot. The gliding angle of this form is also very good.

As far as the author has been able to judge, none of the forms above given are superior to the simple ballasted rectangular plane,

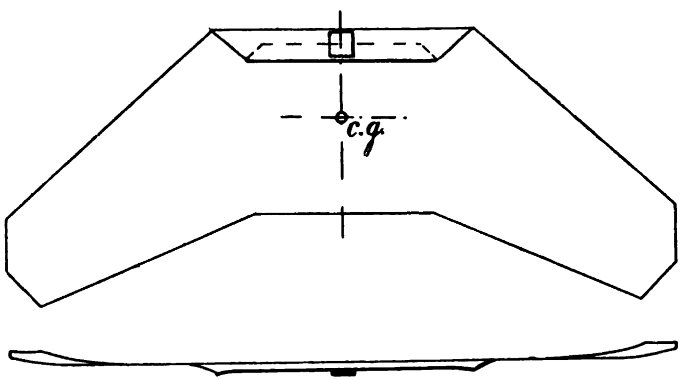


FIG. 12.

and the only obvious reason for the adoption of fancy shapes, such as those of Weiss and Hele Shaw, appears to be found in their common origin as imitations of forms presented by nature.

§ 10. Author's Experiments, 1894.—In 1894 the author commenced the practical investigation of stability in flight, some preliminary theoretical work having shown that it should be possible to obtain automatic equilibrium with a suitably designed apparatus.

At the date in question, although Mouillard's "L'Empire de l'Air" had been published several years, the author was not aware of the automatic equilibrium possessed by the ballasted

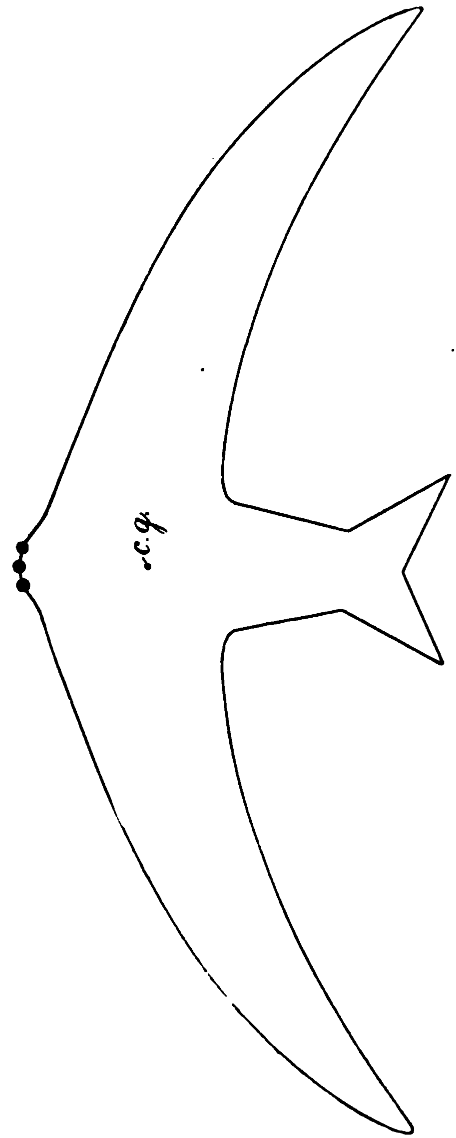


Fig. 13.

aeroplane, or, indeed, of the existence of Mouillard's work at all, and it is not surprising, therefore, that his models were designed and constructed on entirely different lines.

In the author's original model (Figs. 14 to 20), the various functions that are performed incidentally by the ballasted aeroplane in the maintenance of equilibrium are carried on by special organs, and this fact, which arises from the theoretical origin of the appliance, renders it particularly well suited to analytical study.

One of the deductions from the author's preliminary investigations was in effect that the higher the velocity of flight the greater the stability attainable; the models employed were, in pursuance of this conclusion, designed for velocities of some 40 miles per hour and upwards. It was estimated that such a velocity would render the stability independent of any gusts of wind such as would ordinarily be encountered, a result that was fully substantiated by the subsequent experiments.

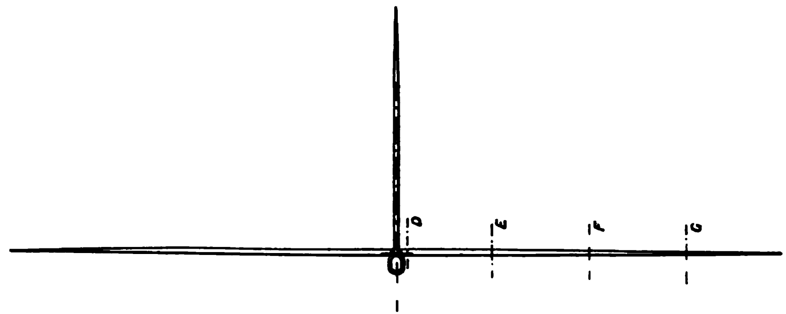
Owing to the high velocities employed the launching had to be effected by means of a catapult, the construction of which is illustrated and described in § 12 (Fig. 21).

§ 11. Author's Experiments (continued). The Aerodone.—The form of aerodone employed by the author in these experiments is drawn to $\frac{1}{10}$ scale in Figs. 14, 15 and 16, and a photograph reproduction is given in Fig. 17. Further details are given in Figs. 18, 19 and 20.

The *aerofoil* sections D, E, F, G (Fig. 19) being of elliptical plan-form 40 in. by 3 in., and the aspect ratio, n being 18·3, the actual area = ·65 sq. ft. On the basis of a parabolic grading,[1] the *effective* area, given by the expression $\frac{2}{3}\frac{l^2}{n}$, = ·55 sq. ft. The centre of gravity of the aerodone is arranged about 1 inch from the front edge of the aerofoil, the latter being the unique organ of sustentation.

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 192.

Fig. 14.



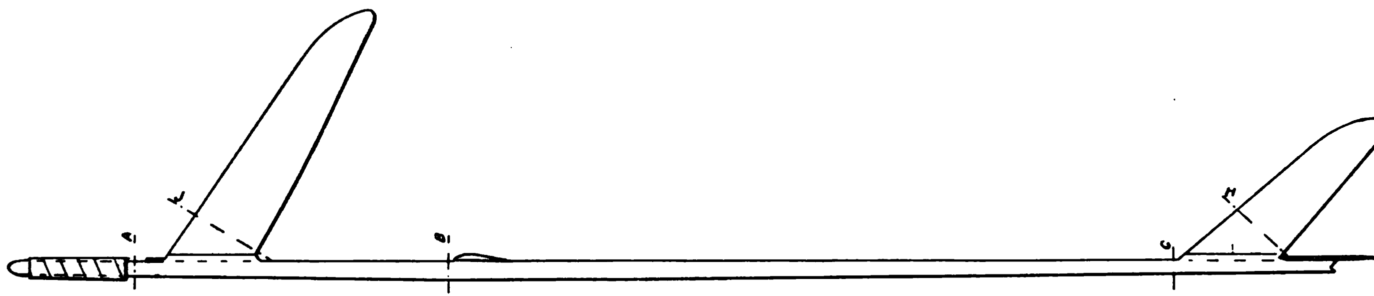


Fig. 15.

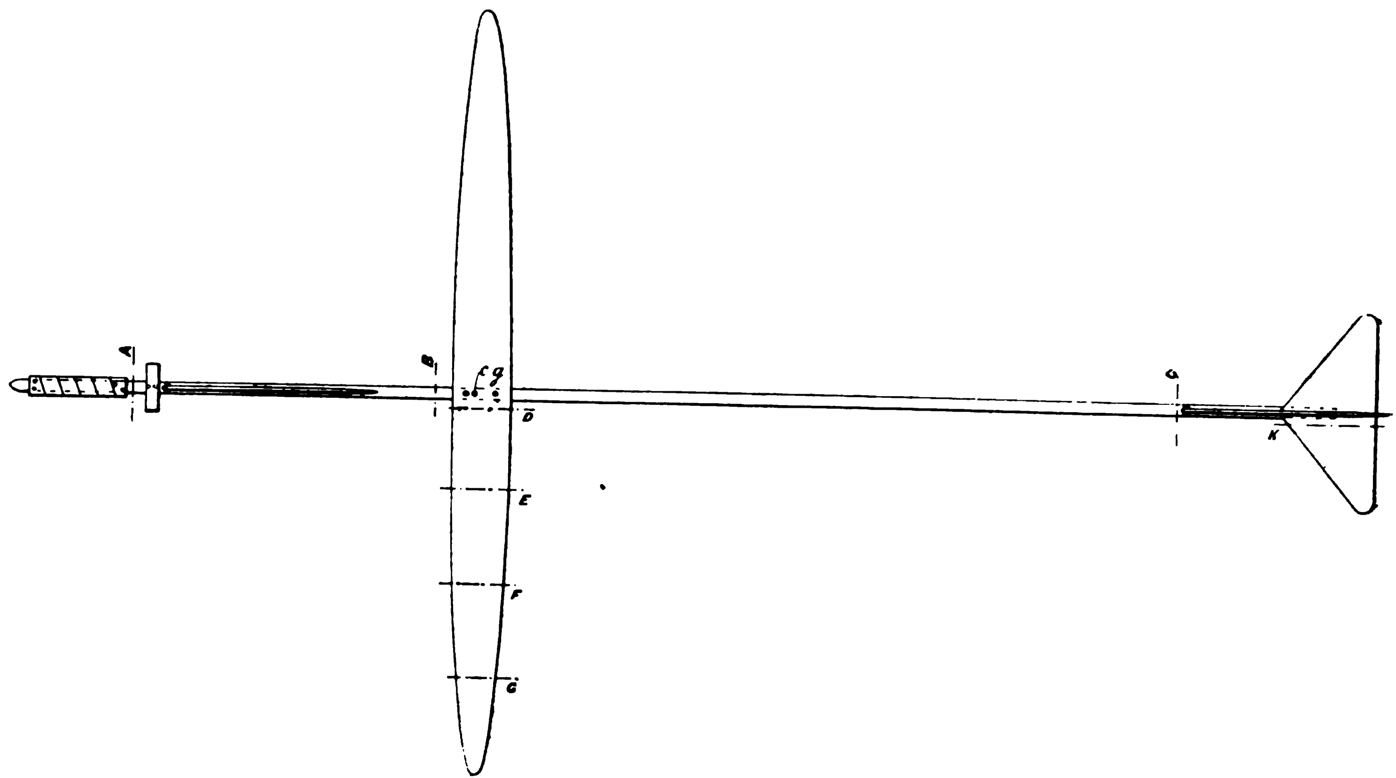


Fig. 16.

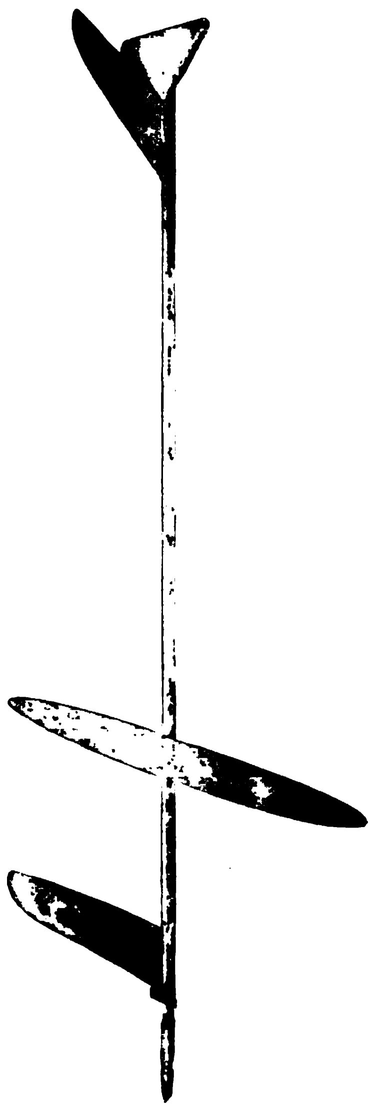


FIG. 17.

The *tail-plane* is of triangular form, and has an area = ·20 sq. ft., *its function is purely directive.* The tail-plane has the same function as is performed by the change in the position of the centre of pressure of the ballasted aeroplane; it preserves

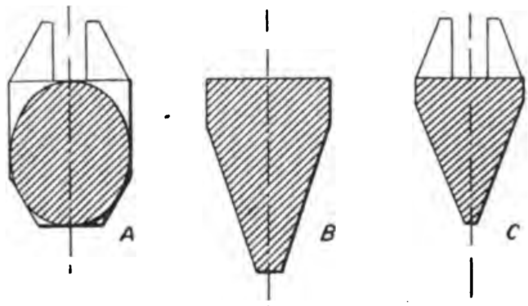


Fig. 18.

the constant *attitude* of the aerodone in relation to its line of flight, in this respect its action being analogous to the feathers on an arrow.

The distance between the centres of pressure of the aerofoil and tail-plane is 3·9 feet approximately.

Two *fins*, sections *H* and *J* (Fig. 20), are provided, of the

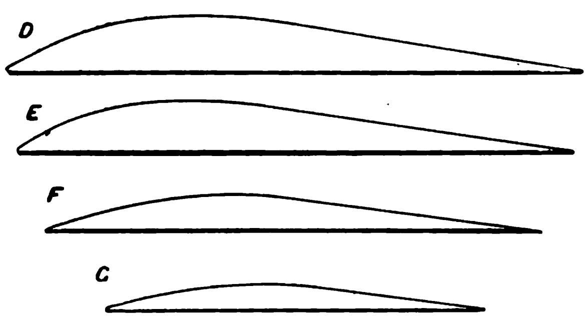


Fig. 19.

form shown in Fig. 15, whose area is approximately ·2 sq. ft. and ·3 sq. ft. respectively. These have for their function the maintenance both of lateral and directional equilibrium, and their action is represented in the ballasted aeroplane by the

end edge effect on the one hand, and the *frictional equilibrium* on the other, assisted it may be by an auxiliary fin, or by the upturned corners, as described in § 6.

The *backbone* (Fig. 18) is of triangular section, and serves for the attachment of the various functional parts, also supporting the ballast, by which the centre of gravity is brought to its correct position.

The construction adopted was of a most substantial description, in no way suggestive of the lightness it is customary to associate with a flying model, the backbone, tail-plane and fins being of white pine, and the aerofoil of thoroughly seasoned and dried pitch pine[1]; the ballast being made from a strip of

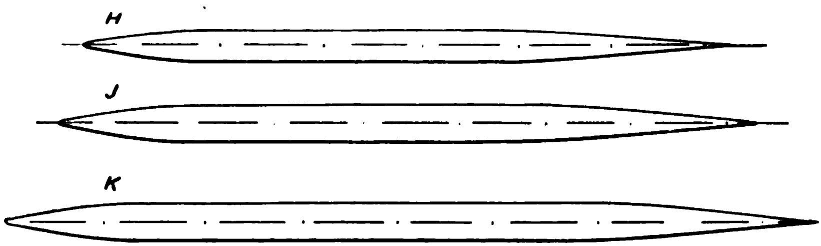


Fig. 20.

sheet lead, wound spirally round the backbone, and nailed or otherwise secured in position.

A small cross-piece serves the purpose of supporting the nose of the aerodone on the runners of the catapult during launching.

§ 12. The Author's Experiments (continued). The Catapult.—The catapult employed for launching the aerodone is illustrated in Fig. 21; briefly this apparatus consists of two runners, on which the aerodone is free to slide, separated by distance pieces arranged at intervals along its length, leaving a groove about 1 inch in width by which the backbone of the aerodone is guided

[1] Pitch pine is not a particularly suitable kind of timber, owing to its being liable to develop a twist. Mahogany would be better in this respect, but it is more easily injured by impact.

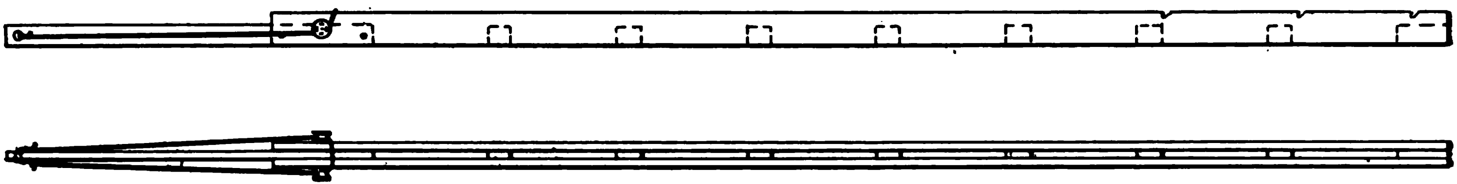


FIG. 21.

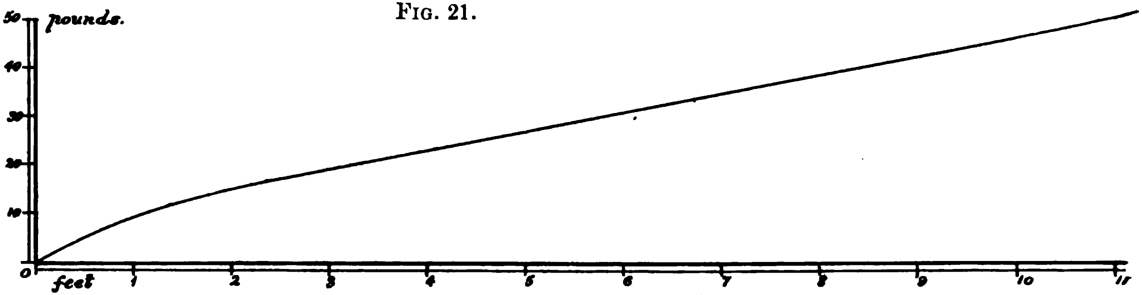


FIG. 22.

during discharge. The "motive power" is furnished by an india-rubber band, usually composed of about four strands of black "elastic."

The india-rubber is secured to a projecting piece and is guided by two pulleys or spools, one on each side of the runners. The rubber is bound with twine at and about its middle point where it engages with the notch in the after extremity of the backbone of the aerodone. When loaded, the rubber is held in a state of extension by notches near the rear end of the runners, and the discharge is effected by prising it out of the said notches by means of a forked lever.

The length of the free portion of the rubber ordinarily used was 3 feet 4 inches, the weight being ½ lb. and the maximum range of extension employed 11 feet 6 inches.

A plotting of the extension diagram is given in Fig. 22, in which abscissae represent extent of elongation and ordinates the corresponding tensions in pounds. The figures given are *loading data*, and the loading energy will be given by the area of the curve for the extension employed. It is improbable that the output energy is more than about 75 per cent. of the total.

§ 13. Site and Date of Experiments.—The site of the experiments under discussion was the then residence of the author, "Fairview," St. Bernard's Road, Olton, Warwickshire; the models being projected from a back first floor window facing west; the point of discharge was approximately 15 feet above the ground level, where the general run of the land falls away at a slope of about 1 in 25.

The situation is represented diagrammatically in Figs. 25 and 26, in which are depicted some of the flights made. A plan is given in Fig. 23 showing the general situation, on this also some of the flights have been roughly laid out.

Experiments were made at different dates during June and July, 1894, the models employed being in most part simple aerodones as described in § 11, some half-dozen in all being made

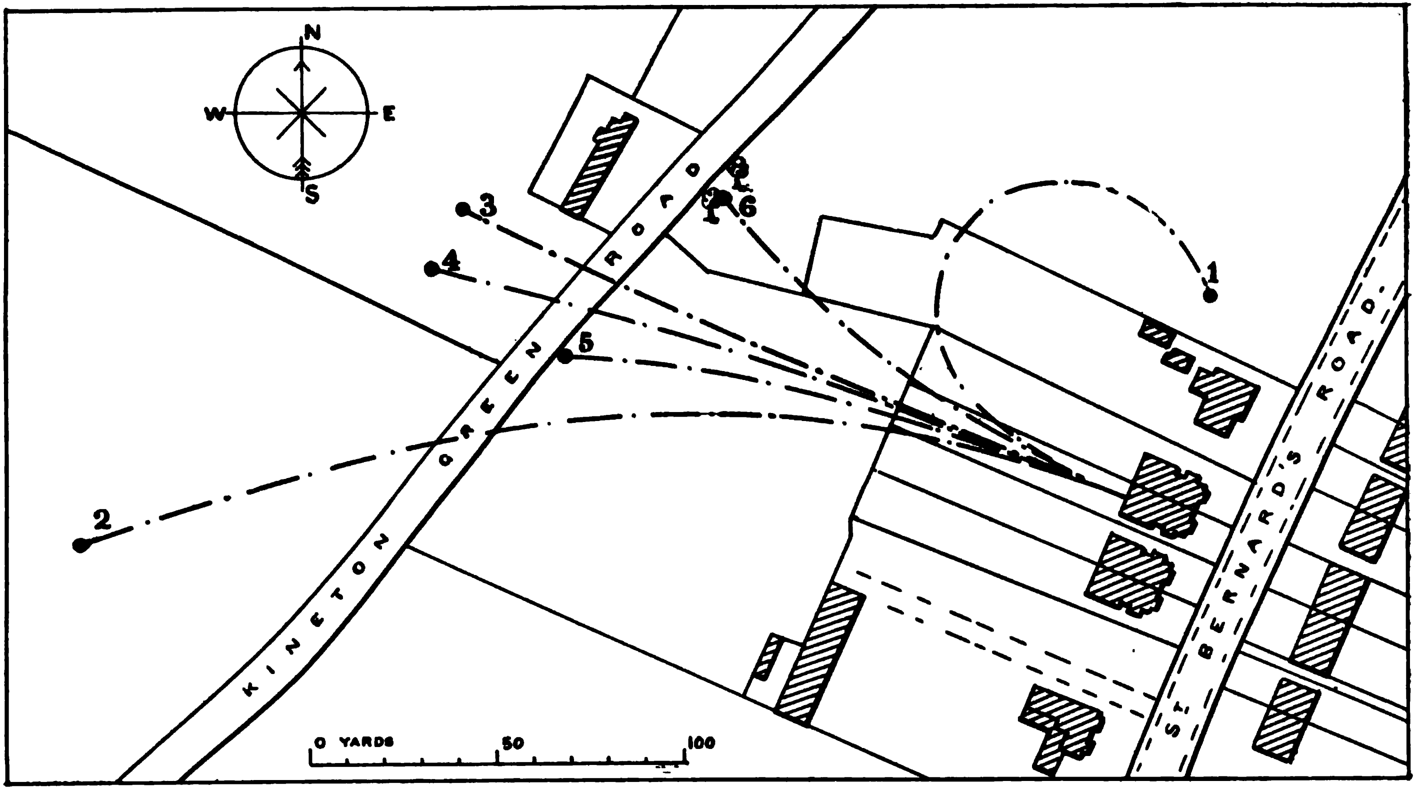

N
W
E
S
KINETON GREEN ROAD
ST BERNARD'S ROAD
1
2
3
4
5
6
0 YARDS
50
100

Fig. 23.

FIG. 24.

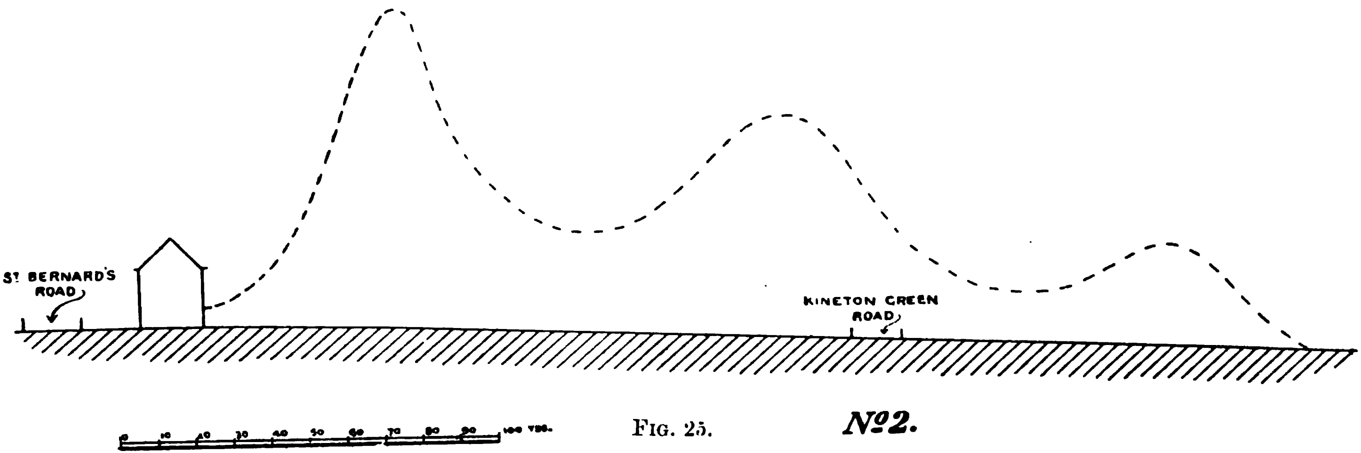

St BERNARD'S ROAD
KINETON GREEN ROAD
0 10 20 30 40 50 60 70 80 90 100 YDS.
No 2.

Fig. 25.

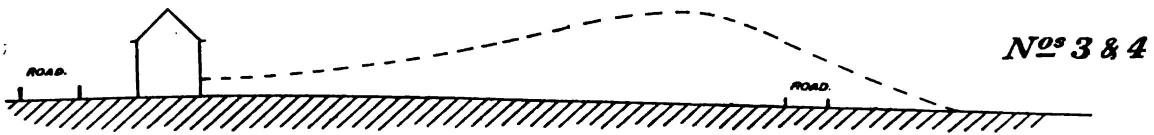

Nos 3 & 4.
ROAD.
ROAD.

Fig. 26.

to the one design, and weighing approximately 1 lb. 7 oz. each. Subsequently a more fully developed model or aerodone (Fig. 24), elastic propelled with twin screws, was successfully flown. This model is fully described in Appendix III.

§ 14. The Author's Experiments. Records. — A considerable number of flights were made, probably some twenty or thirty in all; the records relate only to some half-dozen cases.

The object at the time being merely to *demonstrate* the stability of a high velocity model, the records are not very complete, and the actual velocities and times of flight were not in every case fully noted. The forms of flight path given in the accompanying figures are drawn as they *appeared* to observers present, and

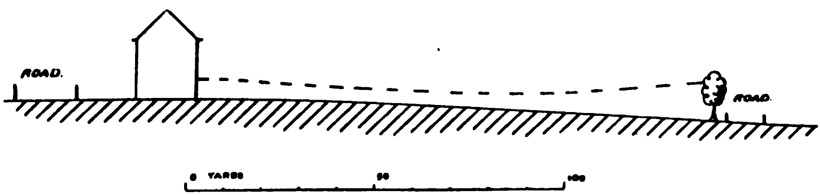


FIG. 27.

cannot be regarded as more than rough approximations to the actual curves; the positions of the points of greatest altitude are, however, fairly near the truth.

Referring to Fig. 28 (plan of site), we have *Flight No.* 1, date not recorded, weight of aerodone entered as $1\frac{1}{4}$ lbs.; very light wind; distance about 200 yards. This is the initial flight made with first model.

Flight No. 2, June 24th (?), weight 1 lb. 7 oz.; distance 280 or 290 yards; high wind with powerful gusts, probably about 30 m.h. direction W.S.W.; time of flight 27 seconds. A magnificent flight, remarkable "switch-back" flight path (Fig. 25), distance, *relative to wind,* probably over 600 yards.

Flight No. 3, June 24th (?), same aerodone as Flight No. 2; distance 200 yards; time of flight $7\frac{1}{2}$ seconds; very light wind. Velocity = 55 m.h. (Fig. 26).

Flight No. 4, same as No. 3, distance 200 yards; time not recorded (Fig. 26).

Flight No. 5, June 24th (?), same aerodone as Flight No. 2; distance 150 yards;[1] time of flight 5½ seconds; very light wind. Velocity = 56 m.h.

Flight No. 6, July 3rd, twin screw aerodrome; weight 2½ lbs.; distance 133 yards; no appreciable wind; time of flight 4½ seconds[2] (Fig. 27).

§ 15. The Author's Experiments. Discussion.—The result of the experiments described was to fully confirm the author's views as to the possibility of securing automatic stability without the employment of any equilibrium mechanism or "brain equivalent." When the weather was calm the aerodones carved their way through the air as if running on invisible metals, without the smallest visible fluctuation or quiver. When on the other hand the flight was made in a gale of wind, the flight path took the form of a bold sweeping sinuous curve, without a momentary suggestion of loss of equilibrium, but rather with the appearance of some set and intelligent purpose.

In general the velocities of flight employed appeared to range from about 50 to 60 miles per hour, and it is certain that at velocities of this sort there is very little to fear from any ordinarily bad weather conditions.

In general the launching velocity was very much higher than the natural velocity, and consequently the gliding angle was entirely masked and remained an unknown quantity; the considerable range of flight obtained was without doubt due to this cause.

One remarkable fact in connection with the above series of experiments is the exceptional time (27 seconds) the aerodone remained in flight when launched in a gale of wind (Flight No. 2). Taking the velocity relatively to the air as that measured in

[1] Model collided with tree.

[2] The accuracy of the timing in this flight is in doubt.

other experiments with the same model, say 55 m.h., this length of time corresponds to a flight of 725 yards.

There are two possible explanations of this anomalous flight. (1) That the aerodone was actually soaring after the manner of an albatros or gull. (2) That the initial kinetic energy, being due to the velocity of the aerodone *relatively to the wind*, is much greater when the aerodone is launched in the teeth of a gale, and so the energy disposable in the flight will be greater in a proportionate degree.

Both these explanations are possible, and either would, within limits, account for an enormous increase in the observed range of flight. At first sight alternative (1) seems most unlikely; it appears to imply an exercise of intelligence not possible for an inanimate thing. Later experiments,[1] however, show that it is by no means impossible for a simple aerodone to perform true soaring evolutions, and that it is not at all improbable that this was actually the case in the flight in question. On the other hand alternative (2) alone is almost sufficient to account for the additional energy as evidenced, so that the correct explanation of the case in point must be regarded as uncertain.

§ 16. Author's Experiments. Remarks and Summary.—If we define the line of flight of an aerodone as the path of its mass centre, it is one feature of the designs employed by the author, and in the models discussed in § 9, that the centre of resistance is substantially coincident with the line of flight; the same is approximately true of the soaring birds.

Attention is directed to this point on account of the fact that many inventors have proposed *acentric* types of aerodrome, in which the weights carried have been suspended some distance beneath the supporting member; this is typically the case in the captive machine of Sir Hiram Maxim, a diagrammatic illustration[2]

[1] Compare Ch. IX., § 158.

[2] Fig. 28 is taken from a Patent Specification. It must not be supposed that Sir Hiram Maxim actually constructed a machine in which the aeroplane was trussed in the manner shown.

of which is given in Fig. 28. In all probability this idea is based on the superficial resemblance an aerodrome possesses to a kite, the suspended weight being considered as the analogue of the kite string; or, perhaps it is from endeavouring to follow the lines of an aerostat that this type has arisen, the aeroplane or aerofoil being regarded as a simple substitute for the gas bag.

The relative advantage of the centric and acentric types of aerodone is a question that has not been thrashed out; it is

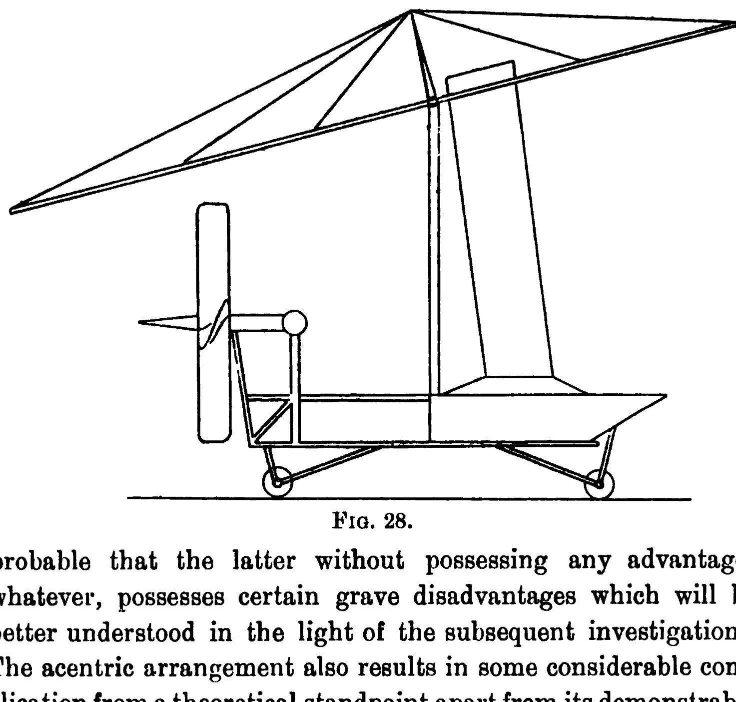


Fig. 28.

probable that the latter without possessing any advantages whatever, possesses certain grave disadvantages which will be better understood in the light of the subsequent investigations. The acentric arrangement also results in some considerable complication from a theoretical standpoint apart from its demonstrable failings.

The acentric type is not found in nature to any marked degree, except perhaps in the case of some birds that are inveterate *rameurs*,[1] when the gliding position is sometimes with

[1] There is no convenient English equivalent for this word in the present sense; the term "wing-flappers" perhaps might be used.

the wings inclined upward at an angle instead of being fully extended; the pigeon is an example.

§ 17. Author's Experiments. Remarks and Summary (continued). —It is of some interest to interpret the forms of flight path made by the author's aerodones, in typical cases, in the light of the ballasted aeroplane experiments described in § 3.

It is evident that the rising and falling curve of the trajectory Fig. 25, and in other similar cases, constitutes a portion of the wave-like flight path referred to in (2) § 3; and if we continue this curve as though the launching had taken place from a greater altitude, that is to say, as if the surface of the earth had not intervened, we should have a curve of the form shown dotted in Fig. 29, oscillating about a mean gliding path represented by the line *a a*. In this figure the curve of flight has also been continued *retrospectively* in order to show more clearly the sinuous flight curve of which the actual trajectory forms but a fragment. It is evident that the angle of the line of mean flight depends upon the design of the aerodone, the less the total resistance of the latter in proportion to its weight, the less the angle γ: this is a matter of aerodynamics.[1] It is further evident that the greater the launching velocity the greater the distance at which the line of mean flight path passes vertically over the point of discharge, *i.e.*, the greater the distance h in the figure.

In the particular flight depicted in Fig. 25, it is evident that, owing to the existence of a head wind at the time of the experiment, the velocity was considerably higher in effect than the ordinary launching velocity; it would in fact under these circumstances be equal to the sum of the launching and wind velocities. In consequence, the height of the mean flight path line is considerably greater in this case as is represented in Fig. 30, and as a result we have nearly three complete "periods" of the flight path oscillation before the aerodone finally comes to earth. The question of whether there was actual *soaring* taking place

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, Chapters VII. and VIII.

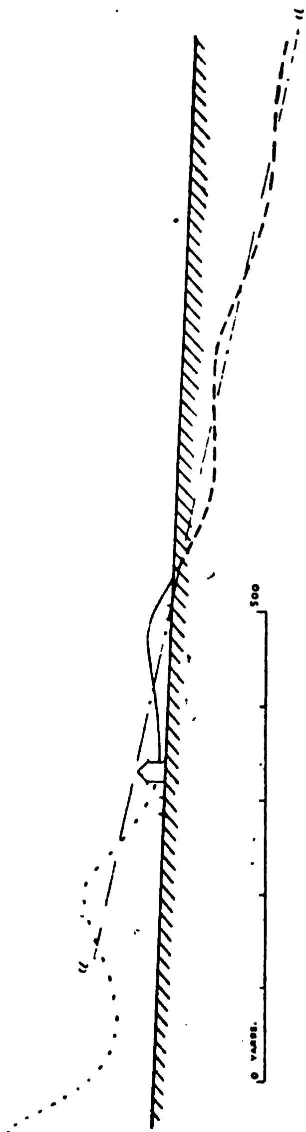


FIG. 29.

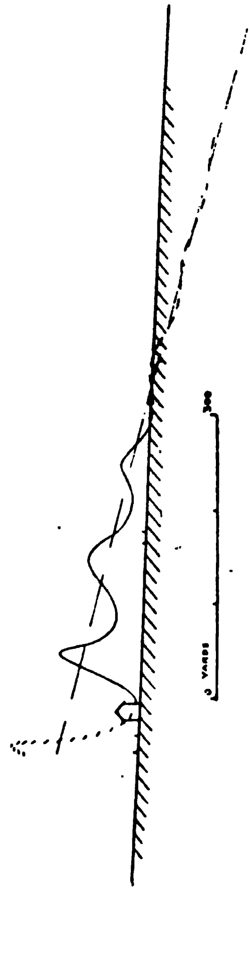


FIG. 30.

during this flight, the alternative explanation of its abnormal duration, is not for the moment under discussion.

The examination of the behaviour of an aerodone in flight has in the foregoing demonstration been carried as far as has been found possible without a mathematical analysis; it now becomes necessary to adopt a more rigid line of treatment and to examine quantitatively the curves of flight and the conditions governing their form and permanence.

CHAPTER II

THE PHUGOID[1] THEORY.—THE EQUATIONS OF THE FLIGHT PATH

§ 18. **Introductory.**—The Phugoid theory deals with the longitudinal stability, and the form and equations of the flight path of an aerodone.

It has been proved by the experiments of the author and others that a simple aerodone can be constructed possessing, within certain practical limits, complete longitudinal stability, and the general considerations on which this stability depends have been stated.[2]

We have seen firstly that there exists a stable state of equilibrium between the attitude of the aerodone and the direction of flight path in the vertical plane; and, further, that there is a complex system of equilibrium of a stable kind maintained between the said direction of the flight path and the velocity of flight, due to an interchange of kinetic and potential energy that automatically takes place if the conditions of uniform gliding are disturbed.

In the investigations that follow the character and form of the flight path of an aerodone in free flight are deduced from purely dynamical considerations, the argument being in the main mathematical, aided by graphic methods, and reasoning of other kinds. Constituting the foundation is a mathematical analysis based on hypothetical conditions, the problem being presented first in its simplest form; the superstructure comprises an extension of the initial theory to deal with the problem in its more complete form, and the interpretation and application of the results achieved.

[1] From φυγη and ειδος (lit. *flight-like*). [2] § 4.

§ 19. Initial Hypothesis.—The first hypothetical condition assumed is that the aerodone is only possessed of the three degrees of freedom involved in its longitudinal stability. Thus it is supposed free to move in any direction in a vertical plane containing the line of flight or to rotate about an axis at right angles thereto.

The above condition we may suppose fulfilled either by

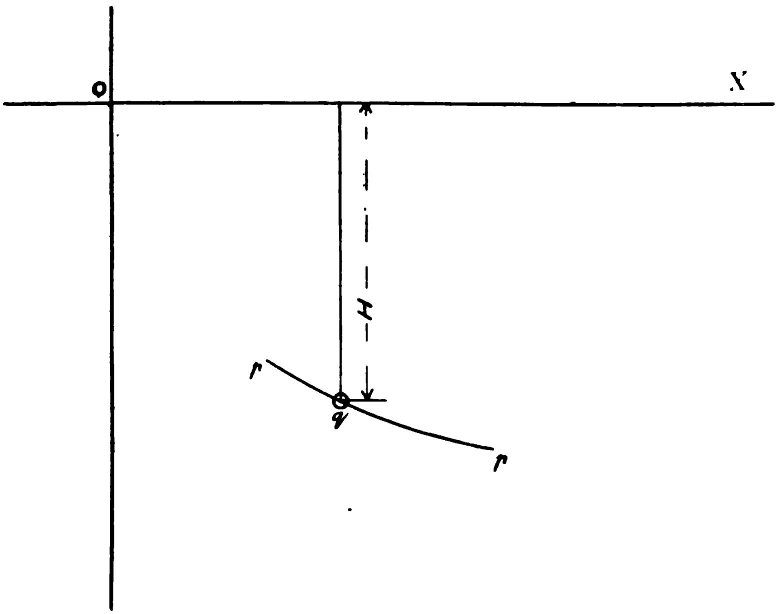


Fig. 31.

imagining the aerodone to be constrained in its motion by two parallel vertical guide planes (as suggested in § 7) or we may suppose that lateral stability and constancy of direction are otherwise accounted for, and maintained.

In the second place it will be assumed that the aerodone *loses no energy* during its flight, either by considering the air as frictionless and the supporting wave[1] as perfectly conserved, or by supposing a force applied in the direction of motion precisely

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 117.

equal, at every instant of time, to the resistance experienced by the aerodone in flight.

It is in the third place assumed that the whole mass of the aerodone is concentrated at the mass centre, so that it shall possess no moment of inertia about a transverse axis.

In the fourth place and lastly it is assumed that the size of the aerodone is small in proportion to the minimum radius of curvature of its flight path.

§ 20. The Phugoid Equation.—In Fig. 31, let $p\ p$ be any portion of the flight path of an aerodone, and let H be the height of free fall corresponding to the velocity of the aerodone at the point q; then the velocity at every point in the path of the aerodone will be that *corresponding to* the ordinate measured from $O\ X$, since no energy is lost. Let this ordinate be reckoned positive measured in a *downward* direction.

Let us take:—

H = distance of aerodone below horizontal datum line $O\ X$.

F = the force per unit mass of the aerodone, due to the reaction of the air on its supporting member (*always of positive sign*).

f = the force due to the inertia of unit mass of the aerodone owing to its change of direction; that is the centrifugal force due to curvature of flight path; measured as positive when in the same direction as F and negative when contrary to F.

W = weight of aerodone per unit mass ($= g$).

V = velocity of aerodone in flight.

L = length of path.

Θ = angle of path to horizon.

t = time of flight.

Between these quantities we have the following relations:—

$$F = n\ V^2 = n \times 2\ g\ H \qquad (1)^{1}$$

where n is a constant.

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 159, also Chap. VIII.

$$f = \frac{d\Theta}{dt} V. \qquad (2)$$

The total force at right angles to the path (*i.e.* the sum of F and f) must equal the resolved part of the weight, so that

$$F + f = W \cos \Theta = g \cos \Theta \qquad (3)$$

$$\therefore \quad g \cos \Theta = n \times 2 g H + \frac{d\Theta}{dt} V, \qquad (4)$$

but $$dt = \frac{dL}{V} \quad \therefore \quad \frac{d\Theta}{dt} V = \frac{d\Theta}{dL} V^2 = 2 g H \frac{d\Theta}{dL}, \qquad (5)$$

and since $$\sin \Theta = - \frac{dH}{dL}, \qquad dL = - \frac{dH}{\sin \Theta}$$

and $$\frac{d\Theta}{dL} = \frac{d\Theta \sin \Theta}{- dH} = \frac{d \cos \Theta}{dH}, \qquad (6)$$

$\therefore$ (4) becomes $$g \cos \Theta = 2 g H \left(n + \frac{d \cos \Theta}{dH}, \right) \qquad (7)$$

$$\therefore \quad \frac{d \cos \Theta}{dH} = \frac{\cos \Theta}{2 H} - n. \qquad (8)$$

This is the differential equation to the curve of flight in terms of H, Θ, and n.

Integrating the solution is—

$$\cos \Theta = \frac{2}{3} n H + \frac{C}{\sqrt{H}} \qquad (9)$$

where C is a constant.

This is the general equation to the curves of flight or the *phugoid equation.*

§ 21. Substitution for the Constant n.—The value of the constant n is defined by (1)—

$$F = n V^2 = n \times 2 g H.$$

Now, we know that there is a certain value of V at which the weight of the aerodone is exactly balanced by its pressure reaction, this is the *natural velocity* of § 3. Let this *natural*

velocity be denoted by the symbol V_n, and let the corresponding value of H (*i.e.* the height of fall that would give rise to a velocity $= V_n$) be denoted by the symbol H_n and termed the *natural height*. Under the conditions of hypothesis, if the aerodone be launched horizontally with a velocity $= V_n$, its path will continue indefinitely as a horizontal straight line, and in this case we have $F = W = g$ and (1) becomes—

$$g = 2\,g\,H_n\,n, \text{ or } n = \frac{1}{2\,H_n}. \tag{10}$$

Equation (9) then becomes—

$$\cos\Theta = \frac{H}{3\,H_n} + \frac{\mathsf{C}}{\sqrt{H}}. \tag{11}$$

The author has not been able to reduce this expression to a form suitable for co-ordinate plotting, but the difficulty of laying out the curves for different values of the constant C is surmounted in an indirect way.[1] In the first place we will find an expression for the *radius of curvature*, and examine a few special cases.

§ 22. Radius of Curvature.—We may express the radius of curvature r for any curve in the form :—

$$r = \frac{dL}{d\Theta}.$$

Now by (10) $n = \dfrac{1}{2\,H_n} = \dfrac{g}{V_n^2}$ (for $V_n^2 = 2\,g\,H_n$ by the ordinary laws of a falling body),
and by (4) and (5)

$$g\cos\Theta = 2\,g\,H\left(n + \frac{d\Theta}{dL}\right)$$

[1] At the time of making the plottings of the curves of flight or *phugoids* given in the present volume (1897), the author imagined the method adopted, described in the subsequent chapter, to be entirely new. It would appear, however, to have been no more than an application of a method originated by Professor C. V. Boys in 1893 to facilitate the plotting of the generating curve of a capillary surface. *Vide* "Phil. Mag." vol. xxxvi., p. 75, "On the Drawing of Curves by their Curvature."

substituting for n

$$\frac{\cos\Theta}{2H} = \frac{g}{V_n^2} + \frac{d\Theta}{dL}$$

or
$$\frac{d\Theta}{dL} = \frac{\cos\Theta}{2H} - \frac{g}{V_n^2} = \frac{V_n^2\cos\Theta - 2Hg}{2HV_n^2},$$

but
$$V_n^2 = 2gH_n \qquad \therefore\ r = \frac{2H}{\cos\Theta - \frac{H}{H_n}}, \tag{12}$$

and since by (11)
$$\cos\Theta = \frac{H}{3H_n} + \frac{\mathsf{C}}{\sqrt{H}}$$

$$r = \frac{2H}{\frac{H}{3H_n} + \frac{\mathsf{C}}{\sqrt{H}} - \frac{H}{H_n}} = \frac{2H}{\frac{\mathsf{C}}{\sqrt{H}} - \frac{2H}{3H_n}},$$

or
$$r = \frac{2}{\frac{\mathsf{C}}{H\sqrt{H}} - \frac{2}{3H_n}}. \tag{13}$$

Thus in addition to the equation for the slope of the curve at every point (11), we have now the expression for the radius of curvature. With these two equations we are in a position to investigate the general characteristics and particular forms of the curves of flight or *Phugoids*, as they may be appropriately termed.

We may note *firstly*, that the value of H_n is a constant for any given aerodone, and *secondly*, that for any aerodone whose H_n is known, and for which simultaneous values of Θ and H are given, the constant C may be calculated, and the course of flight is determined. Thus if H_1 be the value of H when Θ_1 is the value of Θ we have, by equation (11),

$$\mathsf{C} = \sqrt{H_1}\left(\cos\Theta_1 - \frac{H_1}{3H_n}\right). \tag{14}$$

Before proceeding to apply the equations generally to the plotting of curves for arbitrary values, we may consider three special cases in which the results are of particular interest.

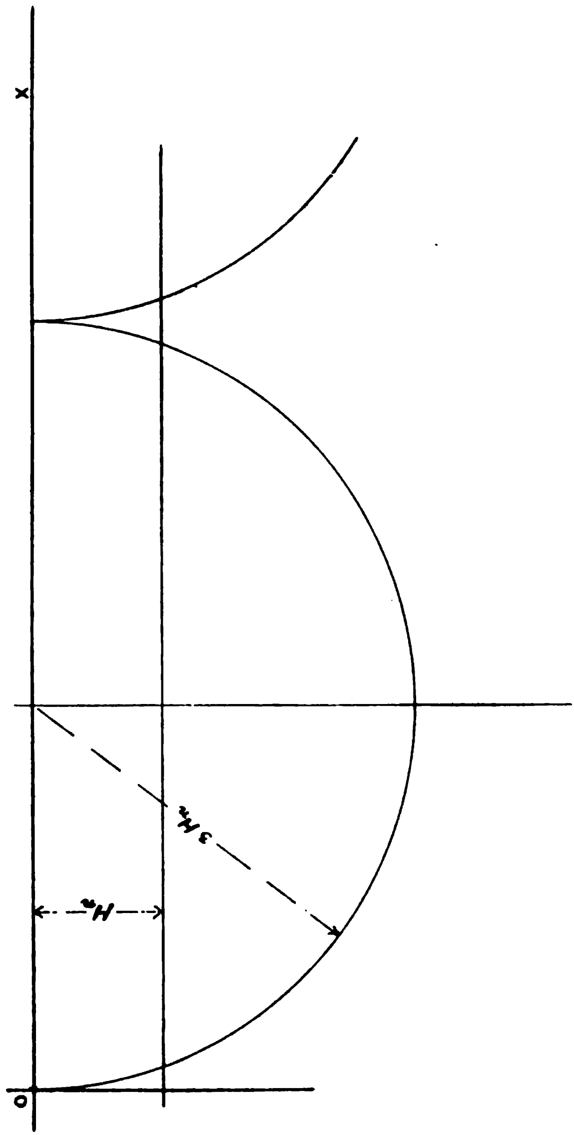


FIG. 32.

§ 23. Some Special Cases of the Phugoid Curve.—(1) Let an aerodone be dropped from rest, head foremost, then, $H_1 = 0$, $\Theta_1 = 90^\circ$ or $\cos\Theta_1 = 0$.

Whence by either (11) or (14) $\mathbf{C} = 0$. (Note: the value of Θ is in this case immaterial.)

And by (13)

$$r = \frac{2}{-\frac{2}{3 H_n}} = -3 H_n,$$

that is to say, r is constant; the path of the aerodone is semicircular, with a radius three times the natural height (Fig. 32).

It is worthy of remark that in this special case the curve is discontinuous, and it is necessary to suppose that the aerodone is artificially inverted on its arrival at a cusp before it can proceed to describe another semicircle. The fact that the radius is of minus sign denotes that the centre of curvature is on the *rarefaction side* of the aerofoil; the results from the convention adopted in respect of F and f.

Case (2).—Let $\Theta = 0$, then $\cos\Theta = 1$, and by (12) we have

$$r = \frac{2 H_1}{1 - \frac{2 g H_1}{V_n^2}} = \frac{2 H_1}{1 - \frac{H_1}{H_n}} = \frac{2}{\frac{1}{H_1} - \frac{1}{H_n}}.$$

Under these conditions, if $H_1 = H_n$ the radius becomes infinite, and the path straight; this is the condition of uniform gliding. If H_1 be less than H_n the radius is of positive sign, which signifies that the centre of curvature is on the compression side of the aerofoil. If H_1 be greater than H_n, r becomes of minus sign, and the centre of curvature is on the rarefaction side of the aerofoil.

Case (3).—In the preceding case $\cos\Theta_1$ was taken as $= 1$. We will now suppose the aerodone to be travelling in the opposite direction, so that $\cos\Theta$ becomes $= -1$; we have

$$r = \frac{2 H_1}{-1 - \frac{H_1}{H_n}} = -\frac{2}{\frac{1}{H_1} + \frac{1}{H_n}}.$$

The radius of curvature is now negative *for all values of* H_1. The signification of this fact will be better understood in the light of the subsequent chapter when the actual curves have been plotted.

It may be noted that in order that *cos* Θ should be $= -1$ the aerodone must be launched *upside down*, the same "hand" must always be kept towards the supposed observer. If the aerodone were launched in the reverse direction *the right way up* this latter condition would be infringed.

CHAPTER III

THE PHUGOID THEORY.—THE FLIGHT PATH PLOTTED

§ 24. Preliminary Considerations.—The form of equations (11) and (13) is such as to indicate that so long as a value of C is chosen proportional to the square root of H_n (the *natural height*), the form of curves for all values of H_n will be geometrically similar, though of different linear scale. This follows from the form of the equation and from dimensional theory, for the constant C is of the dimensions of the square root of a linear quantity. It is thus of no consequence what particular value of H_n be chosen for the purpose of plotting the curves; a series of curves plotted for any one value of H_n applies equally to any other value if read to an appropriate scale.

It has already been pointed out that the phugoid equation does not lend itself to plotting in the ordinary way; the form of the expression $\cos\Theta = \frac{H}{3\,H_n} + \frac{C}{\sqrt{H}}$ is one that, so far as the author is aware, is not susceptible of being reduced to co-ordinate form. It is consequently necessary to employ some other method of plotting, thus taking the equation for the radius of curvature (13),

$$r = \frac{2}{\frac{C}{H\sqrt{H}} - \frac{2}{3\,H_n}}$$

and laying the curve off step by step by means of a trammel the difficulty is overcome.

§ 25. Plotting the Curves. The Trammel.—Having selected any convenient value[1] for H_n, or employing that proper to some

[1] $H_n = 64$ in the plottings given. There is no real reason why this particular value of H_n should have been chosen; it could equally have been made $= 1$.

particular experimental data, first take any simultaneous known values of H and Θ for the curve it is desired to plot. These two values may be those given by the launching data, since at the instant of launching the velocity and angle may be presumed to be known.

Next calculate C from equation (14),

$$\mathsf{C} = \sqrt{H_1}\left(\cos\Theta_1 - \frac{H_1}{3\,H_n}\right)$$

where H_1 and Θ_1 are the simultaneous known values as above.

Now calculate r from equation (13) thus:—

$$r = \frac{2}{\dfrac{\mathsf{C}}{H\sqrt{H}} - \dfrac{2}{3\,H_n}}$$

and repeat this calculation for a series of small increments of H, which must be set out as horizontal lines on the chart.

A trammel is next prepared (Fig. 33), on which these calculated radii are marked off from a fixed tracing point, the extremity of each radius, as marked off, is pierced and indexed with the value of H to which it relates. Thus in Fig. 33 the fixed tracing point is that marked with a cross at the left-hand end of the trammel, and the distances measured off to the points marked 100, 150, etc., are the calculated values for the radius of curvature for the values of H = 100 feet, 150 feet, etc., respectively.

A numerical example of the manner in which a trammel is prepared for any particular curve is as follows:—

Data, $H_n = 64$, $H_1 = 36$, $\cos\Theta_1 = -1$.

Calculation of constant:—

$$\mathsf{C} = \sqrt{H_1}\left(\cos\Theta_1 - \frac{H_1}{3\,H_n}\right) = 6 \times -1\tfrac{3}{16} = -7{\cdot}125.$$

Radius of curvature:—

$$r = \frac{2}{\dfrac{\mathsf{C}}{H\sqrt{H}} - \dfrac{2}{3\,H_n}} = \frac{2}{\dfrac{-7{\cdot}125}{H\sqrt{H}} - {\cdot}01042}.$$

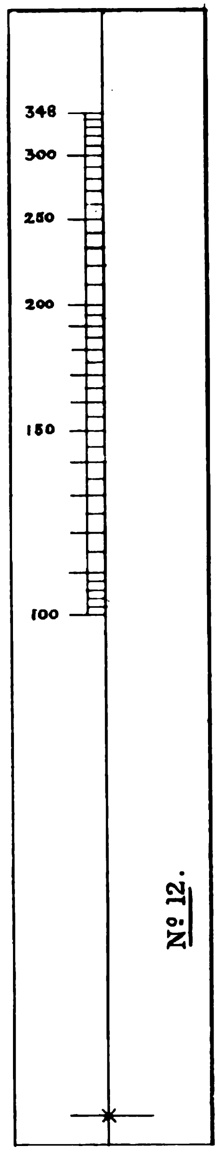


Fig. 33.

Solving this for progressive values of H we have:—

$H.$	$H\sqrt{H}.$	$\frac{7\cdot125}{H\sqrt{H}}.$	$-\frac{7\cdot125}{H\sqrt{H}} - \cdot01042.$	$r.$
36	216	·03300	—·04342	—46·1
38	234	·03045	—·04087	—48·9
40	253	·02814	—·03856	—51·9
42	272	·02620	—·03662	—54·6
44	292	·02442	—·03484	—57·4
etc.	etc.	etc.	etc.	etc.

Fig. 34 is the trammel plotted from the above data.

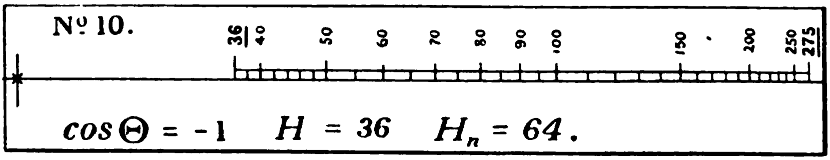


FIG. 34.

Transparent celluloid about $\frac{1}{32}$ inch thick is a suitable material for making these trammels.

§ 26. Plotting the Curves. The Use of the Trammel.—The employment of the trammel in the plotting of the curves is illustrated in Fig. 35, in which the trammel shown in Fig. 33 is being used to set out or plot the curve to which it relates.

The tracing point is first set to the datum value of H (H_1 = 100 in the present example), and the radius line on the trammel is set in a direction at right angles to the flight path (the flight path is horizontal, that is, $\Theta_1 = 180°$ in the example chosen, the aerodone being inverted), in the case given this is vertical. The trammel is now transfixed by a needle at the radius point corresponding to the value of H, that is, where marked 100, and the appropriate element of the curve is marked off. The needle is then moved to the adjacent radius point, and another element of the curve is drawn, and so on. In Fig. 35 the process of

plotting is shown in operation, and Fig. 36 gives the form of the completed curve in the case of the numerical example, of which the trammel is given in Fig. 34.

The method above described is one of astonishing accuracy; the curves, such as that plotted, in which, when the aerodone is at the highest point in the flight path, *cos* $\Theta = -1$, may be easily laid off with an error of not more than one part in 1,000 of their linear measure. It is in fact a question whether the process is not really more convenient than co-ordinate plotting.

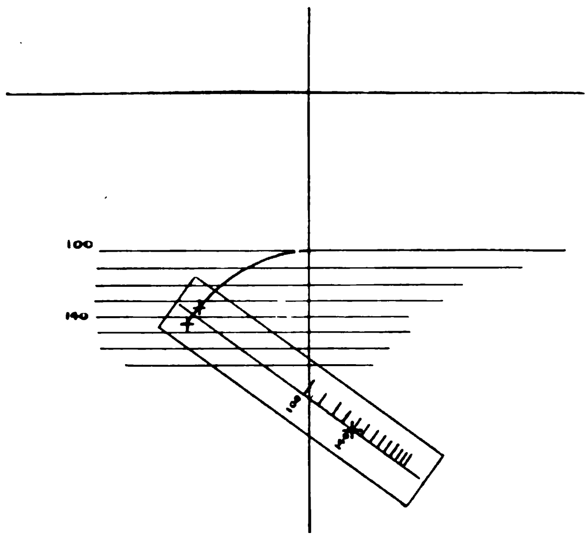


Fig. 35.

Curves such as that plotted, in which *cos* Θ passes through a value $= -1$, may not inaptly be termed *tumbler curves*, from their resemblance to the flight of a tumbler pigeon. These tumbler curves are easy to plot owing to the fact that the radius *r never changes sign*. There is another type of curve which results from the equation, but with a different value of the constant C, in which there is a point of inflection at which the radius changes sign; at this point the value of *r* runs through infinity. A curve of this type is shown plotted in Fig. 37.

It is evident that curves of the inflected type cannot be plotted continuously, in fact in practice it is not possible to approach the point of inflection within less than a certain distance owing

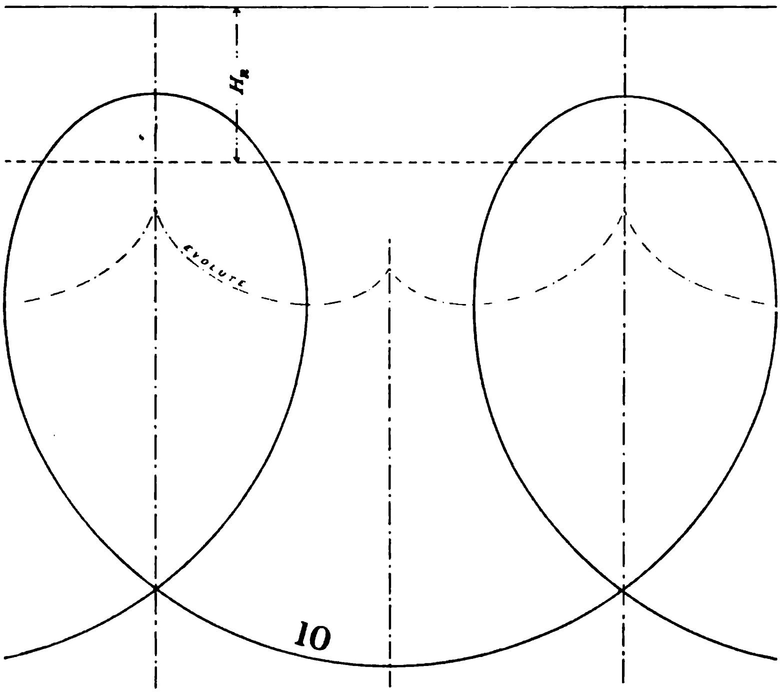


FIG. 36.

to the unwieldy length of the trammel required. A modification of the procedure becomes necessary.

§ 27. Plotting the Inflected Curve.—The first step to be taken in the plotting of an inflected curve is to calculate its limiting values of H, in order that the two parts of the curve in which r is plus and minus respectively may be independently plotted. This done, two trammels are made for the respective portions of the curve of as great length as found workable, and the two portions are then plotted *independently* to within as short a

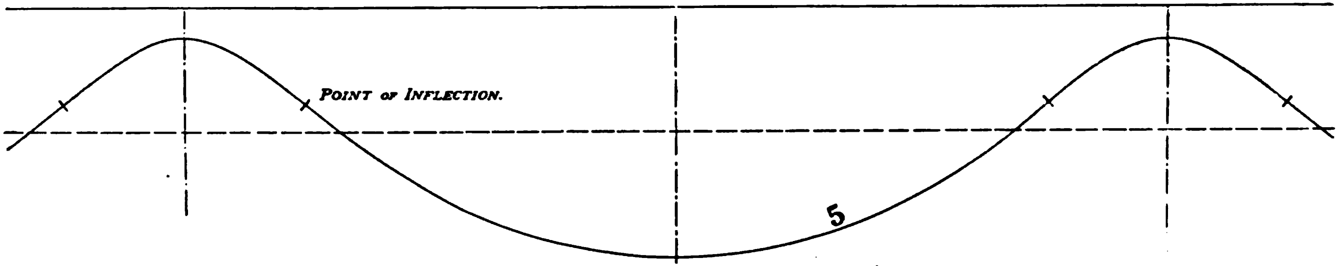


FIG. 37.

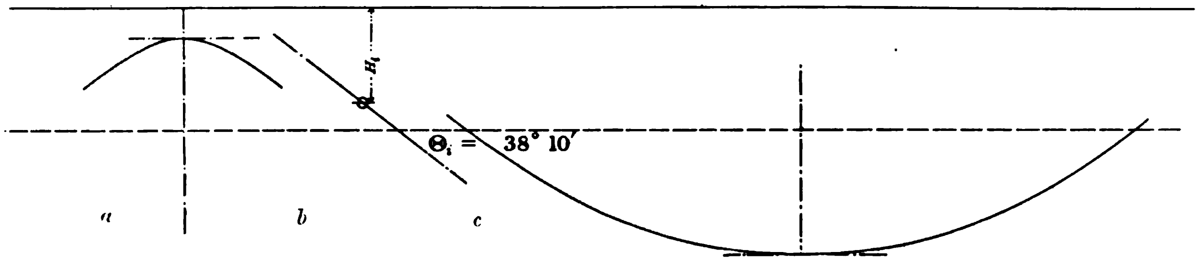


FIG. 38.

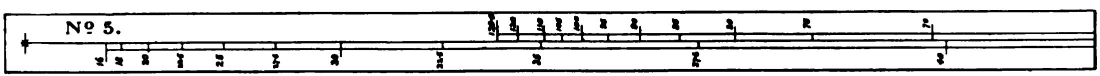


FIG. 39.

distance from the point of inflection as possible, Fig. 38, (*a*) and (*c*). Next the *angle*, that is, the value of Θ, at the point of inflection is calculated, and this angle is laid off on the chart, Fig. 38 (*b*). Finally the three parts of the curve (*a*), (*b*), and (*c*), are adjusted in their relative positions, as nearly as the eye can appreciate, and continuity is given to the three components by drawing a smooth curve (Fig. 37).

It may be remarked that as a means of drawing the actual form of the curve with a sufficient degree of approximation to serve every useful purpose, the above method is quite satisfactory. From the theoretical point of view, however, it would be better if we could find some means of actually plotting the missing portions of the curve round about the point of inflection. It has been suggested to the author that it may be found possible to deduce an approximate co-ordinate expression for this portion of the curve, and to employ the same to fill in the gap left by the radius method ; the matter however is scarcely of sufficient importance to justify the additional mathematical work.

§ 28. Plotting the Inflected Curve. Example.—The following example, plotted in Fig. 37, to which reference has already been made, illustrates the manner in which the plotting data are calculated.

Example.

$$H_n = 64 \qquad H_1 = 16 \qquad \cos\Theta_1 = 1.$$

By (14),

$$\mathbf{C} = \sqrt{H_1}\left(\cos\Theta - \frac{H_1}{3\,H_n}\right) = 4 \times \left(1 - \frac{16}{192}\right) = 3{\cdot}666.$$

Substituting for $\mathbf{C}$ in equation (14), also for $\cos\Theta$ for highest and lowest points, that is, $\Theta = 0$, $\cos\Theta = 1$, we have

$$3{\cdot}666 = \sqrt{H}\left(1 - \frac{H}{192}\right)$$

or

$$\frac{H}{192} + \frac{3{\cdot}666}{\sqrt{H}} = 1,$$

the solution of this equation[1] gives two values of H:—

$H = 16$, highest point on curve (already known).

$H = 129{\cdot}9$, lowest point on curve (value required).

We now require the inflection data, *i.e.*, the values of H and Θ at the point of inflection. Let us denote these values by the symbols H_i and Θ_i respectively. We have

$$r = \frac{2}{\dfrac{C}{H_i \sqrt{H_i}} - \dfrac{2}{3 H_n}} = \infty$$

$$\therefore \quad \frac{C}{H_i \sqrt{H_i}} - \frac{2}{3 H_n} = 0, \text{ or, } H_i = \left(\frac{3 H_n C}{2}\right)^{\frac{2}{3}} \qquad (15)$$

Further, by equation (11)

$$\cos \Theta_i = \frac{H_i}{3 H_n} + \frac{C}{\sqrt{H_i}}. \qquad (16)$$

Thus in the present example,

$$H_i = \left(\frac{192 \times 3{\cdot}666}{2}\right)^{\frac{2}{3}} = 49{\cdot}8$$

and

$$\cos \Theta_i = \frac{49{\cdot}8}{192} + \frac{3{\cdot}666}{7{\cdot}06} = {\cdot}779,$$

whence $\Theta_i = 38° \, 50'$.

The data for the laying off of the two scales on the trammel, Fig. 39, are as follows:—

H.	$H\sqrt{H}$.	$\dfrac{3{\cdot}666}{H\sqrt{H}}$	$\dfrac{3{\cdot}666}{H\sqrt{H}} - {\cdot}01042$.	r.
16	64	·05740	·04698	42·55
18	74·2	·04950	·03908	51·18
20	89·4	·0411	·03068	65·24
22·5	etc.			
and				
129·9	1480	·00247	— ·00795	251·8
120	1315	·002788	— ·007632	262
110	1153	·00318	— ·00724	276·0
105	1076	·003415	— ·007005	285·5
100	etc.	etc.		

[1] See Appendix, IV.

It will be understood from the preceding section that the two portions of the curve are plotted separately by means of the trammels as above, beginning at the highest and lowest points respectively and continuing until they can be carried no further, when their path angles will mutually approach the inflection value Θ_i (Fig. 38).

§ 29. Phugoids of Small Amplitude.—In the case of the flight path in which the launching velocity of the aerodone differs but little from the natural velocity (Θ_1 being zero), the curves become so flat that the foregoing method of plotting must be considered impracticable; for such cases some other process must be devised. We will investigate the conditions that obtain when the variations in Θ are very small, that is to say when *cos* Θ only differs from unity by a small quantity of the second order.

Starting from the initial hypothesis we have by (1)

$$F = 2\,g\,H\,n \quad \text{where by (10)} \quad n = \frac{1}{2\,H_n}$$

$$\therefore \quad F = \frac{g\,H}{H_n}.$$

When $H = H_n$ we have as before (§ 21), $F = g$; let H differ from H_n by a small amount $= h$, so that $H = H_n + h$, and let F become $F + f$, that is, $g + f$. Then,

$$g + f = F \times \frac{V^2}{V_n^2} = g \times \frac{H_n + h}{H_n} = g + \frac{g\,h}{H_n}$$

or $f = \dfrac{g\,h}{H_n}$, or $\dfrac{f}{h} = \dfrac{g}{H_n}$, which is constant.

Hence f varies as h, that is to say, the force varies with the displacement. Therefore *for small amplitude the vertical component of the motion of the aerodone is harmonic.* The time period of this harmonic motion is given by the equation,

$$t = 2\,\pi\sqrt{\frac{H_n}{g}} \tag{17}$$

Now, dealing with the horizontal component of the disturbance, that is, with the excess or deficit of velocity above or below the mean, let $V_n + v$ be the velocity of the aerodone when its ordinate is $H_n + h$, then since

$$V^2 = 2\,g\,H$$

$$(V_n + v)^2 = 2\,g\,(H_n + h) \qquad (a)$$

but

$$V_n^2 = 2\,g\,H_n \qquad (b)$$

subtracting (b) from (a) we have

$$2\,V_n\,v + v^2 = 2\,g\,h,$$

but for small amplitude v^2 is negligible, hence

$$V_n\,v = g\,h \qquad (c)$$

or $\frac{v}{h} = \frac{g}{V_n}$, which is constant, therefore the excess or deficiency of horizontal velocity v is at all times proportional to the vertical displacement h, and it at once follows that, since the latter has been proved harmonic, the velocity variation, or superposed motion in the line of flight, is also harmonic.

§ 30. Phugoids of Small Amplitude. Form of Orbit.—The simplest view to take of the problem, at the present stage, is that the motions of the aerodone are plotted relatively to itinerant ordinates, the origin being supposed to travel with a uniform velocity $= V_n$ along the axis of flight.

We have so far proved that the displacement in the direction of the axis of y (that is, the variation of H), is proportional to the force by which it is produced, and is therefore harmonic. We have further shown that the velocity along the axis of x (the axis of flight), measured from the itinerant origin, is proportional at every instant to the displacement on the axis of y, and that this also is consequently harmonic; it also follows from the latter result that the two harmonic motions differ by an interval of one quarter phase. It remains for us to determine the amplitude ratio of the two harmonic motions, and the orbit and consequently the course of the aerodone is known.

Let v_1 be the maximum velocity of the horizontal component of the orbit motion, and let t be the time of one complete orbit. Then by equation (17) $t = 2\pi\sqrt{\frac{H_n}{g}}$. Let x_1 be the half amplitude of the horizontal component and let h_1 be the similar measure (the half amplitude) of the vertical component.

We have $$x_1 = \frac{v_1 t}{2\pi} = v_1\sqrt{\frac{H_n}{g}},$$

and by §29, equation (c)

$$V_n v_1 = g\, h_1,$$

but $V_n = \sqrt{g\, H_n}$; substituting, we obtain

$$h_1 = v_1 \frac{\sqrt{2\, g\, H_n}}{g} = \sqrt{2} \times v_1\sqrt{\frac{H_n}{g}}$$

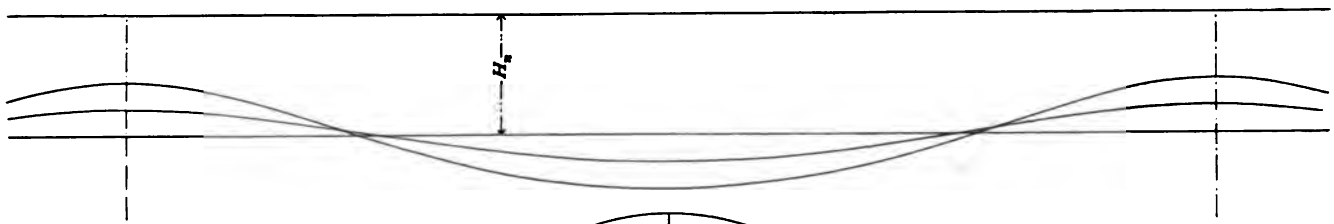
or $$h_1 = \sqrt{2} \times x_1.$$

That is to say, the vertical amplitude of the orbit is $\sqrt{2}$ times the horizontal amplitude. And we already know that both the horizontal and vertical components of the orbit are harmonic. Consequently the form of the orbit is an ellipse whose major axis is vertical and is greater than the minor axis in the ratio $\sqrt{2}$ is to 1.

§ 31. Phugoids of Small Amplitude. Plotting.—In the foregoing sections we have proved that the "evanescent" form of the phugoid, that is, the form of the phugoid curve when almost merging into a straight line, is of trochoid-like form, in which the orbit of the tracing point is elliptical instead of being circular, and that the axes of this ellipse are in the relation one to the other, the vertical to the horizontal, as $\sqrt{2}$ is to 1.

The plottings of two such trochoidal approximations are given in Fig. 40, and a convenient method of plotting (which is in any case but a matter of simple geometry) is illustrated in Fig. 41. In plotting these trochoid-like approximations the length of a complete period, or "phase length," is first calculated from

Fig. 40.



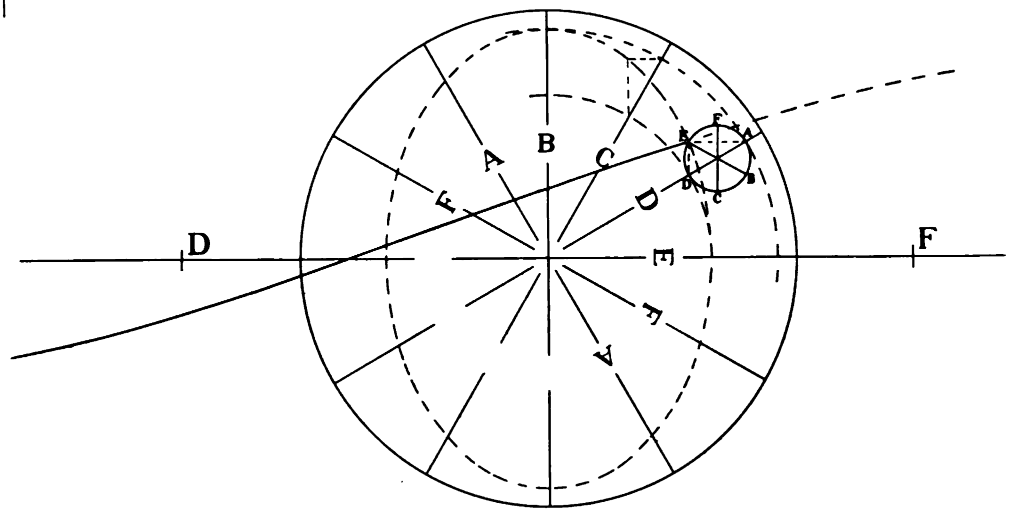


Fig. 41.

equation (17) and from the known value of V_n, this is laid off and divided into a convenient number of equal parts. A celluloid dial (Fig. 41), is next prepared with as many angular divisions as there are linear divisions in the phase length, and on this dial two circles are scribed having radii as $1 : \sqrt{2}$, and a small tracing circle is drawn, touching both. This small circle is divided into half as many divisions as there are in the dial, the plotting is then performed in the following manner:—

Presuming that we begin with the highest point on the curve, the dial is placed with the diameter passing through the centre of the tracing circle vertical, the latter being at the top of the dial. The point A is then pricked off, and the dial is moved bodily from left to right through one of the linear divisions, and rotated through one angular division *counter-clockwise;* the next point reckoned *clockwise*, B, is taken on the tracing circle and pricked off; the process being repeated until the curve is completed.

It is evident from geometrical considerations that the tracing point derives its location along the horizontal axis from the smaller circle, and vertically from the larger one, and so the required compound motion is obtained.

§ 32. **The Phugoid Chart.**—The plotting of the phugoid curves forming a complete series, from the straight line representing the path of uniform gliding, to the tumbler type of curve with constants varying to any desired degree, may be termed a *phugoid chart.* In Fig. 42 such a series is given, including in all some twelve plottings[1]; these, beginning with the straight flight path No. 1, include two trochoidal approximations, Nos. 2 and 3, plotted by the dial method; three true phugoids of the inflected type, Nos. 4, 5, and 6; the special case of the semicircular flight path No. 7 (this may be looked upon either as an inflected curve or as a tumbler curve according to which way the aerodone is turned over at the cusp), and five tumbler curves Nos. 8 to 12 inclusive. The calculations and data of these plottings are given in Appendix V.

[1] Also given to reduced scale in frontispiece.

The *evolutes* of the curves, as developed in plotting, are given so far as the size of the plate permits, and the scale shown is that employed in the calculations, though any other scale may be employed without affecting the geometrical shape of the curves.

The value of H_n has been taken as 64 feet, and is, of course, represented by the distance of the straight flight path below the datum line; the appropriate scale for any other value of H_n is obtained by simply substituting the new value on the chart.

The whole of the curves are plotted to one value of H_n from one horizontal datum line, and the point of the curve forming the starting point of the plotting has in every case been referred to a single vertical axis 0 y, at which the value of $\cos \Theta$ has been taken either $+ 1$ or $- 1$, and the value of H as minimum.

Each curve has its position of mid-phase marked (the point at which its velocity is greatest), and a curve has been drawn through the points so obtained, which is called the *phase curve;* this evidently should be a smooth curve, and it should depart from the path of uniform gliding (curve No. 1) normally and approach the axis of y asymptotically.

The phase curve shows well the manner in which the trochoidal approximations become inaccurate as the amplitude increases; it also serves as a check on the proper joining of the inflected curves. In the present case the form of the curve suggests that these inflected curves have been slightly "telescoped" in the process of plotting. The phase curve, once its form is carefully determined, will enable inflected phugoids to be plotted with greater certainty; there will no longer be any necessity for the process of adjustment.

§ 33. The Time Period of the Phugoid Path. — The complete period of an aerodone, that is to say, the time taken by it to travel from crest to crest of its path, is a quantity for which the author has so far been unable to obtain a general expression;

in all probability the mathematical integration of the phugoid curve in respect of *time* is a matter of very considerable difficulty.

As a general solution the author has, for want of a better method, employed a step by step graphic process, the working of which is as follows. An *inverse velocity curve* (see Phugoid Chart, Fig. 42) is first laid out, this gives by its abscissæ the time taken to traverse an arbitrary small unit of length at a velocity corresponding to the height H of the ordinate. The phugoid, whose period it is required to measure, is then divided into a number of linear elements of the size of the unit chosen, and an ordinate is drawn on the inverse velocity curve, corresponding with the mean position of each unit; these ordinates are then measured, and their sum gives the total period required.

The above method has been applied very carefully to curves 5, 7, 8, 9, 10, 11, and 12, and the results plotted as the "Time Curve" shown. The degree of accuracy may be judged from the deviations of the plottings from the averaging line drawn as a smooth curve through them. It may be noted as a remarkable feature of this curve, and one that is of the greatest possible service in the experimental verification of the theory, that the time period for all curves lying between the straight line and the semicircle is approximately independent of the amplitude; that is to say, for a given value of H_n the time period for the inflected phugoids is sensibly constant.

§ 34. The Time Period of the Phugoid Path. Special Cases.—In certain special cases, for example, in the almost straight flight path, considered in §§ 29, 30 and 31, the time period may be calculated, and in this case is given by equation (17).

$$t_1 = 2\pi\sqrt{\frac{H_n}{g}}.$$

The special case of the semicircular path also is one that may be calculated, the time period being that of a pendulum of length

$3\ H_n$ swinging through an arc of 180 degrees. The expression in this case is

$$t_2 = \pi \sqrt{\frac{l}{g}} \times x,$$

where $l = 3\ H_n$, and where the value of x is a function of the arc of swing defined by the expression,

$$x = 1 + \left(\frac{1}{2}\right)^2 \sin^2 \frac{A}{2} + \left(\frac{1 \times 3}{2 \times 4}\right)^2 \sin^4 \frac{A}{2}$$

$$+ \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 \sin^6 \frac{A}{2} + \left(\frac{1 \times 3 \times 5 \times 7}{2 \times 4 \times 6 \times 8}\right)^2 \sin^8 \frac{A}{2} + \text{etc.},$$

where $2\ A$ is the total arc of swing.

For a value of $2\ A = 180^\circ$ we have $\frac{A}{2} = 45^\circ$ and calculation gives $x = 1{\cdot}1795$, so that the time period will be

$$t_2 = \pi \sqrt{\frac{3\ H_n}{g}} \times 1{\cdot}1795.$$

We may thus express the relation as

$$\frac{t_2}{t_1} = \frac{1{\cdot}1795\ \pi \sqrt{\frac{3\ H_n}{g}}}{2\ \pi \sqrt{\frac{H_n}{g}}} = \frac{1{\cdot}1795\ \sqrt{3}}{2}$$

$$= \frac{2{\cdot}04295}{2} = 1{\cdot}02147,$$

that is to say, the time period of the semicircular path is greater than that of the nearly straight path, to an extent only of about 2 per cent. These two special cases constitute the limits of the inflected series, for which the graphic method gives the time period as sensibly constant, and thus we have in substance a confirmation of the previous conclusion.

The actual time periods for the two special cases under discussion are as follows :—

(1) The nearly straight path,

$$t_1 = 2\ \pi \sqrt{\frac{64}{32{\cdot}2}} = 8{\cdot}85 \text{ secs.}$$

(2) The semicircular path,

$$t_2 = 1{\cdot}1795 \times 1\,\pi \sqrt{\frac{192}{32{\cdot}2}} = 9{\cdot}04 \text{ secs.}$$

As figured on the chart the time curve has not been corrected for these calculated results.

§ 35. The Time Period and Form of the Phugoid Path. Special Cases (continued).—As the amplitude of the phugoid is increased beyond the limits of the semicircle, there is a rapid shortening of the time period as shown by the manner in which the *time curve* falls away almost immediately after that critical form of flight path is passed. The time period of the tumbler curves is in every case less than that of the semicircle, and the time period diminishes continuously as the value of the constant C gets less and less. It becomes of interest to examine the fate of the time curve when outside the range of our plotting.

Let the velocity of the aerodone become very great so that W is a negligible quantity, then the orbit will, in the limit, become circular, since in this case V becomes sensibly constant, and F is therefore also constant. Under these conditions we write $W =$ zero, so that $f = F = \frac{V^2}{r}$.

But by (1) $$F = n\,V^2.$$

$$\therefore \quad n\,V^2 = \frac{V^2}{r}, \text{ that is, } r = \frac{1}{n},$$

but by (10)

$$n = \frac{1}{2\,H_n}. \qquad \therefore \quad r = 2\,H_n.$$

That is to say, if we select phugoids of greater and greater velocity, for a given aerodone the form of the tumbler curve approximates more and more closely to a circle of radius $= 2\,H_n$.

If we suppose that we are investigating depths at which the

phugoids are sensibly circular in form, that is, down where the phase curve approximates very closely to the axis of y, then

$$V = \sqrt{2 g H},$$

and we know length of path $= 4 \pi H_n$.

Hence $t = \dfrac{4 \pi H_n}{\sqrt{2 g H}}$ or $t^2 H = 8 \dfrac{(\pi H_n)^2}{g}$, which is constant, thus giving the form of curve to which the time curve continually approximates as the depth becomes greater and greater.

§ 36. Relations of Time Period, Phase Length, and Velocity for Phugoids of Small Amplitude.—We may now return to consider the simple relationship that exists between time period, phase length, and velocity, in phugoids of small amplitude. A clear statement of this relationship is desirable in view of the subsequent experimental investigations, that are presented later in the work, in confirmation of the phugoid theory.

Let L_1 = complete phase length in feet.
t_1 = time period as before.
V_n = natural velocity.

We have

$$t_1 = 2 \pi \sqrt{\frac{H_n}{g}}$$

and

$$V_n = \sqrt{2 g H_n}.$$

Whence

$$L_1 = t_1 V_n = 2 \pi \sqrt{\frac{H_n}{g}} \times \sqrt{2 g H_n}$$

$$= 2 \sqrt{2} \times \pi H_n$$

or

$$L_1 = 8{\cdot}88\, H_n.$$

Taking $g = 32{\cdot}2$, t_1 may be expressed in terms of H_n thus,

$$t_1 = 1{\cdot}105 \sqrt{H_n}.$$

The experimental verification is given in Chap. VI.

§ 37. Variations in the Value of C for Geometrically Similar Curves. The Constant K.—It has been already remarked that

systems of phugoids based on different values of H_n are geometrically similar, and consequently a *phugoid chart* (such as Fig. 42), may be read to any scale appropriate to the value of H_n chosen.

Then, for any particular curve and point on that curve as defining the position of the aerodone, we have for different values assigned to H_n the following relations:—

$$\frac{H}{3\,H_n} = \text{constant; also } \cos\Theta = \text{constant.}$$

$$\therefore \quad \cos\Theta - \frac{H}{3\,H_n} = \text{constant.}$$

But we know from equation (14),

$$\frac{\mathsf{C}}{\sqrt{H}} = \cos\Theta - \frac{H}{3\,H_n},$$

hence $\frac{\mathsf{C}}{\sqrt{H}}$ is constant for the point chosen, whatever the value of H_n, that is to say, whatever the scale assigned to the curve, and $\frac{\mathsf{C}}{\sqrt{H_n}}$ may be taken as an invariable constant relating to the particular curve and is independent of the scale to which the curve is read. We will represent this constant by the symbol K.

This constant K will be termed the *amplitude constant*, and all curves for a given value of K are geometrically similar.

Also for the straight path ($H = H_n$) we have $\mathsf{K} = \frac{2}{3}$ and $\mathsf{C} = \frac{2}{3} H_n$, this is the condition of the maximum value of K.

The variations of K with H and $\cos\Theta$ for any given value of H_n follow the same law, and curve, as variations in C. K is in fact the constant C deprived of its dimensionality.

CHAPTER IV

ELEMENTARY DEDUCTIONS FROM THE PHUGOID THEORY

§ 38. **Permanence of Stability.**—The theoretical investigations of the preceding two chapters constitute the proof, under the restricted conditions of hypothesis, of the longitudinal stability of an aerodone in flight, the whole demonstration being a quantitative analytical version of the theory of stability enunciated in §§ 3, 4 (Chap. I.).

The permanence of stability, according to the present proof, rests definitely on the fact that the phugoid curve consists of a succession of phases repeated, without change of form, an indefinite number of times. If the substitution of real conditions for those of hypothesis be found to result in any progressive change in the form of the curve, that is, if the constant C be found to undergo a change (for a given value of H_n), or more broadly, if the constant K suffer any progressive change of value, then the question of permanence of stability is greatly complicated, and further investigation will be required.[1]

There is one particular case of the phugoid curve, even under the supposed conditions, in which the stability must be regarded as indeterminate; this is the semicircle when C $=$ zero. The fourth condition of hypothesis, § 19, is "that the size of the aerodone is small in proportion to the minimum radius of curvature of its flight path." Now, for the purposes of the theoretical investigation, this condition may be supposed fulfilled by the aerodone being assumed to have no size at all, but in the case of the semicircular flight path this is not sufficient, for the "cusp" must be considered to be a portion of the flight path

[1] This further investigation forms the subject of Chap. V.

of infinitely small radius; we have to suppose the aerodone artificially *turned over* after each semicircular flight before it can begin the next semicircle.

If the aerodone be a thing of substance, then not only do we meet with difficulty in the particular case of the semicircle, but also in the case of all curves approximating thereto, whether of the inflected or the tumbler variety; a point is reached sooner or later when the aerodone is sensibly embarrassed in turning over by its fore-and-aft dimension, in other words by the length of its tail, and the theoretical form of the curve ceases to hold good.

§ 39. Unstable Conditions. The Danger Zone on the Phugoid Chart.—From the foregoing considerations we must regard the environment of the cusp of the semicircular phugoid as constituting a *danger zone*, on entering which an aerodone is liable to complete loss of stability. The manner in which this loss of stability takes place becomes apparent if we examine the case of an aerodone of some considerable tail length on a flight approximating to the semicircle; as the aerodone approaches nearer and nearer to the cusp it gradually loses its velocity until, as it arrives at the datum level, it finally comes to rest, *and then commences to run backwards.* Now the flight of an aerodone in the contrary direction to that for which it is designed must be regarded as denoting that the limits of stability have been exceeded, for this condition is quite inadmissible either from a theoretical or practical standpoint. We consequently appreciate that any cumulative change in the constant, either C or K, tending towards zero value, is fatal to the permanence of longitudinal stability.

So long as the aerodone is of reasonably small size in comparison to the phase length L_1, the whole of the inflected phugoids of moderate amplitude, including those which may be approximated by the elliptical trochoids, and perhaps such as numbered 4 and 5 on the chart, may be regarded as curves of stable flight

path, but as the semicircle is approached the conditions of instability supervene.

If we go beyond the semicircle, we again find stability in the tumbler form of flight path; this state, however, is more a matter of scientific than of practical interest; from the point of view of the problem of flight we may regard any approach to the semicircle as nearing the limit of stability; the danger zone in the neighbourhood of the cusp prescribes the limit permissible to the amplitude of the flight path.

§ 40. **Stability in Face of a Disturbing Cause.**—One immediate consequence of the phugoid theory is that the stability of an aerodone, in face of a given disturbing cause, increases with its velocity of flight. Thus it is evident that if an aerodone, travelling at its natural velocity in still air, were to meet with a gust of wind travelling in the opposite direction, it would immediately begin to rise, just as if its value of H had suddenly been increased, which in effect it has. Its subsequent path, evidently, then depends upon the velocity of the gust in relation to the value of V_n; should the velocity of the gust reach $(\sqrt{3} - 1)$ V_n the aerodone will find itself in the middle of a semicircular swoop, and on rising will pass into the danger zone and lose its equilibrium. Consequently it is the relation of the velocity of the gust of wind to the natural velocity of the aerodone that constitutes the criterion, and the higher the natural velocity the higher (in the same proportion) is the velocity of the gust of wind that can be encountered with impunity.

The same naturally applies in passing from an adverse gust into still air, or, that which is the same thing, if the gust of wind take the aerodone from behind. In either case if the velocity of the gust be equal to that of the aerodone the latter will find itself at rest relatively to the air, and consequently its stability is in jeopardy; we may regard an aerodone under these conditions as at the cusp of a semicircular phugoid in the act of being turned over.

Thus, a change in the velocity of the wind taking effect as a gust either with or against the direction of flight, may cause instability, the magnitude of the wind change required to bring the constant C to zero being ·73 V_n in the case of an adverse gust, or V_n in the case of a gust in the direction of flight.

§ 41. The Constant C as an Index of Stability.—We have seen that in the particular case of C = zero, the semicircular phugoid constitutes the worst possible case of unstable flight path, and that as the constant C increases to its maximum value (when the path of the aerodone becomes a straight line) the flight path is further and further removed from the danger zone. We may consequently regard the constant C as an *index of the stability.*

Stability is not a thing that we have learnt to measure, so that it is impossible to assert that the stability is in any way proportional to C; we can merely learn to associate an increase in C with an increase in stability and *vice versâ.*

When we are concerned with the tumbler curves, we find that the converse is the case, for the further removed the tumbler curve becomes from the semicircle, the lower is the value of C. We can, since the values of C are here of minus sign, rectify the anomaly by making C^2 the index of stability, when the rule applies for tumbler and inflected curves alike.

As a gauge of stability C^2 is more appropriate than K^2, for K undergoes no alteration in the face of a change in the value of H_n, and we know from the preceding section that stability is improved by an increase in H_n. The importance of K in dealing with the maintenance of stability will appear from its mode of employment in the subsequent chapter.

§ 42. Wind Fluctuation. A Question of Relative Motion.—The supposititious case of wind fluctuation dealt with in § 40 is merely one example of this kind of disturbance. The turbulent component of wind motion is a matter of great complexity at present but little understood; in all probability a wind of any

considerable velocity consists very largely of vortex motion, and an aerodone, in passing from one part of such a "vortex pack" into another part, experiences many and varied changes both as to velocity and direction. It has been demonstrated experimentally by Prof. Langley that the higher the wind velocity, the higher in proportion becomes the velocity of turbulence reckoned either as the maximum or the mean velocity difference. Thus the variations are not confined to horizontal fluctuations as discussed in § 40, but include vertical components of motion distributed in an irregular manner.

The question from the point of view of an aerodone in flight is entirely one of *relative motion.* As the aerodone passes from one mass of air into another, possessed of a different velocity and direction of motion, it is the difference of the two velocities with which the aerodone is concerned, and not the absolute velocity of either. Thus the main motion of translation of the wind has no influence whatever except in the case of an aerodone at the time of launching from a fixed point.

The above fact simplifies the consideration of the problem materially. We have already dealt with the case of a change of wind velocity in the direction of flight, we will now, on similar lines, deal with the more general case of a change of velocity and direction, and show how the use of a fully plotted phugoid chart enables any case, however complex, whose data are defined, to be dealt with to any desired degree of approximation.

§ 43. Wind Gusts of changing Velocity and Direction.—Owing to the fact that we have assumed the whole mass of the aerodone to be concentrated at its centre of gravity, and that it consequently possesses no moment of inertia, we can suppose that its directive organ acts instantaneously; thus, when an aerodone passes from one mass of air into another, having a different relative direction of motion, it will instantly adapt itself to the new conditions in respect of its attitude to the line of flight.

The influence of distributed mass is more fully discussed

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later, but it may be remarked parenthetically that the general effect of moment of inertia about a transverse axis is to hinder and delay the adaptation of the attitude to the change of relative

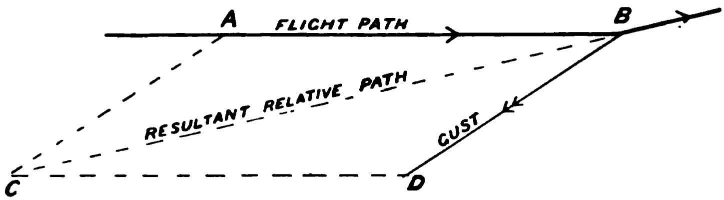


FIG. 43.

direction, and the greater the moment of inertia the larger the directive organ or tail plane required; beyond this the present argument is not seriously affected.

Let us suppose, as a general proposition, that the motion of

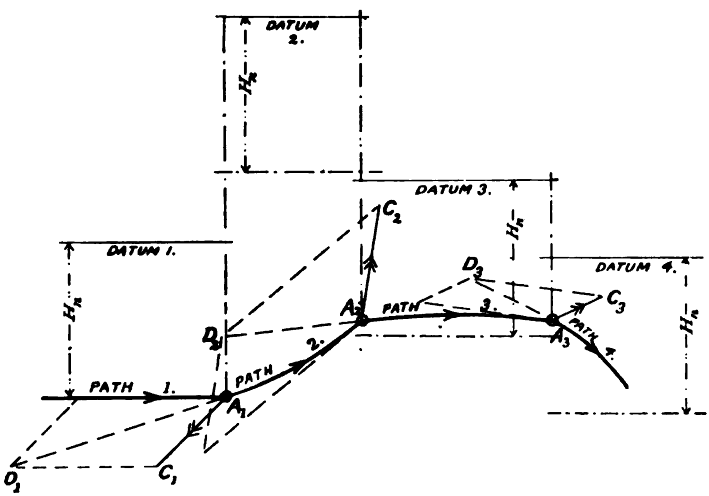


FIG. 44.

the aerodone through (relatively to) the air at any instant is represented both as to direction and magnitude by the line $A\ B$ (Fig. 43). Let now a change take place in the velocity and direction of the wind, which we will represent by the line $B\ D$.

It is evident that the new relative velocity and direction may be obtained by a parallelogram of velocities, and will be given by the line $C\ B$; for, if we suppose the path of the aerodone represented by $C\ D$, a particle of air from B and the aerodone from C will, if the velocities and directions be maintained for a short period of time, reach the point D simultaneously.

Now, referring to Fig. 44, since we know the velocity of the aerodone at the point A_1, we know its value of H; let this be the height measured from datum line No. 1 and $= H_n$. And since we know the angle of the path we can plot (or lay off from a fully plotted chart), the phugoid which constitutes the path of flight for the time being; this is marked "Path 1," and under the given conditions is a uniform glide.

Let $A_1\ C_1$ represent the change of wind at the instant the aerodone arrives at the point A_1, then draw the parallelogram and obtain the new line of direction and velocity $D_1\ A_1$. Next from the value of V so obtained calculate the new value of H and draw datum line No. 2, and, as before, lay off flight path (2) from phugoid chart.

Let $A_2\ C_2$ be a further wind fluctuation encountered at the point A_2, then, proceeding as before, we obtain the new relative flight path (3), and so on.

§ 44. Stability in the Face of Wind Fluctuations.—In the actual changes that constitute wind fluctuation the gusts do not arise with absolute suddenness as depicted; it is evident, however, that we may make the individual steps as small as we please, and so approximate, as closely as we wish, to any known set of conditions.

The absolute flight path is not in any case a jagged curve, as shown in Fig. 44, which represents the curve plotted relatively to the changing gusts of wind. If plotted to some fixed coordinates the sudden changes of direction and velocity would not exist; this kind of plotting would, however, be very tedious, and, fortunately, would serve no useful purpose.

It is evident that observations made at a fixed observatory of the curves of wind fluctuation would not of necessity apply to an aerodone in flight, under identical wind conditions. If, however, the fluctuations are due to some kind of organised turbulence, as would be the case if the wind consists of a kind of *vortex pack*, we could, from time observations made at a stationary point, plot a typical sample *on a linear basis*, and it would be fair to assume that the changes so plotted represent the average experience of the aerodone in its flight path.

§ 45. Stability in the Face of Wind Fluctuations (continued).— The method of the foregoing sections enables us to plot the relative flight path of an aerodone in a known fluctuating aerial field to whatever degree of approximation may be desired, and this plotting may be taken as prescribing the stability or otherwise of the flight path. If the aerodone, in the course of its career, find itself at the cusp of a semicircular phugoid, or if in practice it be even in the near vicinity of the cusp, it must be regarded as having lost its equilibrium, and the wind conditions must be deemed dangerous to flight. Likewise, if at any time the value of **C** falls below zero, so that the path of flight is a tumbler curve, the conditions of stability have, by our definition, been violated.

It has been shown in § 40 that for a single gust of defined magnitude the stability of an aerodone may be assured by increasing to a sufficient degree its *natural velocity*, this involving a structural alteration of the aerodone or an increase of its weight in accordance with established aerodynamic principles.

With a certain reservation, that will be discussed later, it is evident that the same principle applies where the disturbing cause comprises a number of lesser fluctuations, and that the greater the magnitude of the wind turbulence the greater will be the value of V_n necessary to ensure stability. In other words safety in flight depends upon the employment of a sufficiently high velocity for the conditions that obtain.

§ 46. The Practical Limit of Stability.—A question arises that is very difficult to approach from a theoretical standpoint; namely the extent of the danger zone in the vicinity of the cusp. This is, it has already been pointed out, dependent upon the *size*, principally the tail length of the aerodone, and the subject is not one that can at present be dealt with on rigid lines.

Referring to the phugoid chart, it is evident that an aerodone travelling on the curve entitled No. 6, would be getting perilously near the region of the cusp, whilst curves Nos. 2, 3, and 4 do not give rise to any anxiety.

Dealing with the question on the basis of § 40, we know that it is an adverse gust of any given magnitude that has the greater influence on the value of **C**; if the aerodone is safe in face of an adverse gust of given magnitude, a greater gust may come up from behind without causing instability. We also know that the velocity of an adverse gust, that will change an aerodone from the straight line gliding path to the semicircle is $(\sqrt{3}-1)\,V_n$, or $= \cdot 73\,V_n$, and we can express the values of the gust (relative) velocity for the other curves plotted as follows.

Curve No.	Gust Velocity, in Terms of V_n.
2	$\cdot 103\ V_n$
3	$\cdot 200\ V_n$
4	$\cdot 335\ V_n$
5	$\cdot 425\ V_n$
6	$\cdot 520\ V_n$

Thus it is evident that if v be the gust velocity relatively to the air in which an aerodone is gliding tranquilly, the relation $v = \cdot 73\ V_n$ is definitely dangerous; $v = \cdot 5\ V_n$ is uncomfortably near the limit; and $v = \cdot 33\ V$ has every appearance of being reasonably safe.

When we later pass on to consider the damping influences that are at work we shall see that it is possible to design an aerodone whose stability shall be immune to considerably greater

disturbances than above stated. When any influence is at work to continuously damp out the amplitude, that is, to raise the value of C, the change that takes place during one half-phase may be quite sufficient to carry the aerodone from an unstable to a stable path, and it may consequently be necessary for it to be on a tumbler curve in the trough of its flight path in order that it should arrive on the semicircular phugoid at the moment of cresting, and so experience instability.

The practical limit of stable flight path is one that in all probability may be best investigated by model experiments; the rules relating to scale models, and the laws of corresponding speed and similarity, are dealt with later in the work.[1]

§ 47. The Influence of a Periodic Disturbance.—Wind fluctuations are sometimes met with having a definite periodicity. There is perhaps no evidence that wind fluctuation in general is periodic in character; on the contrary it may frequently consist of an irregular turbulent motion. In certain cases, however, especially in the vicinity of deep sea waves, there is unquestionably at times a definite *period* both in the frequency and in the form of the fluctuations.

The condition of an aerodone in the presence of regular aerial waves or pulsations is closely analogous to that of a vessel rolling in a seaway; so long as the two periods, that of the vessel or aerodone and that of the wave motion, do not approximately synchronise, the conditions call for no special comment; once, however, synchrony is established, the stability of the aerodone will be in jeopardy.

In the case of the rolling ship we have a damping factor in the skin friction and other resistances the hull offers to rolling motion to prevent the oscillation from becoming excessive. It is evident that we must look for some equivalent means of exercising a similar influence on the aerodone in its phugoid path.

The analogy between the motions of an aerodone and the

[1] §§ 126, 127, *et seq.*

rolling of a ship is strictly limited to the sense of its application. In the case of the aerodone the oscillatory disturbance is neither pitching nor rolling in the nautical sense, both these being dynamically rotational in character. The phugoid oscillation cannot be strictly analogous to either of these since the aerodone as at present defined has no moment of inertia.

The form of the phugoid oscillation for small amplitude consists, as has been shown, in motion in an elliptical orbit. This motion is dynamically the same as could be produced if the aerodone were suspended in space by a spring system having equal elasticity in all directions; as such the motion in question is one form of a possible infinite number; it is evident that in such a system anything in the nature of a series of impulses planted periodically, with about the required frequency, may eventuate in the amplitude exceeding the limit permissible. This dynamic *picture* of the condition of the aerodone, which is founded on the phugoid theory, gives the most vivid impression of the susceptibility of an aerodone in flight to vibrational disturbance, that is, to a decrease in the value of the phugoid constant. The problem of meeting disturbances having a synchronising period, resolves itself into finding a means of applying a "dash-pot" to damp out the oscillations as fast as they are generated.

§ 48. Unaccounted Factors in relation to the Flight Path.—There are certain factors excluded by the hypothesis of Chapter II. which have a very far-reaching influence on the behaviour of an aerodone in flight. Referring to the conditions laid down in §19, we have firstly the exclusion of all motions relating to lateral stability; this we will allow to stand. We have secondly the supposition that the aerodone loses no energy, and thirdly that it possesses no moment of inertia about a transverse axis; these two conditions involve the "unaccounted factors" of which it is now necessary that we should take stock.

It has been set forth in the statement of hypothesis that

condition (2) may be supposed complied with in two distinct ways, namely, either by supposing the air as frictionless and the supporting wave as perfectly conserved, or by supposing a force of propulsion in the direction of motion precisely equal, at every instant of time, to the resistance experienced by the aerodone in flight.

It has further been shown [1] that the law of the phugoid path, so far as weight supported and resistance are concerned, is that all variations are as the velocity squared, that is to say, both the aerodynamic resistance and the frictional resistance vary as V^2, the latter to the usual degree of approximation. It is evident that the influence of resistance is indissolubly associated with the question of propulsion, for if we could supply propelling mechanism whose thrust varied as the velocity squared the conditions of hypothesis in this respect would be complied with. When we have finally to consider the full aerodromic problem of the flying machine, we shall have occasion to discuss the idiosyncrasies of various means and machinery of propulsion, but for the purposes of aerodonetics pure and simple, we are concerned essentially with the behaviour of the gliding model, in which the propulsion is supplied as a component of gravity and is therefore to be regarded as *constant*; if we supply a constant horizontal force acting on the aerodone, this force and that of gravity will give a resultant invariable both in magnitude and direction.

It is evident, however, that the gravity component is not the exact equivalent of a constant force of propulsion, for, from the above argument, the component of gravity must be regarded as acting in the constant direction of the mean gliding path, whereas a force of propulsion must act at every instant in the changing direction of the line of flight. In the further development of the subject, owing to the extreme difficulty of the problem in its entirety, the flight path is assumed to be of small amplitude, so that the distinction under discussion becomes unnecessary, the difference being negligible.

[1] §§ 20, 110, and 111; also "Aerial Flight," Vol. I., *Aerodynamics*, § 159.

§ 49. Unaccounted Factors (continued).—It is shown in the subsequent chapter that, under the conditions of natural gliding or uniform propulsion, the effect of resistance is to exercise a damping influence on the phugoid oscillations, tending to a continued increase in the value of C, and, unless adverse influences are at work, the flight path continually approximates more and more closely to the straight line or path of uniform gliding. This result is one that might well be anticipated from the action of the supposititious dynamic model of § 47. If we imagine a constant force applied to the aerodone parallel to the minor axis of its orbit in one direction, the supposed direction of flight, and a variable force in the opposite direction whose value is greater when opposed to the orbit motion, and less when in its favour; then it is evident that the mass in oscillation will do work in respect of the variable applied force, and therefore its amplitude will diminish.

The effect of taking into account the moment of inertia of the aerodone about a transverse axis is also dealt with in the chapter that follows, which constitutes an extension of the phugoid theory, carrying it to a point at which it becomes of the greatest practical utility. The influence of moment of inertia is the reverse to that of resistance, for it tends to an increase in the amplitude of the flight path oscillation. The result of this further investigation is quantitative and culminates in an equation which correlates every factor and constant essentially involved in the flight of a real material aerodone for phugoids of small amplitude.

CHAPTER V

STABILITY OF THE FLIGHT PATH AS AFFECTED BY RESISTANCE AND MOMENT OF INERTIA

§ 50. **Introductory.**—In the present chapter we shall discuss the influence of departures from the original hypothesis in respect of *resistance* and *moment of inertia*, both of which were initially assumed to be absent.

It has not been found possible to achieve any results by including these quantities in the original investigation of Chapter II.; the form of expression is far too unwieldy, and the integration and plotting become impossible.

It is consequently necessary to deal with the matter as a kind of correction, and to confine ourselves more especially to the investigation from a less general point of view, especially directing our attention to the *practical* aspect of the problem, as concerned in flight under normal conditions.

In the particular case of the uniform straight line phugoid, supposing a constant propelling force, or that which amounts to the same thing, an appropriately inclined path,[1] it is evident that the conditions are those of equilibrium, for the resistance is at every instant balanced by a propulsive force, and since there is no rotary movement of the aerodone about a transverse axis, the existence or otherwise of moment of inertia can make no difference. Although the condition is one of equilibrium, it does not follow that the path of flight is stable, it is possible that it may be a condition of unstable equilibrium so that on the least

[1] A simple resolution of forces proves the identity of these alternative suppositions, except so far that the applied propelling force results in the total reaction being slightly greater than the weight.

disturbance taking place, the resulting phugoid tends to a continual increase of amplitude.

The conclusions of the present chapter have been to some extent anticipated by the statements made in §§ 48 and 49; in short, it will be shown, firstly, that the effect of resistance is to damp out oscillations, whereas that of moment of inertia is the converse; and, secondly, it will be shown that for small amplitude these two influences may be pitted one against the other so as to leave the amplitude of the phugoid path unaffected. This condition marks the limit of stability of the gliding flight path.

§ 51. Influence of Resistance on Amplitude.—It is, as before, assumed that the total resistance varies as the square of the velocity of flight, and that the propelling force is constant.

Let R be total resistance.

" V, V_n, H and H_n stand as before.

" Q = uniform applied thrust.

" $h = H_n - H$ = height of aerodone above path of mean gliding.

" $q = Q - R$ = difference of thrust and resistance representing net force of propulsion acting on aerodone.

Then—

$$R/V^2 = Q/V_n^2$$

or

$$R/H = Q/H_n$$

$$\therefore Q - R \propto H_n - H$$

or

$$q \propto h.$$

Referring to Fig. 45, we have the path of mean gliding $p\ p$, and the phugoid path $p_1\ p_1$, separated by a varying distance h to which we have shown q proportional. Now, when q is positive as when the aerodone is "cresting" (in the vicinity of the crest of its wave-like path), it is receiving energy, and its H value is consequently increasing; this is equivalent to the aerodone being

Fig. 45.

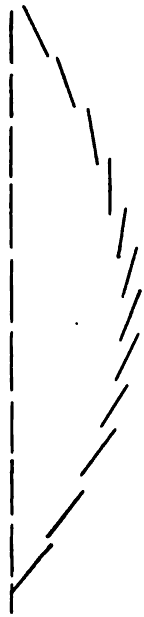


Fig. 47.

Fig. 46.

transferred downward on the phugoid chart as it progresses (Fig. 46). On the other hand, when the aerodone is "troughing" (in the trough of a flight path undulation), it is losing energy, for q is negative, and consequently the value of H is decreasing; this is equivalent to the aerodone being transferred upward on the phugoid chart from one curve to another as illustrated by Fig. 47.

There is, of course, no discontinuity of path such as that shown in the figures; the correct procedure would be to conceive the phugoid chart to move in space, and that the air and aerodone retain their existing relationship. Thus, in Fig. 48, the variation in H is provided for by an undulating datum line: this

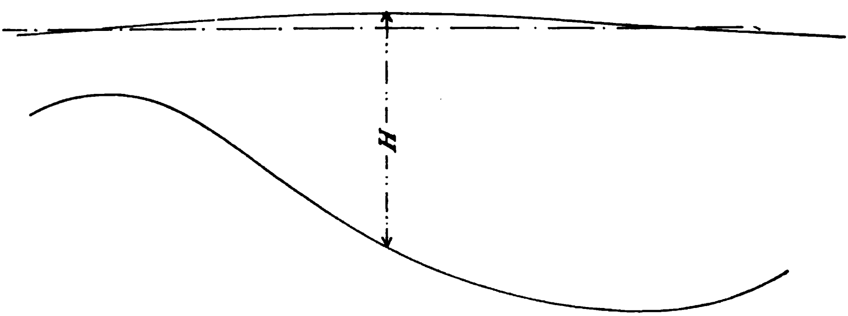


FIG. 48.

represents the locus of a datum point laid off from instant to instant.

Referring now to Figs. 46 and 47, we see that the transference that takes place in the position of the aerodone on the phugoid chart, due to the action of the force q, has the effect, both during "cresting" and "troughing," of diminishing the amplitude of the phugoid path. This is particularly clear at the points of phase and mid-phase where $\Theta = 0$; any transference at either of these points in the direction indicated gives rise to an obvious diminution of amplitude that, figuratively speaking, could be measured with a foot rule.

We thus see that the effect of resistance is to damp out or

diminish the amplitude of the phugoid undulation, and so results in an increase in the value of **C**. Before following the matter further, we will pass on to a preliminary survey of the influence of moment of inertia.

§ 52. Influence of Moment of Inertia.—An aerodone in flight pitches through an angle that increases with any increase of the path amplitude. If the angle at the point of inflection be, as before, represented by the symbol Θ_i, then the total angle ranged by the aerodone is $2\ \Theta_i$. Knowing the law of the change of angle in respect of time, the total angular amplitude ($= 2\ \Theta_i$), the time period—that is, the phase time of the phugoid—and the moment of inertia of the aerodone, we could calculate the resulting torque or couple for all values of Θ.

Now this couple is supplied by some reaction received from the air on the supporting and directive surfaces; for it is a couple applied to the aerodone from without, and the material atmosphere is the only external agent which we are prepared to recognise.

Further, the organ of the aerodone whose duty it is to preserve the *functional aspect* or *attitude* of the aerodone to its line of flight, is the tail, and since it is the preservation of the functional aspect that necessitates the angular oscillations, we may look to the tail to provide one of the forces by which the couple is constituted. In other words, as the aerodone "pitches" in its phugoid path, the tail plane supplies one of the varying forces necessary to impart to, and take away from, the aerodone, the rotational motion involved.

Now this varying force on the tail plane involves that its upper and under surfaces are alternately under compression and rarefaction, which means that the tail plane moves obliquely to the line of flight, with an obliquity and sense varying with the magnitude and direction of the torque.

Moreover, since this force on the tail must be sustained by some equal and opposite reaction, there will be an equal and

oppositely varying force on the aerofoil itself.[1] It is these two forces that constitute the couple.

§ 53. Influence of Moment of Inertia (continued).—In the phugoid theory it arises from the initial hypothesis that there is absolute constancy of functional aspect or *attitude*, and the condition that the reaction varies strictly as V^2, that is the application of the V^2 law, rests on this fact.

We now find that when an aerodone possesses moment of inertia, such perfect constancy does not exist. Now to vary the

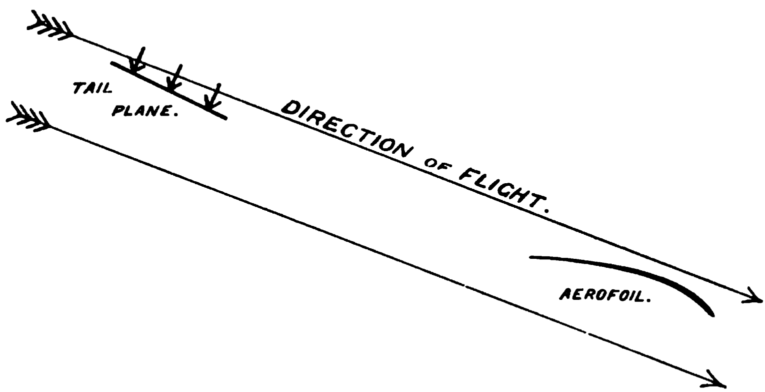


FIG. 49.

functional aspect of the supporting member of an aerodone (the aerofoil) means varying the fundamental constant, the n of the differential equation (8), § 20, or the H_n of equation (11); for any change of functional aspect means in effect a variation of the aerodynamic angle β, and our knowledge of aerodynamics tells us that this means an increase or reduction of the supporting power of the aerofoil for any given speed.

Now a variation in the value of H_n means an *alteration of*

[1] This does not mean that the aerofoil is at any time sustaining a negative (*i.e.*, downward) reaction, but rather that the upward reaction undergoes variation as due to the added downward or upward pressure consequent on the oscillations that are taking place.

scale of the phugoid chart; it therefore remains for us to determine in what way such an alteration of scale will operate. We have seen that the swing of the aerodone terminates at each point of inflection, that is to say, at a point of inflection the aerodone has no rotational motion, the condition being that of greatest angular displacement. Consequently, it is at or about such points that the torque has its greatest value, and the value of H_n is at one of its extremes.

Let us examine the condition on the downward inflection; at this point the inertia torque is clockwise (the direction of flight being from left to right), and the equal and opposite *applied* torque is counter-clock. This signifies that the tail of the aerodone is moving obliquely as in Fig. 49, and the effective angle β is less than its normal value, and therefore H_n is *increased*. Beyond this the pressure developed on the upper surface, and negative pressure on the under surface, of the tail, the two constituting the tail plane reaction, is experienced by the aerofoil as additional load; this being the equal and opposite force constituting the couple. There is thus a further cause tending to an increase in the value of H_n.

Likewise it can be shown that in the region of the upward inflection H_n will be diminished.

Consequently, we find that during *troughing* the value of H_n is *diminishing*, that is to say, *the phugoid chart is contracting;* whilst on the other hand, during *cresting*, the value of H is *increasing*, and the *phugoid chart is expanding.*

§ 54. Influence of Moment of Inertia (continued).—Thus we find that whereas *resistance* takes effect on the value of the variable H, the influence of *moment of inertia* manifests itself by a change in the magnitude of the constant H_n. Moreover, the changes due to moment of inertia have no influence whatever on the value of H for the time being of the aerodone, since there is no energy being given or taken away, so that the conditions introduced by

moment of inertia are that H remains unaffected in the face of changes in H_n, that is, changes of scale of the phugoid chart; therefore if the chart *contracts* during troughing, as has been proved to be the case, the amplitude is *increasing*; likewise when the chart *expands* during cresting the amplitude is again *increased*. Thus the influence of the moment of inertia of the aerodone about a transverse axis, if not counteracted in some way, is to render the gliding flight path unstable, any disturbance results in the aerodone describing a phugoid path of continually increasing amplitude that must sooner or later bring the flight path into the danger zone when loss of equilibrium must occur.

The word *amplitude* as used in the preceding section does not of necessity mean the amplitude as absolute linear quantity, although as such the argument applies equally, but rather the *relative amplitude*, that is, in relation to the scale of the phugoid chart. There is a relationship between the relative amplitude and the absolute amplitude, analogous to that between the constants K and C, comp. § 37.

When we have to deal with changes in the value of H, as where we are concerned with the effects of resistance, it is unimportant whether we regard the changes in amplitude as involving changes in C or K, these two constants being in constant proportion. When, however, we are concerned with changes in H_n, and therefore in the scale of the phugoid chart, there is an essential difference, for C is of the dimensions $\sqrt{\mathrm{L}}$ and its value is affected by the change of scale as well as by the alteration of amplitude. It is therefore necessary to renounce the employment of C in favour of K, when the effects of moment of inertia are under discussion.

The results of the preliminary investigation may be presented as follows. We have ascertained that resistance and moment of inertia have opposite effects on the value of the amplitude constant K, and whereas resistance has a benign influence involving an increase of K and a damping down of the oscillation, the effect of moment of inertia, on the contrary, is detrimental, and if

unchecked results in a decrease of K and a progress towards instability.

The experimental demonstration of these results will be given in a later chapter.

§ 55. **The Quantitative Problem.**—The quantitative aspect of the present branch of the subject is one of considerable difficulty. It would appear possible to deal with the influence of resistance or of moment of inertia, either separately or together, by plotting the curve step by step, making the corrections due to these two disturbing causes stage by stage, and so approximating as closely as found practicable to the continuous variation that in reality takes place. It is improbable, however, that the labour involved in such a process would be justified by the results to be obtained, unless perhaps, for the purpose of comparison with curves obtained photographically, as a master check on the whole theory.

If we take the special case of the nearly straight flight path, that is, with the amplitude bordering on the evanescent, and suppose firstly that there is no resistance to flight, but that the aerodone possess moment of inertia, then from the foregoing sections the amplitude will gradually increase, that is, the constant K will diminish. If now, when the amplitude has reached some definite value, we suppose the aerodone deprived of its moment of inertia, and that it experience resistance to flight,[1] the amplitude will forthwith begin to diminish, that is, K will increase, and we may suppose this to go on until the original condition is restored.

Let us now suppose the aerodone to possess moment of inertia, and also to experience resistance to flight, then it is evident that the proportions may be such that K will diminish from the one cause faster than it will increase from the other, in which case the conditions tend to instability, or the reverse may be the case when the gliding flight path will become definitely stable.

[1] Following the V^2 law.

The investigation that follows has for its object to ascertain the circumstances under which the one effect will exactly balance the other, so that we shall be able to determine when the path is on the point of becoming unstable, and specify in such a form as will permit of the theory being checked by means of models of the critical proportion.

§ 56. The Form of the Nearly Straight Phugoid.—It has been shown in Chap. III., § 30, that the form of the phugoid of small amplitude is a curve of trochoidal type, it differing from a true trochoid only in respect of its elliptical $1 : \sqrt{2}$ orbit instead of the circular orbit of the regular curve. This curve of trochoidal type may be regarded as taking a position intermediate between a true trochoid and a sine curve; it is the projection of a trochoid on a plane at 45°.

It is a property of the trochoid, and one may say of this whole family of trochoid-like curves, that as the amplitude diminishes the distinction between one and another or between a trochoid and a sine curve tends to vanish. Thus referring to Fig. 50, in which the curve $p\ p$ is a trochoid, and $p_1\ p_1$ is a sine curve of the same amplitude and phase length, the maximum horizontal distance separating the two curves $A\ B$ is equal to half the radius of the trochoidal orbit, and if Θ_1 be the path inclination at this point we have $\frac{A\ C}{A\ B} = tan\ \Theta_1$. Let h_1 be the maximum displacement, or half amplitude of the curve, then $h_1 = A\ B$, but Θ_1 varies directly as the half amplitude h_1, or for small values of Θ_1, $tan\ \Theta_1 \propto h_1$, hence

$$\frac{A\ C}{A\ B} \propto h_1, \text{ or } A\ C \propto h_1^2.$$

Consequently when h becomes small $A\ C$ becomes negligible.

In the case of the actual phugoid approximation we can arrive at the same result from another point of view.

It has been shown, § 29, that for phugoids of very small

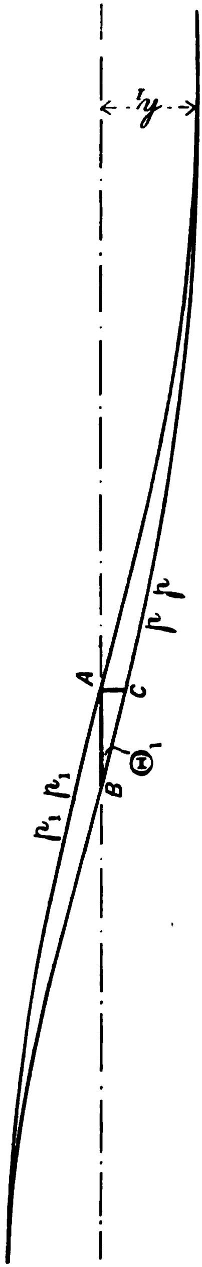


FIG. 50.

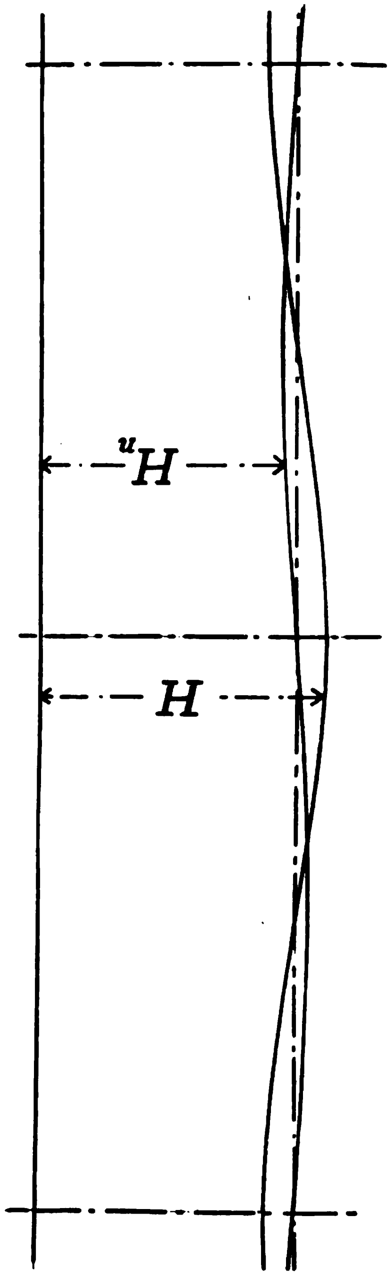


FIG. 51.

amplitude the vertical component of the motion of the aerodone is harmonic ; consequently if the rise and fall of the aerodone be plotted *against time*, that is with time as abscissæ, the result is a sine curve. The reason that the actual path is not a sine curve is that abscissæ = distance.

If the velocity were uniform instead of fluctuating then the curve of h by L would be the same as the curve h by T, and the velocity becomes more and more nearly uniform as the amplitude is diminished. Therefore, for amplitudes of very small magnitude the h by L and h by T curves may be taken as identical.

Hence for phugoids bordering on the limiting case of the straight line, we may regard the flight path as a sine curve.

§ 57. On the Form of the Change in H due to Resistance.—The assumption involved when dealing with the influence of resistance is that the aerodone is propelled by a uniform force, and that the net resistance is due to the differences that arise from point to point in its path between this uniform force and a gross resistance proportional to V^2.

It has been shown (§ 51) that the net resistance under these conditions is proportional to the quantity h, measured positive in a downward direction.[1]

Now the value of H is continually undergoing variation, in accordance with the phugoid equation. Any changes due to resistance may be looked upon as superposed on the equational changes.

Resistance changes may be conceived to take effect on the value of H *at the opposite end of the ordinate*, so to speak, to the equational changes, that is to say, we may imagine the said changes to be provided for by a vertical movement of the phugoid chart.

The simplest conception is to suppose the datum line to

[1] This is the same thing as taking the *net propulsive force* proportional to h measured positive *upward*.

undergo vertical displacement, and, as in § 51, to abandon the datum line in favour of a datum point (Fig. 48), this datum point being supposed to travel with the ordinate H, marking its zero from instant to instant. The ordinate H will then be measured *by the amount cut off between two curves*, the upper, or datum, curve giving the variations due to resistance, and the lower, or flight path curve, variations due to the phugoid equation.

Let h_1 be the departure of the datum point from the axis of x, due to the resistance, measured positive in a downward direction.

Let L be the linear measure of the flight path. Owing to the assumption of small amplitude L is also the linear measure along the horizontal axis.

Then $W L \frac{dh_1}{dL}$ is the rate at which the aerodone (of weight $= W$) is doing work against resistance, as measured by its loss of "head." And the work done against resistance is proportional to the net resistance $\times$ L, that is, $h\ L$ where h is the ordinate of the flight path as in Fig. 45 (§ 51). Or, $\frac{dh_1}{dL} \propto h$.

That is to say, the tangent of the angle of the datum curve is proportional to the ordinate of the flight path. But the latter is a sine curve. Therefore the datum curve is a sine curve differing by one quarter-phase.[1]

§ 58. On the Form of the Change in H due to Resistance (continued).—There is a point of some subtlety to which it is necessary to devote attention. The relationship shown in the preceding section between the flight path curve and datum curve, if postulated as correct at any stated point, will become less and less accurate as time goes on, owing to the reaction of the one curve on the other. Thus we know that the displacement of the datum point results in a reduction in the amplitude

[1] The well-known result of the differentiation of a sine curve.

of the flight path, whereas we have assumed the flight path amplitude to be unaffected. Given, however, that the relative positions of the aerodone, flight direction, and datum point, are correct at any particular instant, then the datum curve for that instant is correctly indicated by the sine curve, as set forth.

At the present stage it is perhaps desirable to make clear the reason for allowing a result so transparently slipshod to pass. Let us suppose that we have at command means of some kind by which the value of H can be varied in the reverse manner to that just demonstrated, and suppose that, laying down a sine curve flight path, we are able to prove that our new means of acting on the value of H acts on the datum point in just the reverse way to that in which resistance acts, that is, by causing a harmonic variation of opposite phase, then we can so adjust the relative magnitude of the two disturbing causes as to have no effect on the datum point whatever.

Under these conditions it is of not the least consequence whether or no either of the individual disturbances is liable to an accumulated error, for this error can only arise when the flight path is disturbed by datum variations, and if the two disturbing causes act simultaneously no datum variation will arise. Thus, if we examine any small increment of the flight path, since no change takes place in the datum level while the aerodone traverses that increment, the relation of flight path to datum will hold good at the end of the increment as if neither disturbing cause existed, and the same will apply to the next increment and so on indefinitely.

In the present investigation, the two disturbing causes do not act in precisely the manner set forth above, but the same considerations apply.

§ 59. On the Changes in the Magnitude of H_n due to Moment of Inertia.—The present problem resolves itself into two portions; firstly, we have to establish the relationship between a torque or couple acting on the aerodone, and the resulting change in the

value of H_n; secondly, we have to investigate the nature of the changes in the torque that arise from the rotational inertia of the aerodone in flight.

We know that the forces that constitute the components of the couple consist of a reaction on the tail plane on the one hand, and a reaction added to the load on the aerofoil on the other. We consequently require to employ aerodynamic data, the expression $W \propto V^2 \beta$ being for the moment all that is necessary.[1]

Since $V^2 \propto H_n$ we have, $W \propto H_n \beta$ or $H_n \propto \frac{W}{\beta}$ consequently,

$$dH_n = dW \frac{H_n}{W} - d\beta \frac{H_n}{\beta},$$

or approximately for small increments,

$$\Delta H_n = \Delta W \frac{H_n}{W} - \Delta\beta \frac{H_n}{\beta}.$$

But ΔW is the change in the load on the aerofoil due to the torque, and is therefore proportional to the torque.

And $\Delta \beta$, the change in the angle β, is the angle made by the tail plane to its line of flight; this is proportional to its pressure reaction, and therefore proportional to the torque.

Consequently,

$$\Delta H_n \propto \tau \left(\frac{H_n}{W} - \frac{H_n}{\beta}\right),$$

and for small variations the latter term is sensibly constant, hence,

$$\Delta H_n \propto \tau,$$

thus the change in the magnitude of H_n is proportional to the torque which produces it.

Now as to the variations of the torque as related to the flight path. Since we are assuming that the flight path consists of a sine curve, the variations of *tan* Θ will be harmonic. But the values of Θ concerned are well within the definition of a *small angle*, hence the variation of Θ will also be harmonic. Now for

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 159.

a harmonic angular motion we know that the torque at every instant is proportional to the angle, therefore the torque τ and the variation of H_n are harmonic.

Since the variations of H_n from the above reasoning follow *tan* Θ, the curve of variation of H_n is a differentiation of the flight path, and therefore differs from it by one quarter phase (Fig. 51).

§ 60. Statement of Case.—It has now been demonstrated that the two disturbing causes under discussion, *resistance* and *moment of inertia*, result in variations of H on the one hand, and H_n on the other; and, further, that for small amplitude the variations of both quantities are of the same form, *i.e.*, *harmonic*. The object of the further investigation is to show in what manner the two kinds of variation can be made to cancel out.

Now the quantities H and H_n are not directly related to one another in the phugoid equation in such a way as to render their interdependence in the present connection quite obvious; in fact, in the equation, where H is a variable, H_n is a constant.

We cannot destroy a variation of H_n by any conceivable variation in H, or *vice versâ;* neither can we eliminate the effect, for any variation of H_n results definitely in a change of scale of the phugoid chart, a matter in which variations of H have no influence whatever.

The key to the position is found in the fact that if H and H_n be made to vary simultaneously in like ratio, the amplitude constant K undergoes no change, the flight path being represented as a single phugoid curve, from which the aerodone never deviates, *but whose scale undergoes a regular periodic change.*

Now, since we are dealing with curves of very small amplitude, H is sensibly equal to H_n, and we can express the condition of $\mathsf{K} =$ constant in the form,

$$\frac{dH}{dH_n} = 1.$$

And the variation of both quantities is harmonic in form,

and in effect alike in phase, hence if the conditions are fulfilled at any one point in the curve, they will be fulfilled for all points.

§ 61. The Equation of Stability.—Employing British absolute units,

Let W = weight of aerodone (poundals).
,, V = velocity of flight (ft./sec.).
,, K = aerofoil constant ($W = K\,V^2$).
,, $\mathbf{I}$ = moment of inertia of aerodone about transverse axis (lbs. × ft.^{2}).
,, a = tail area (sq. ft.).
,, C and c be aerodynamic constants for tail plane, as defined in Vol. I., § 177.
,, β = aerofoil trail angle (as in Vol. I.).
,, γ = natural gliding angle (radians).
,, Θ_1 = maximum path angle (radians).
,, l = length of tail, *i.e.*, from centre of pressure of aerofoil to that of tail plane (ft.).
,, τ = turning moment about a transverse axis (poundals × ft.).
,, τ_1 = maximum turning moment about a transverse axis (poundals × ft.).
,, L = distance in the line of flight.
,, L_1 = complete phase length (ft.).
,, t_1 = complete phase time (secs.).
,, ρ = density of air (lbs. per cu. ft.).
,, H, H_n, h, etc., stand as before.

Let us deal firstly with the effect of resistance, and express the *maximum rate* of variation of H in terms of flight path as $= \frac{dH}{dL}$; the quantity thus expressed being thus defined as the rate of variation at a particular point on the flight path.

We must first relate Θ_1 to the path amplitude. Let h_1 be the maximum value of h, then on the harmonic basis,

$$h_1 = \frac{tan\ \Theta_1\ L_1}{2\ \pi}.$$

But when Θ_1 is small, tan $\Theta_1 = \Theta_1$,

or, $$h_1 = \frac{\Theta_1\ L_1}{2\ \pi}. \qquad (1)$$

Now the rate of loss of "head" per foot of horizontal flight path at V_n velocity, *on the basis of no propulsive force* is $= tan\ \gamma$; or conversely, the steady rate in terms of L at which an increase of H must be provided by a propulsive force, in order that it should remain constant $= tan\ \gamma$. And we know that the loss of H per unit flight path varies as H itself; therefore $\frac{H_n + h_1}{H_n} \times tan\ \gamma$ represents the loss of head per foot if no propulsive force is applied; hence with uniform propulsion we have,

$$\frac{dH}{dL} = \frac{H_n + h_1}{H_n}\, tan\ \gamma - tan\ \gamma,$$

or $$\frac{dH}{dL} = \frac{h_1}{H_n}\, tan\ \gamma.$$

Substituting (1) for h_1 the expression becomes,

$$\frac{dH}{dL} = \frac{\Theta_1\ L_1}{2\pi\ H_n}\, tan\ \gamma.$$

But by § 36 $\quad L_1 = 2\sqrt{2}\,\pi\ H_n$,

$$\therefore \qquad \frac{dH}{dL} = \sqrt{2}\ \Theta_1\ tan\ \gamma. \qquad (2)$$

§ 62. The Equation of Stability. The Investigation continued. We have next to deal with variations of H_n, due to moment of inertia.

Let us denote the increment of variation of H_n by the symbol δH_n, and in accordance with § 59 let us regard it as consisting of two parts $\delta_1 H_n$ due to the change of *attitude* of the aerofoil,

and $\delta_2 H_n$ due to the change of effective load on the aerofoil. Let ΔH_n, $\Delta_1 H_n$, and $\Delta_2 H_n$ be the maximum values of these quantities respectively. Likewise let $\delta\beta$ be the increment of variation of the angle of trail of the aerofoil, (β), of which the maximum value is $\Delta\beta$; $\Delta\beta$ therefore is also the angle of inclination of the tail plane with respect to the incident stream.

From aerodynamic considerations we may write, pressure reaction on tail plane

$$= C \rho a V_n^2 \times c \, \delta\beta$$

and torque corresponding to this reaction,

$$\tau = C \rho a V_n^2 c \, \delta\beta \, l$$

or,
$$\delta\beta = \frac{\tau}{C \rho a V_n^2 c l}$$

but
$$\frac{\delta\beta}{\beta} = -\frac{\delta_1 H_n}{H_n}$$

$\therefore$
$$-\delta_1 H_n = \frac{H_n \tau}{C \rho a V_n^2 c l \beta}$$

or for maximum value,

$$-\Delta_1 H_n = \frac{H_n \tau_1}{C \rho a V_n^2 c l \beta}. \tag{3}$$

Now as to $\delta_2 H_n$, *i.e.* the variation due to change of the effective load on aerofoil, due to component of couple, we have

$$\tau = l \, \delta W,$$

and since $H_n \propto W$,
$$\frac{\delta_2 H_n}{H_n} = \frac{\delta W}{W},$$

$\therefore$
$$\tau = \frac{l W \delta_2 H_n}{H_n}.$$

Let K represent the aerofoil constant, so that $W = K V_n^2$, then

substituting
$$\tau = \frac{l K V_n^2 \delta_2 H_n}{H_n},$$

or,
$$\delta_2 H_n = \frac{H_n \tau}{l K V_n^2}.$$

Writing this for maximum value we have

$$\Delta_2 H_n = \frac{H_n\,\tau_1}{l\,K\,V_n^{\,2}}. \tag{4}$$

Now,

$$\Delta H_n = \Delta_1 H_n + \Delta_2 H_n = \frac{H_n\,\tau_1}{C\,\rho\,a\,V_n^{\,2}\,c\,l\,\beta} + \frac{H_n\,\tau_1}{K V_n^{\,2}\,l},$$

or,

$$\Delta H_n = \frac{H_n\,\tau_1}{V_n^{\,2}\,l}\left(\frac{1}{C\,\rho\,a\,c\,\beta} + \frac{1}{K}\right), \tag{5}$$

and this variation takes place in one quarter-phase length of the flight path.

Now on the harmonic basis, maximum rate of change, *i.e.*, $\frac{dH_n}{dL}$ is $\frac{\pi}{2}$ times the mean rate; hence

$$\frac{dH_n}{dL} = \frac{\pi}{2} \times \frac{4\,\Delta\,H_n}{L_1},$$

but $$L_1 = 2\sqrt{2}\,\pi H_n.$$

Substituting (5) for $\Delta\,H_n$ and for L_1, we have

$$\frac{dH_n}{dL} = \frac{\pi}{2} \times \frac{4\,H_n\,\tau_1}{2\sqrt{2}\,\pi\,H^n\,V_n^{\,2}\,l}\left(\frac{1}{K} + \frac{1}{C\,\rho\,a\,c\,\beta}\right)$$

$$= \frac{\tau_1}{\sqrt{2}\,V_n^{\,2}\,l}\left(\frac{1}{K} + \frac{1}{C\,\rho\,a\,c\,\beta}\right). \tag{6}$$

§ 63. The Equation of Stability. The Investigation concluded.— The next step is to express the value of the maximum torque τ_1 in terms of the moment of inertia **I** and the angular amplitude of the flight path.

The magnitude of τ_1 is the maximum moment of the couple generated in causing the aerodone to oscillate harmonically about its transverse axis through a total angle equal to twice the maximum inclination of its path.

Now the total angle of oscillation is $2\,\Theta_1$, hence τ_1 will be given by the expression,

$$\tau_1 = \frac{2\,\mathbf{I}\,\pi^2}{t_1^{\,2}} \times 2\,\Theta_1 = \frac{4\,\Theta_1\,\mathbf{I}\,\pi^2}{t_1^{\,2}}.$$

Substituting in (6) we have

$$\frac{dH_n}{dL} = \frac{4\,\Theta_1\,\mathbf{I}\,\pi^2}{\sqrt{2}\,l\,V_n^2\,t_1^2} \times \left(\frac{1}{K} + \frac{1}{C\,\rho\,a\,c\,\beta}\right). \quad . \ (7)$$

And the conditions of stability are defined by the equation

$$\frac{dH}{dL} > \frac{dH_n}{dL},$$

$\therefore$ by (2) and (7),

$$\sqrt{2}\,\Theta_1\,tan\,\gamma > \frac{4\,\Theta_1\,\mathbf{I}\,\pi^2}{\sqrt{2}\,l\,V_n^2\,t_1^2}\left(\frac{1}{K} + \frac{1}{C\,\rho\,a\,c\,\beta}\right),$$

or

$$tan\,\gamma > \frac{2\,\mathbf{I}\,\pi^2}{l\,V_n^2\,t_1^2}\left(\frac{1}{K} + \frac{1}{C\,\rho\,a\,c\,\beta}\right).$$

But $V_n = \frac{L_1}{t_1}$, $\therefore$ by § 36, $V_n^2\,t_1^2 = 8\,\pi^2\,H_n^2$.

Hence we may write the expression—

$$\Phi = \frac{4\,l\,H_n^2\,tan\,\gamma}{\mathbf{I}\left(\frac{1}{K} + \frac{1}{c\,C\,\rho\,a\,\beta}\right)} > 1,$$

which is the equation of stablity, Φ being termed the coefficient of stability, which, as the equation shows, must be greater than unity.

There is one factor that has not been taken into account in the foregoing analysis—the influence of the "wash" of the aerofoil on the angle of the tail plane. It is evident that since the aerofoil communicates downward motion to the air coming within its *sweep*, the tail plane will be situated in a down current, and consequently its attitude of zero pressure reaction will be inclined to the line of flight. The extent of this inclination will depend upon the distance of the tail plane astern of the aerofoil. If the two are in close proximity the inclination of the tail for zero reaction will be but little less than β itself. If, on the other hand, the tail plane be arranged far astern it will only require to

accommodate the residuary down current of the peripteral system, which will be given by the expression

$$\beta \times \frac{\beta - a}{\beta},$$

which may be otherwise expressed as $\beta\ (1 - \epsilon)$ where ϵ is the constant ratio $\frac{a}{\beta}$ proper to the aerofoil.[1]

Now so long as there is no change in the value of β the present considerations have no influence on the problem, for the tail plane is initially designed to occupy its position of zero pressure reaction, and there the matter ends. When, however, any change takes place in β, as due to the moment of inertia, this reacts on the effective tail angle, so that if $\delta\beta$ be the initial variation, the relative direction of the *tail current* or "wash" will change through an angle $\delta\beta\ (1 - \epsilon)$, and the consequent displacement of the tail will be through an equal angle. This will result in a further increment $\delta\beta\ (1 - \epsilon)$ being added to the effective value of β. This in turn will give rise to a further change in the tail angle $= \delta\beta\ (1 - \epsilon)^2$, and so on, the total resulting addition to the increment $\delta\beta$ being the sum of a series

$$\delta\beta\ (1 - \epsilon) + \delta\beta\ (1 - \epsilon)^2 + \delta\beta\ (1 - \epsilon)^3, \text{ etc.},$$

which is given by the expression,

$$\delta\beta\, \frac{1 - \epsilon}{\epsilon},$$

consequently the increment as given in the foregoing analysis requires to be multiplied by a factor $1 + \frac{1 - \epsilon}{\epsilon}$, that is $\times\ \frac{1}{\epsilon}$, in order to correct for the influence of "wash."

On applying this correction, equation (3) of § 62 becomes,

$$\Delta_1 H_n = \frac{H_n\, \tau_1}{C \rho\, a\, V_n^2\, c\, l\, \beta} \times \frac{1}{\epsilon},$$

[1] "Aerial Flight," Vol. I., *Aerodynamics*, Ch. VIII.

and, carrying the correction through, we reach finally,

$$\Phi = \frac{4\, l\, H_n^2 \tan \gamma}{\mathrm{I}\left(\frac{1}{K} + \frac{1}{c\, C\, \rho\, \epsilon\, a\, \beta}\right)} > 1$$

which is the *complete equation of stability.* It will be noted that the correction has been based on the supposition that the tail is *long,* that is to say that the tail plane is well out of the immediate influence of the aerofoil, and is only affected by the residual peripteral motion.

CHAPTER VI

EXPERIMENTAL EVIDENCE AND VERIFICATION OF THE PHUGOID THEORY

§ 64. Preliminary. The Importance of the Experimental Verification.—The experimental verification of the conclusions of the preceding chapters is evidently a matter of very great moment, for the results obtained from theoretical considerations, if experimentally established, form a definite proof, not only of the aerodonetic portion of the work, but also of the aerodynamic fabric in which the phugoid theory has its foundation.

The checks that have so far been applied to the theory are of various kinds. In the first place there is the general qualitative resemblance of the curves of flight, both as determined and illustrated by other workers and by the author himself, to the curves as plotted from the phugoid equation. Secondly, there is the direct measurement of the phase length, phase time, and velocity relationship as a check on the results given in § 36. Thirdly, there is the experimental determination of the condition of neutral stability for the phugoid of small amplitude, as a check on the investigation given in Chapter V., and the demonstration of the conditions of flight path stability and instability by means of model experiments. Lastly, there is the post-mortem examination of birds as throwing some light on the extent of the regard paid by nature to the requirements of theory; in this category may also be included the examination of gliding machines and models, made empirically in the absence of any knowledge of the equation of stability.

It will be shown in the present chapter that the theory is able to withstand every test that has so far been applied.

It is important to point out that although the model experiments employed for the purpose of demonstration are on a small

scale, on a very small scale in some cases, the theory makes no discrimination in the direction of a superior limit of size, and that therefore the results will apply also, within determinate limits of error, to full-sized models or even flying machines of many tons weight. There is a difference between reasoning from empirical data from the small to the large, and in the application of a generalised quantitative theory, proved true in respect of any one sized model, to any other sized model that may be desired.

§ 65. Penaud's Experiments.—Some successful experiments were made by Penaud about the year 1870, with small paper winged models, propelled by twisted rubber and a screw propeller. Mention has already been made of these experiments in § 9, and a Penaud model has been illustrated in Figs. 9 and 10. One of these models is stated to have crossed a pond 40 or 50 yards wide, and to have shown complete stability.

The theory of longitudinal stability held by Penaud as set forth is not satisfactory, but in spite of this fact the author feels that the defective argument, in the form presented, may be largely a matter of faulty description, and that perhaps Penaud had actually in his mind some idea, even if of rather an indefinite kind, of the theory subsequently propounded by the author in 1894, mentioned in Patent Specification 3608 of 1897, and given in § 3 of the present volume. In order that this point may be judged on its merits, the text of Penaud's argument is given in Appendix I.

Only slight mention is made of the phugoid oscillation; a note occurs as follows calling attention to the point. Penaud says:—"On observe alors assez souvent des oscillations dans le vol, comme nous en voyons decrire aux passereaux et principalement au pic-vert." [1]

[1] The *Green Woodpecker*. The author has remarked and commented on the undulating flight of this bird in Vol. I., Appendix IV. The resemblance traced by Mons. Penaud is more apparent than real, the undulations in the flight path of the Green Woodpecker are not those of the phugoid theory as is the case with an inanimate model.

§ 66. Mouillard's Observations.—M. Mouillard, in his " L'Empire de l'Air " (1881), describes at some length the behaviour of a rectangular aeroplane both ballasted and otherwise, and gives diagrams representing the forms of motion observed. Unfortunately no information is given from which the relative scale of the curves can be estimated, and the representations are at best but sketches.

Fig. 52 is from a diagram given by Mouillard as typical of the motions of the ballasted aeroplane, but the corresponding

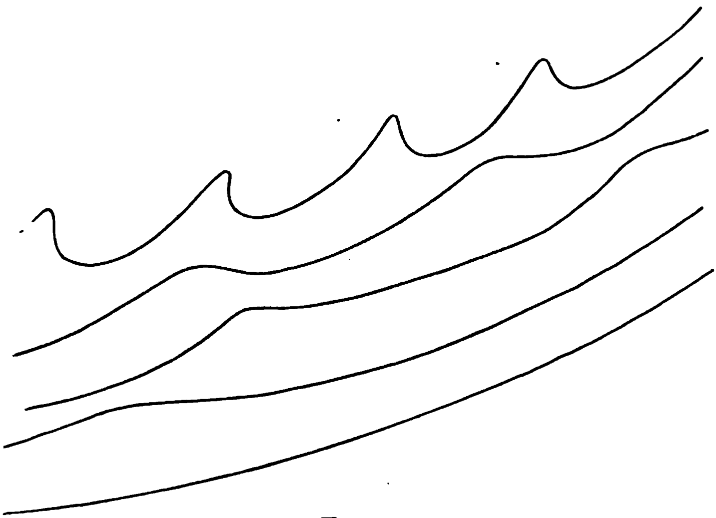


FIG. 52.

reference in the text is insufficient to make clear the nature of the motions that are depicted. The author was at one time inclined to think that Mouillard had actually seen and endeavoured to record the phugoid oscillations, but it is by no means certain that such is the case. The whole of the description is obscured by theories of an impossible kind, and it is only here and there that passages occur that are really relevant.

It would appear from the description that Mouillard's cardboard aeroplanes were very inferior to the mica planes described in Chapter I. and in Vol. I., § 162, and it is quite possible that

the oscillations recorded by Mouillard were merely vibrations about the attitude of equilibrium, that is to say, oscillations of much quicker pitch than those of the flight path, due to some initial disturbance of the balance between the *attitude* of the plane and the position of its centre of pressure. This possibility

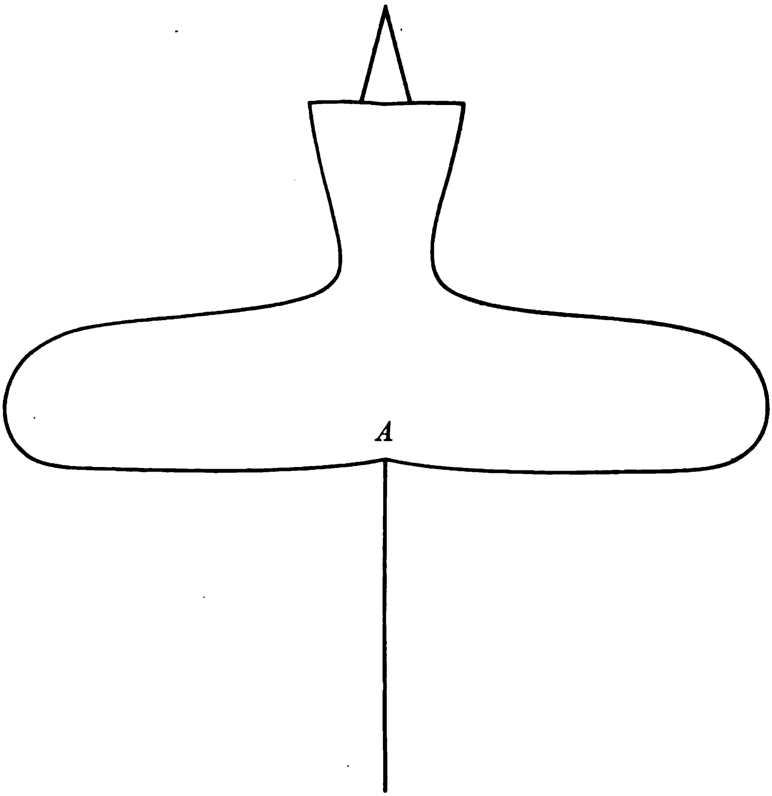


FIG. 53.

is suggested by the following passage, " Très souvent un aéroplane produit, *a part ces mouvements imperceptibles*, on pourrait dire théoriques, d'autres grands mouvements de chutes et de rèlevements: cela tient a ce que la correction du gouvernail n'est pas parfaite." This passage occurs where Mouillard discusses the effect of a fold (*pli*) made in the rear portion of the aeroplane

to minimise the extent of the oscillations: this fold he refers to as acting as a rudder (*gouvernail*).

The author has noticed quick pitch vibrations occasionally when experimenting with mica models, which are in no way related to the phugoid oscillation.

It would appear from the foregoing that Mouillard's observations must be considered a doubtful quantity in the present connection.

§ 67. **Marey's Experiments.**—Mons. E. J. Marey, in his "Vol des Oiseaux," describes an experiment made with a paper

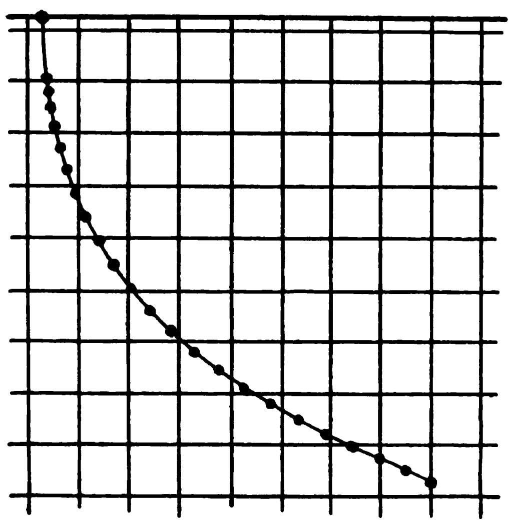


FIG. 54.

gliding model of a pattern devised by Mons. J. Pline in which a photochronographic record is made of the flight path when the model being suspended from its tail is let fall from rest.

The form of the model employed is illustrated in Fig. 53, drawn from the representation given in Marey's work. It is described as made from a single sheet of paper, folded in some manner not made clear, and ballasted by means of a needle or

Fig. 55.

wire that may be slid forward or backward in order to shift the centre of gravity as may be desired.[1]

The flight path described on being released head foremost is given in Fig. 54, the point indicated in successive positions being that marked "A" in Fig. 53; in the original in Marey's work the motion of the model is given as photographically recorded.

According to the phugoid theory we should have to deal here with the special case of the semicircle, but though the first portion of the curve is approximately the arc of a circle, the latter portion tends to become a straight line. It may also be noted that the model undergoes very little acceleration during the latter portion of the flight recorded, the "pitch" of the photographic images being nearly constant.

It is evident that in the latter part of the flight the model has practically settled down to its path of uniform gliding, in which case the damping of the phugoid oscillation must be very rapid indeed, so much so that the aerodone is to all intents and purposes "dead beat." This is quite in harmony with the fact that the gliding angle γ (taking the latter portion of the flight as the gliding angle) appears to be about 1 in 2, which betokens a resistance nearly 50 per cent. of the weight, and consequently a very rapid decrease of the phugoid amplitude.

Marey does not state the number of images per second employed in the case in question, but he gives the scale of the diagram by division into 10 cm. squares. Taking the radius of the circular path as 80 cm., the value of H_n will be 27 cm. approximately or $=$ ·9 ft., the corresponding velocity is $V_n =$ 7·5 ft./sec., and the pitch of the images on the steady gliding portion of the curve is 5·5 cm. By division we obtain 40 as the number of images per second. If any record exists of this experiment it would be interesting to know the actual number of exposures employed.

[1] A drawing of a model made by the author from Mons. Marey's description has already been given, Fig. 11.

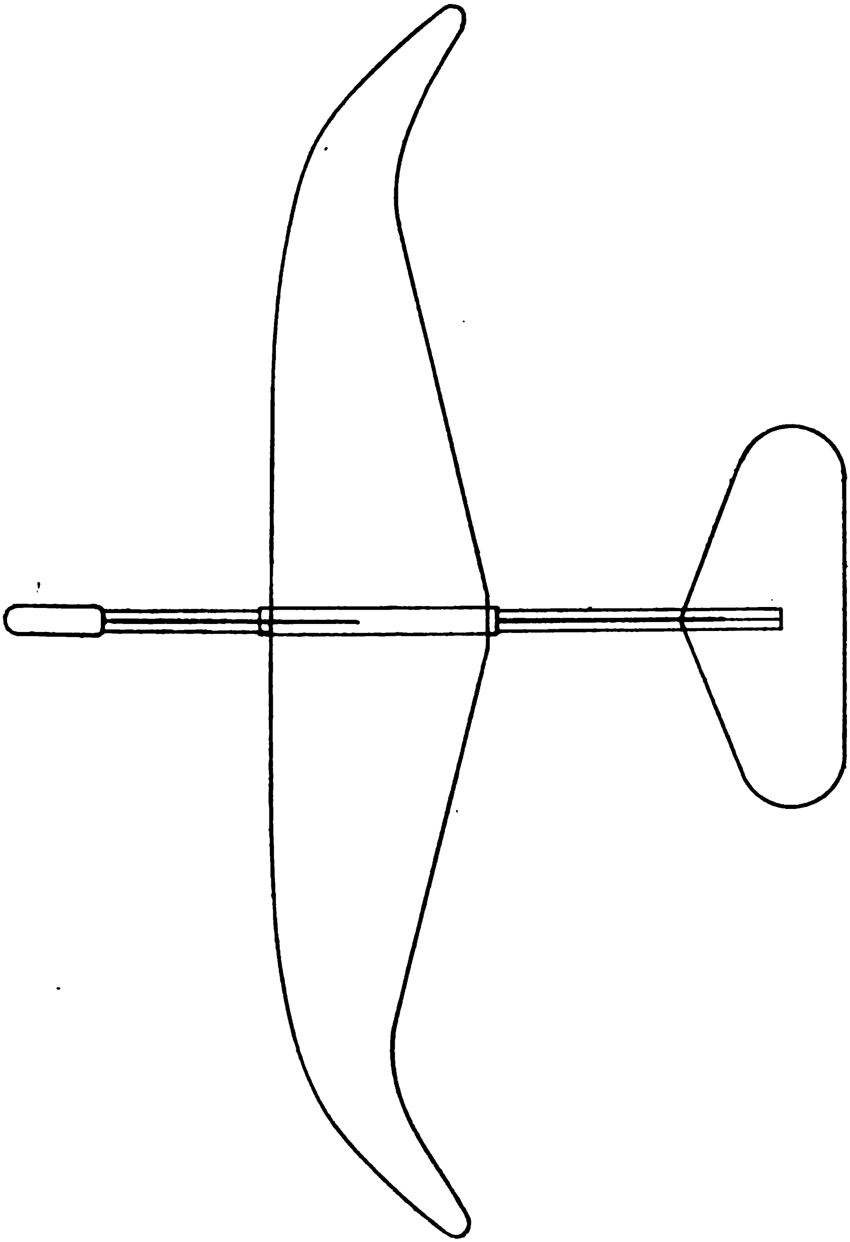


FIG. 56 ($\frac{3}{4}$ full size).

§ 68. The Author's Experiments in Confirmation of the Phugoid Theory.—By the use of thin laminæ of mica in combination with other improvements, the author has succeeded in bringing flight models of small size and low velocity to a degree of perfection not previously obtainable, and during the last few years, 1905-7, a considerable amount of work has been accomplished by means of these mica aerodones in the confirmation and extension of the Phugoid Theory.

A photograph of a mica aerodone is given in Fig. 55, and scale drawings of some of the models used for the purposes of quantitative investigation are given in Figs. 56, 57, 58, etc.

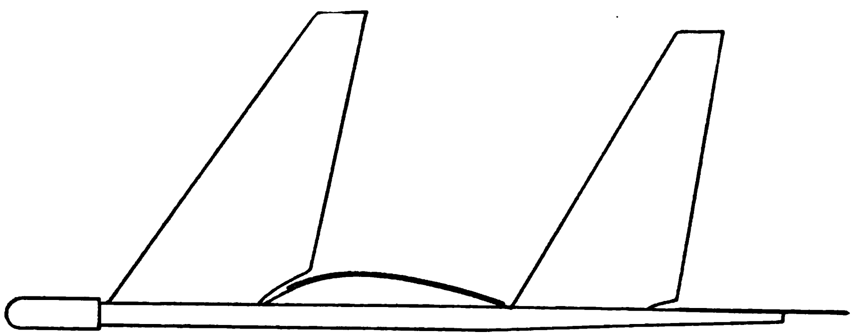


FIG. 57 ($\frac{2}{3}$ full size).

The method of construction is as follows. A mica lamina of a few thousandths of an inch in thickness, or even less than one one-thousandth of an inch in some cases, is cut to the shape required to form the *aerofoil* and fastened to a "bolster," which is a piece of wood of the necessary form to give to the aerofoil the curvature desired. The tail plane and the fins are also of mica, and are attached by fish glue or other adhesive to a wooden backbone, which also carries the necessary ballast, in the form of lead foil, at its forward extremity.

With an aerodone constructed as above described the whole of the properties of the phugoid curve can be readily demonstrated in a large room or lecture theatre, from the simple case

of the steady glide to the curves of tumbler type. With a certain amount of practice with any individual model it becomes quite easy, by varying the initial velocity imparted, to get any recognised type of flight path at will, and by varying the proportions of the aerodone in accordance with the requirements of theory, the flight path may be rendered stable or otherwise as desired.

§ 69. Confirmation of the Phugoid Theory as to Time, Velocity and Phase Length.—By means of these mica models, the author has been able to demonstrate experimentally the relations established in Chapters II. and III. as summarised in § 36.

In these experiments, the first series of which took place in 1905, soon after the method of making these low speed models had been developed, the largest model employed did not exceed five grams, and the smallest was about one-tenth this size. In spite of this fact, the agreement with theory was found to be almost perfect.

Now since if there is a departure from the conditions of the hypothesis, this departure will be greatest for small models and low velocities where the V^2 law of resistance begins to break down, it is evident that for birds of any size, and still more for aeronautical flying machines, the main results of the phugoid theory may be taken as rigidly applicable.

Example.

Mica aerodone. Series A. Model No. 2. Weight, 4·7 grams (Figs. 56 and 57).

Observer, author, assisted by Mr. A. J. Hill, July 3rd, 1905.

Velocity. A flight of 85 ft. occupied 6 secs., or

$$V_n = 14 \text{ ft. sec.}$$

Phase length. The distance occupied by five complete oscillations measured 129 ft., or (§ 36) $L_1 = \frac{129}{5} = 26$ ft. approximately.

Time period. On semicircular flight path (aerodone let fall and time taken to first *crest*[1] of flight path), three observations[2],

$$\begin{array}{r} 1{\cdot}9 \\ 1{\cdot}9 \\ 2{\cdot}0 \\ \hline 3\ )\ 5{\cdot}8 \\ \hline 1{\cdot}93 \end{array}$$

On undulating flight path (aerodone launched at slightly over its natural velocity),

$$\begin{array}{rl} 2{\cdot}0 & \\ 2{\cdot}0 & \\ 1{\cdot}8 & \\ 3{\cdot}6 & \text{(double period)} \\ 3{\cdot}6 & \text{(double period)} \\ 4{\cdot}0 & \text{(double period)} \\ 2{\cdot}0 & \\ \hline 10\ )\ 19{\cdot}0 & \\ \hline 1{\cdot}9 & \text{secs. average.} \end{array}$$

Thus, $t_1 = 1{\cdot}9$.

In order that the above experimental determinations may be used as a check on the theoretical investigation, they require to be compared on the basis of § 36. This may conveniently be done by the calculation of H_n from the three different sources; the substantial identity of the resulting values constitutes the proof.

(1) From the measured velocity, by the ordinary equation to the falling body,

$$H_n = \frac{196}{64{\cdot}4} = 3{\cdot}04 \text{ feet.}$$

[1] Owing to the damping factor the flight path does not form a *cusp*, but rather a crest, the curve becoming inflected

[2] The time being taken by an ordinary stop watch, the division into less than $\frac{1}{5}$ second is probably not justifiable.

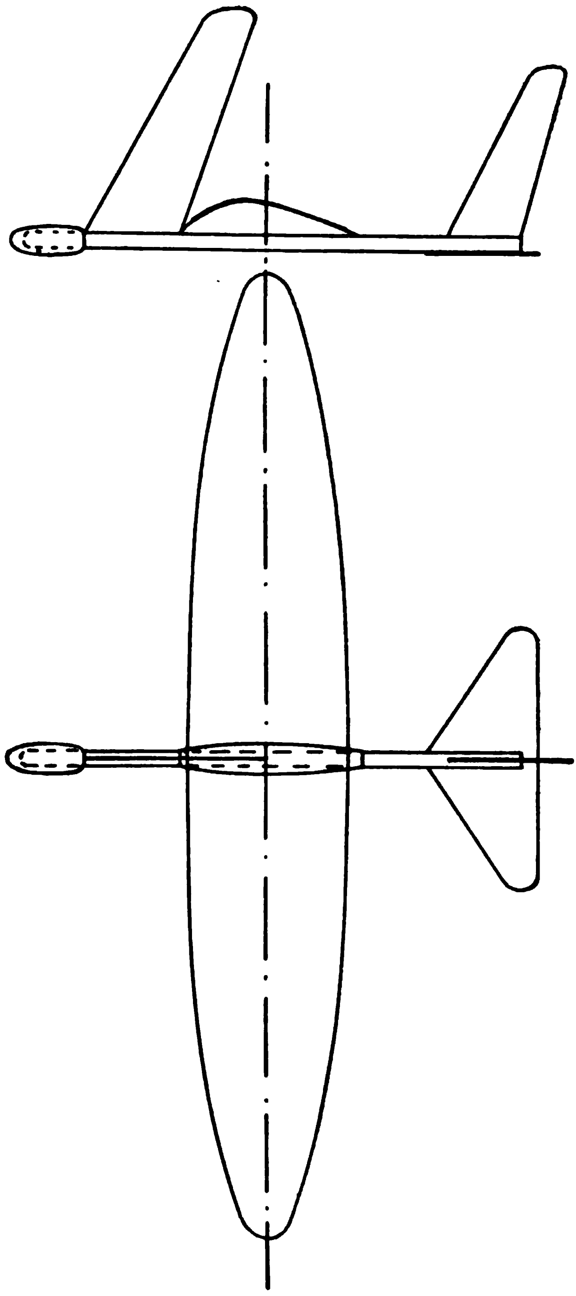


FIG. 58 ($\frac{2}{3}$ full size).

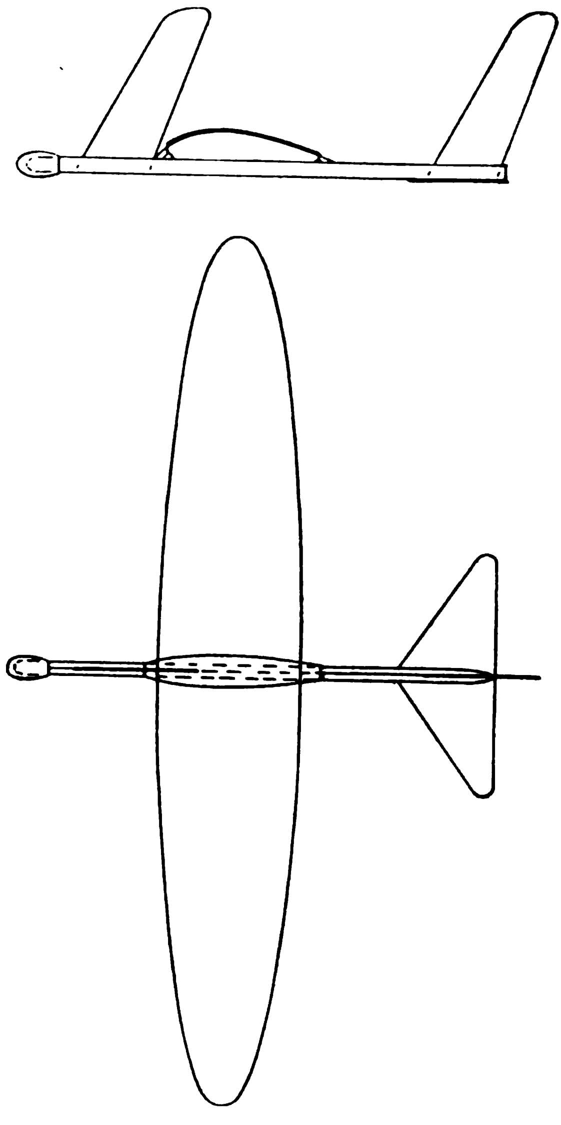


FIG. 59 (full size).

(2) From the phase length (by § 36),

$$H_n = \frac{26}{8 \cdot 88} = 2{\cdot}93 \text{ feet.}$$

(3) From the time period (by § 36),

$$H_n = \left(\frac{1{\cdot}9}{1{\cdot}105}\right)^2 = 2{\cdot}95.$$

The whole of the above results are within 4 per cent., which does not exceed the probable magnitude of experimental error.

Again. Series B. No. 2. Fig. 58.

V_n determined as = 14 ft. sec.

t_1 „ „ = 1·83 (average of 3 trials).

From V_n we obtain $H = 3{\cdot}04$ feet.

„ t_1 „ „ $H = 2{\cdot}74$ „

Again. Series C. No. 1. Fig. 59.

V_n determined as 10 ft. sec.

t_1 „ „ 1·37 (average of twelve trials).

From V_n we obtain $H_n = 1{\cdot}56$ feet.

„ t_1 „ „ $H_n = 1{\cdot}54$ „

§ 70. Verification of the Equation of Stability.—In the verification of the equation of stability, the considerations discussed in § 64 still apply, so that the range of the equation should be restricted only by some inferior limit of size; hence the importance of small scale models and low velocities. It is at the same time desirable to extend the range of the experimental verification as far as possible in the direction of larger sizes and higher velocities as representing more nearly the actual conditions of practical aeronautics.

Up to the present the author's quantitative experiments have been conducted indoors, with models ranging up to about one half ounce weight, but the application of the equation to the author's 1894 models, and to the gliding machine of the late Herr Lilienthal, forms an effective continuation.

The sizes and velocities suitable for employment vary with the circumstances under which experiments can be conducted; thus, experimenting indoors in a room 20 ft. by 30 ft., an aerodone having a natural velocity of about 5 or 6 ft./sec. is found to be most suitable; the short phase length corresponding to these low velocities—some 4 or 5 feet—renders it possible to observe six or eight undulations of the phugoid path, and so to determine whether the amplitude is subject to change, or whether it is sensibly constant. When experiments are conducted out of doors a natural velocity of about 14 ft./sec. is found to be appropriate; this, whilst not excessive from the point of view of ease of observation or of convenience of launching by hand, is sufficient to render the aerodone consistent in its behaviour on a calm day.

Models of more than a few ounces weight require to have a higher natural velocity, and it becomes desirable to employ a launching device such as the catapult described in § 12; it is then no longer any advantage to keep the natural velocity down to the minimum, and velocities of 25 or 30 ft./sec. may appropriately be employed. In this case the phase length will amount to something like 100 to 140 feet and a flight of two or three hundred yards is desirable, involving a drop of about a hundred feet. Thus the experiments should be conducted from the edge of a cliff or other elevated point.

§ 71. Experimental Verification of the Equation of Stability. —For the purposes of the present investigation four models have in all been employed. These, in order of size, are nominally as follows:—

12 gram model:	Figs. 55, 60, and 61.
4 ,, ,,	Fig. 62.
$\frac{1}{4}$,, ,,	Fig. 63.
"One grain" model:	Fig. 64.

The above aerodones were in the first instance designed in

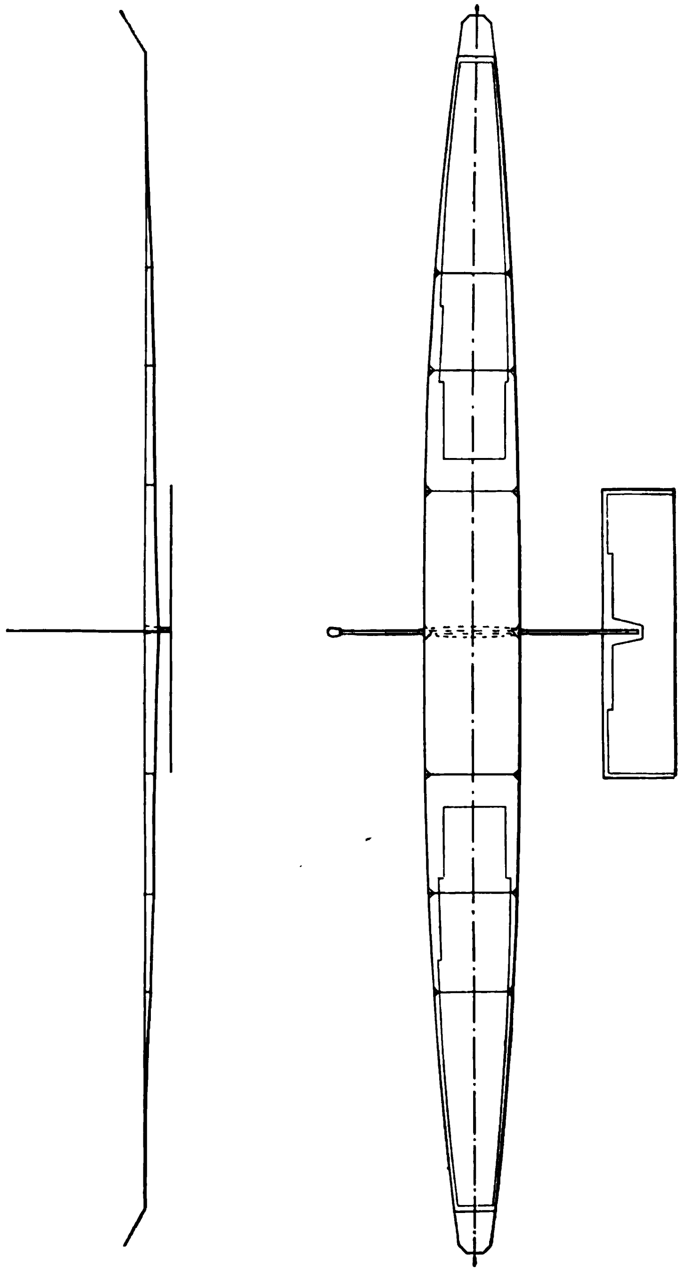


FIG. 60 (¼ full size).

accordance with the principles set forth in Vol. I. for minimum resistance, and rough calculations were made in advance in order to ensure that the stability factor Φ should be approximately equal to unity. The models were then constructed and adjusted to the critical condition by observation of the flight path, the final measurements being then made, and so the data obtained for the final calculations.

In the case of the larger two models the adjustment was made *from the unstable to the stable*, by adding ballast in the immediate vicinity of the centre of gravity; in the smaller models the adjustment was made in the opposite direction, *i.e.*, *from the stable to the unstable*, by the reduction of the tail area.

Incidentally the present experiments demonstrate that, generally speaking, an aerodone that is *just stable* for paths of small amplitude is unstable if the amplitude is great, though whether this is universally the case is not certain.

The whole subject of model construction and adjustment is dealt with in Chapter X., where the necessary detail instructions will be found to enable a repetition of these experiments to be made.

§ 72. Experimental Verification of the Equation of Stability (continued).—The data relating to the 12 gram model, Figs. 55, 60, and 61, are as follows:—

$$\text{Mass} = 12{\cdot}3 \text{ grams} = {\cdot}0267 \text{ lbs.}$$

$$\therefore \quad W = {\cdot}86 \text{ poundal.}$$

$$V_n = 9{\cdot}3 \text{ ft./sec. (observed value)}$$

$$\therefore \quad H = \frac{9{\cdot}3^2}{64{\cdot}4} = 1{\cdot}35 \text{ ft.}$$

$$K = \frac{W}{V^2} = \frac{{\cdot}86}{86{\cdot}5} = {\cdot}01.$$

$$\beta = {\cdot}24.$$

$$Tan\ \gamma = {\cdot}13 \text{ (observed value).}$$

Tail data,

l = ·28 ft.

a = ·0625 sq. ft.

Constants,

ρ = ·078.

C = ·7 } (tail data)
c = 2·27 } n = 4.

ϵ = ·75 { (aerofoil) n = 12.

Moment of inertia,

I = ·00035 lb. ft.² (by measurement and calculation).

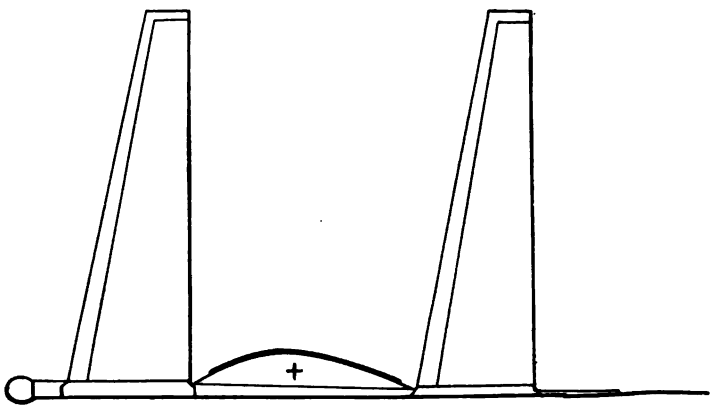


FIG. 61 (½ full size).

Condition of flight path observed for three successive oscillations, recorded as *just stable for small amplitude.*

Calculation of Φ—

$$\Phi = \frac{4\ l\ H_n^2 \tan \gamma}{I\left(\frac{1}{K} + \frac{1}{c\ C\ \rho\ \epsilon\ a\ \beta}\right)}$$

$$= \frac{4 \times \cdot 28 \times 1{\cdot}82 \times \cdot 13}{\cdot 00035\ (100 + 716)}$$

$$= \frac{265}{285} = \cdot 93.$$

Now the observed condition of flight path corresponds to $\Phi = 1$, so that the agreement is within 7 per cent. which is as close as is to be expected. The discrepancy, such as it is, may be due to some imperfection in the equation, for undoubtedly imperfections exist; there are factors of secondary importance of which the theory takes no account. It is equally possible, however, that it is due to inaccuracies of observation and measurement, the possible error in the determination of the natural velocity, V_n, alone might easily account for a difference of greater magnitude than that recorded.

Again, 4 gram model, Fig. 62, the data are as follows :—

$$\text{Mass} = 4{\cdot}52 \text{ grams} = {\cdot}01 \text{ lbs.}$$

$$\therefore \quad W = {\cdot}32 \text{ poundal.}$$

$$V_n = 9{\cdot}5 \text{ ft./sec.}$$

$$\therefore \quad H_n = \frac{9{\cdot}5^2}{64{\cdot}4} = 1{\cdot}4 \text{ ft.}$$

$$K = \frac{W}{V_n^2} = \frac{{\cdot}32}{90} = {\cdot}0035.$$

$$\beta = {\cdot}2.$$

$$Tan\ \gamma = {\cdot}15.$$

Tail data,

$$l = {\cdot}25 \text{ ft.}$$

$$a = {\cdot}028 \text{ sq. ft.}$$

Constants,

$$\rho = {\cdot}078.$$

$$\left.\begin{array}{l} C = {\cdot}7 \\ c = 2{\cdot}27 \end{array}\right\} \begin{array}{l} \text{(tail data)} \\ \mathrm{n} = 4. \end{array}$$

$$\epsilon = {\cdot}71 \text{ (aerofoil) } \mathrm{n} = 9{\cdot}3.$$

Moment of inertia,

$$\mathbf{I} = {\cdot}000133 \text{ lb. ft.}^2 \text{ (by measurement and calculation).}$$

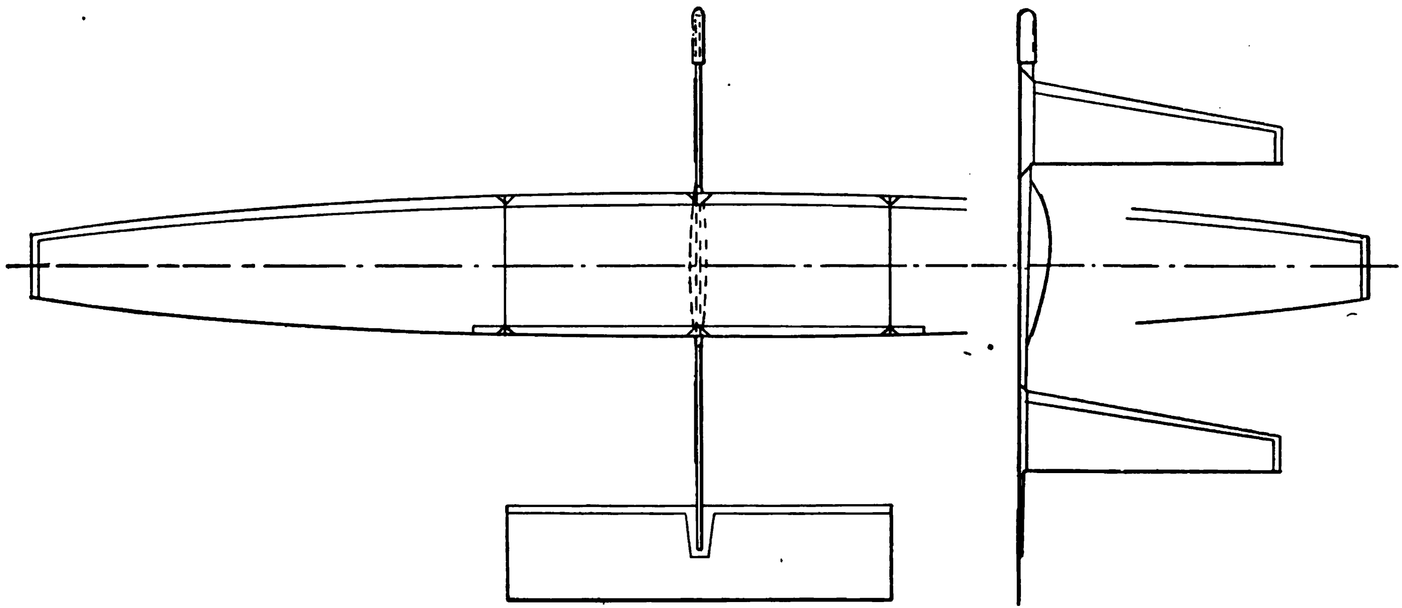


FIG. 62 ($\frac{1}{2}$ full size).

Condition of flight path observed for three successive oscillations, recorded as *nearly stable* for small amplitude.

Calculation of Φ—

$$\Phi = \frac{4 \times \cdot 25 \times 1\cdot 96 \times \cdot 15}{\cdot 000133\,(285 + 2040)}$$

$$= \frac{\cdot 294}{\cdot 309} = \cdot 95$$

which is substantially in agreement with the observed condition.

§ 73. Verification of the Equation of Stability (continued). Small Scale Experiments.—In order to ascertain if possible the extent to which the theory may be limited in its application to models of small size, measurements were made of an aerodone of nominally $\frac{1}{4}$ gram weight, Fig. 63, with the following results:—

$$\text{Mass} = \cdot 245 \text{ gram} = \cdot 00054 \text{ lb.}$$

$$\therefore\ W = \cdot 0174 \text{ poundal.}$$

$$V_n = 5 \text{ ft./sec.}$$

$$\therefore\ H_n = \frac{25}{64\cdot 4} = \cdot 39.$$

$$K = \frac{\cdot 0174}{25} = \cdot 000695.$$

$$\beta = \cdot 25.$$

$$Tan\ \gamma = \cdot 265 \text{ (mean of two observations).}$$

Tail data,

$$l = \cdot 105 \text{ ft.}$$

$$a = \cdot 00184 \text{ sq. ft.}$$

Constants,

$$\rho = \cdot 078.$$

$$C = \cdot 67.$$

$$c = 2\cdot 0 \text{ (Tail data; n = 2 approx.).}$$

$$\epsilon = \cdot 7 \text{ (Aerofoil data; n = 9).}$$

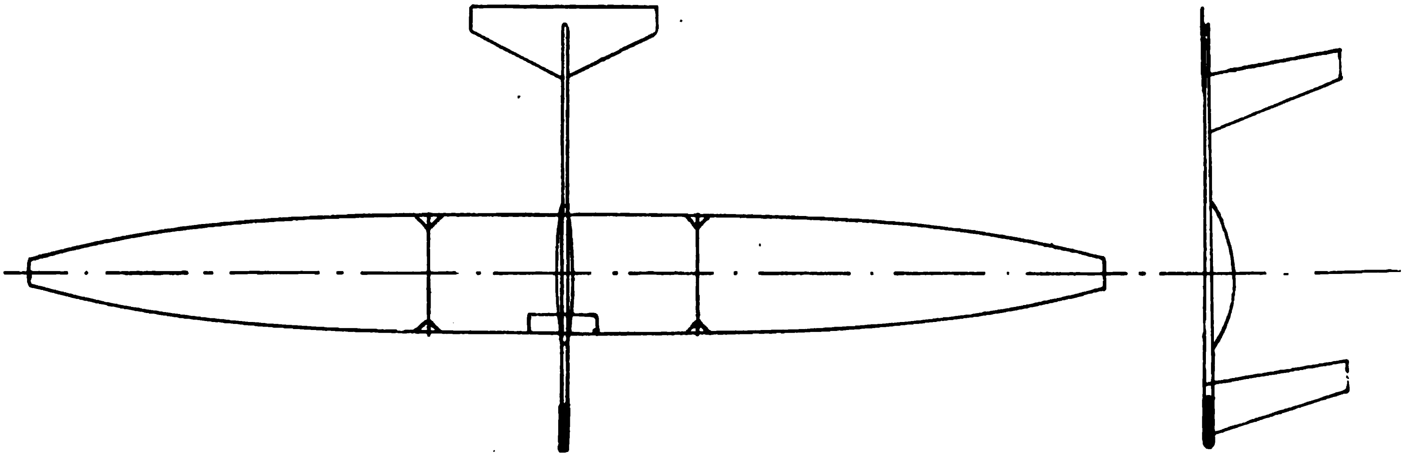


FIG. 63 (full size).

Moment of Inertia,

$\mathbf{I} = \cdot000{,}000{,}835$ lb. ft.^{2} (by measurement and calculation).

Condition of flight path recorded as just stable for small amplitude.

Calculation of Φ—

$$\Phi = \frac{4 \times \cdot105 \times \cdot152 \times \cdot265}{\cdot000{,}000{,}835\ (1{,}440 + 29{,}700)}$$

$$= \frac{\cdot0169}{\cdot026} = \cdot65.$$

In this case the agreement is not so close as in the preceding examples, the stability according to direct observation being apparently 50 per cent. better than as computed from the equation. Further trials of the model failed to show any error in the observation data. An inaccuracy of 10 per cent. in the velocity measurement would be required to account for the discrepancy; it is unlikely that an error of one quarter this amount would have escaped notice.

It is most probable that the estimate of the moment of inertia is in error. The measurement of so small a model is a matter of some difficulty without special apparatus; but it is also possible that the viscosity of the air has a steadying influence on models as small as the present example.

§ 74. Verification of the Equation of Stability. Small Scale Experiments (continued).—Having thus ascertained that the laws on which the aerodonetic theory is based hold good, as a rough approximation, down to a model weighing but one quarter of a gram, it was decided to carry the investigation to a still finer point. To this end an aerodone was designed to weigh only *one grain* (·065 gram) and to have the same velocity as the previous model,[1] namely, 5 ft./sec. (Fig. 61).

On completion, and after adjustment, the actual weighing gave

[1] It was thought preferable to vary only the size of the aerodone and not its velocity, to make sure that the conditions are other than those of corresponding speed, § 126.

·062 grams, or a trifle under that intended, but the natural velocity was exact to within a possible 5 per cent. error. The adjustment made in the first instance gave a degree of stability sufficient to render the flight path steady enough for the purpose of observation, the author estimating, *from experience with other models*, that the stability factor = 2 or thereabouts. The actual value was then calculated from the equation, as below, the result being 1·67. The tail plane was then altered to reduce the stability factor, and the further flight trial served as a final confirmation

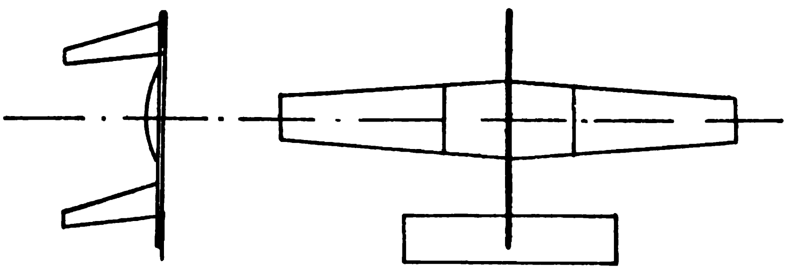


FIG. 64 (full size).

of the truth of the equation; with $\Phi = {\cdot}84$ the flight path had become capricious.

The measurement of the moment of inertia of so small a model presented some difficulty with the rather primitive apparatus at the author's command, and in the present case the value of this quantity was arrived at by calculation, the component parts being weighed separately, and their moment of inertia individually and collectively assessed.

One grain model (Fig. 64). Data as follows:—

$$\text{Mass} = {\cdot}062 \text{ gram} = {\cdot}000137 \text{ lb.}$$
$$\therefore\ W = {\cdot}0044 \text{ poundal.}$$
$$V_n = 5 \text{ ft./sec.}$$
$$\therefore\ H_n = {\cdot}39 \text{ ft.}$$
$$K = \frac{{\cdot}0044}{25} = {\cdot}000176.$$
$$\beta = {\cdot}24 \text{ (mean value).}$$
$$Tan\ \gamma = {\cdot}3.$$

Tail data,

$l = \cdot 052.$

$a = \cdot 0019.$

Constants,

$o = \cdot 078.$

$$\left.\begin{aligned} C &= \cdot 7 \\ c &= 2\cdot 3 \end{aligned}\right\} \text{(Tail data).}$$

$\epsilon = \cdot 63$ (Aerofoil data).

$\mathbf{I} = \cdot 000{,}000{,}17$ lb. ft.^{2} (computed from weight of components).

Observed flight path quite stable, Φ estimated about $= 2$.

Calculation of Φ—

$$\Phi = \frac{4 \times \cdot 052 \times \cdot 152 \times \cdot 3}{\cdot 000{,}000{,}17\ (5{,}700 + 27{,}800)}$$

$$= \frac{\cdot 0095}{\cdot 0057} = 1\cdot 67.$$

The alteration made with a view of reducing the flight path to the "just stable" condition consisted in cutting the tail down to approximately half its lateral breadth, the portions cut off being made to adhere by capillarity to the remaining portion so that the balance should be unaffected. The alteration involved new data as follows:—

$$a = \cdot 001$$
$$c = 2\cdot 0.$$

Re-calculating we have,

$$\Phi = \frac{4 \times \cdot 052 \times \cdot 152 \times \cdot 3}{\cdot 000{,}000{,}17\ (5{,}700 + 61{,}000)}$$

$$= \frac{\cdot 0095}{\cdot 0113} = \cdot 84.$$

The actual flight path of the altered model was found to be scarcely stable; it was impossible to obtain flights of a sufficiently consistent character to make a redetermination of either V_n or γ. Unfortunately, this model was on the point of developing

a lateral oscillation (see § 90), which shows itself first in the region of the crest of the phugoid flight path. For this reason the instability that arose was of a mixed kind, but the main conclusion was not in doubt.

We are thus able to conclude that, even in the case of a model of but one grain weight, the departure from the ordinary laws of flight is comparatively unimportant, and that the influence of viscosity does not give rise to a new *régime* till the size is still very much further reduced. This conclusion can only be, in reality, an approximation, for the author has failed, in spite of repeated endeavour, to obtain gliding angles (*i.e.*, values of γ) for these very small models to approach those easily obtainable with models of say even a few grams weight. Thus for a *one grain* model, $n = 6$, the best result so far obtained is that given in the example $tan\ \gamma = \cdot 3$; whereas for the model of ·25 grams weight $tan\ \gamma = \cdot 26$; for a model of ·6 gram weight (aspect ratio $= 6$), $tan\ \gamma = \cdot 2$, and for a model weighing 30 grams, $\gamma = \cdot 17$ is a value easily obtainable. It is evident, therefore, that in these very small sizes we are approaching the point when the influence of viscosity begins to materially affect the law of resistance, in spite of the agreement with the equations.

§ 75. The Stability of Birds in Flight.—The application of the stability equation to birds in flight is not to be undertaken without some preliminary examination of the conditions. The circumstances surrounding the problem as presented by nature are such as to obscure to some extent the more essential factors. Thus the directive organ, the tail, is not, as is the case in the gliding models employed by the author, a clearly defined plane whose constants are known, but rather a prolongation of the streamline form constituted by the body of the bird. In addition to this the problem is complicated by the question of *active flight.* We do not know precisely what may be the effect on stability of the mobility of the aerofoil, that is to say, the flapping of the wings practised for the purpose of propulsion. Beyond these

two points there is also the question of the flexibility of the after edge of the aerofoil, which evidently renders the angle β variable

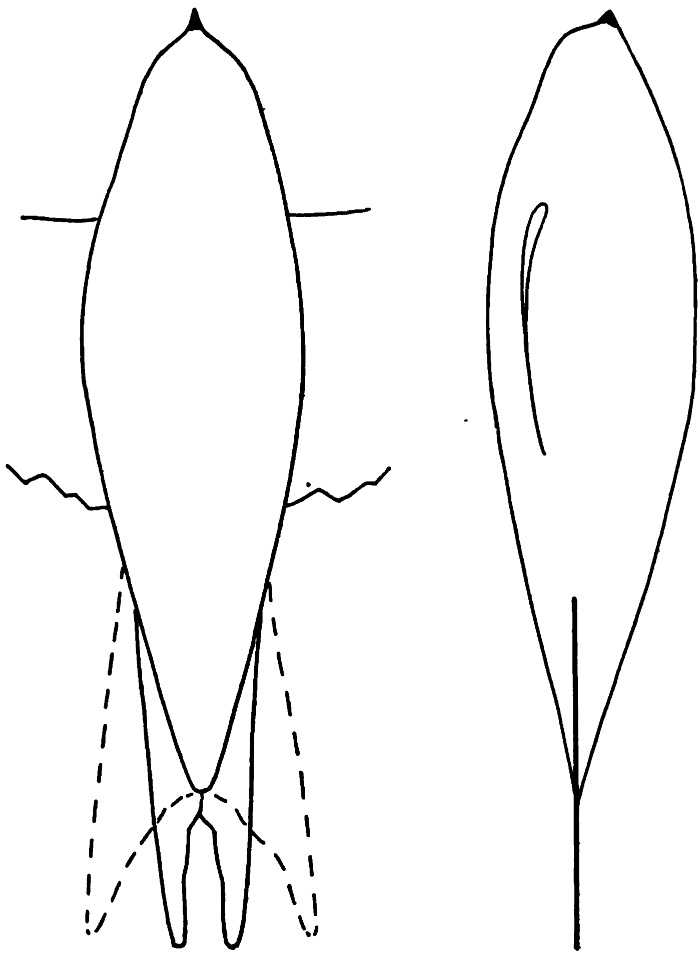


Fig. 65.

under changes of effective load such as arise in the course of the phugoid path.

Dealing with these points in the order raised, we have first to discuss the influence of the position of the tail on its effective

value. It is evident that as a generalisation of the conditions it would be necessary to abandon the notion of treating the tail-plane as a separate entity altogether, in favour of an expression denoting the directive value of the whole streamline body, tail and all.

It would seem, however, that a restriction exists that renders a generalised treatment unnecessary. It is believed from the considerations discussed in Ch. VII., § 101, that the centre of lateral resistance for small inclination to the line of flight must approximate very closely to the centre of gravity, in order that the lateral and directional equilibrium should be stable.[1] It follows from this that the streamline form constituting the body must be almost in *neutral equilibrium* about a vertical axis. But with the exception of the tail, an organ which may usually be clearly defined, the form of the body of a bird is, so far as it is possible to estimate, a solid of revolution; hence it will, if divested of the tail plumage, be also in neutral equilibrium about a horizontal axis. Therefore we may regard the whole directive value of the combination as due to the tail itself, taking due account of its position in relation to the flow of the stream in its vicinity.[2]

§ 76. The Stability of Birds in Flight (continued).—As illustrating the method of treatment under discussion, diagrams are given of the body and tail of the common swift (*hirundo apus*), Fig. 65, and the wandering albatros (*diomedea exulans*), Fig. 71.

Referring to Fig. 65, which is a ⅔ life-sized representation, it is evident that the tail may be regarded as consisting of two triangular fins projecting from the surface of a streamline solid of revolution into the surrounding stream. Most birds have the power, which they exercise to a very great extent, of expanding or contracting the tail area at will; this capacity is well developed in the swallow family, and is indicated in the figure

[1] Compare §§ 95 *et seq.*

[2] See "Aerial Flight," Vol. I., *Aerodynamics*, Chaps. I. and III.

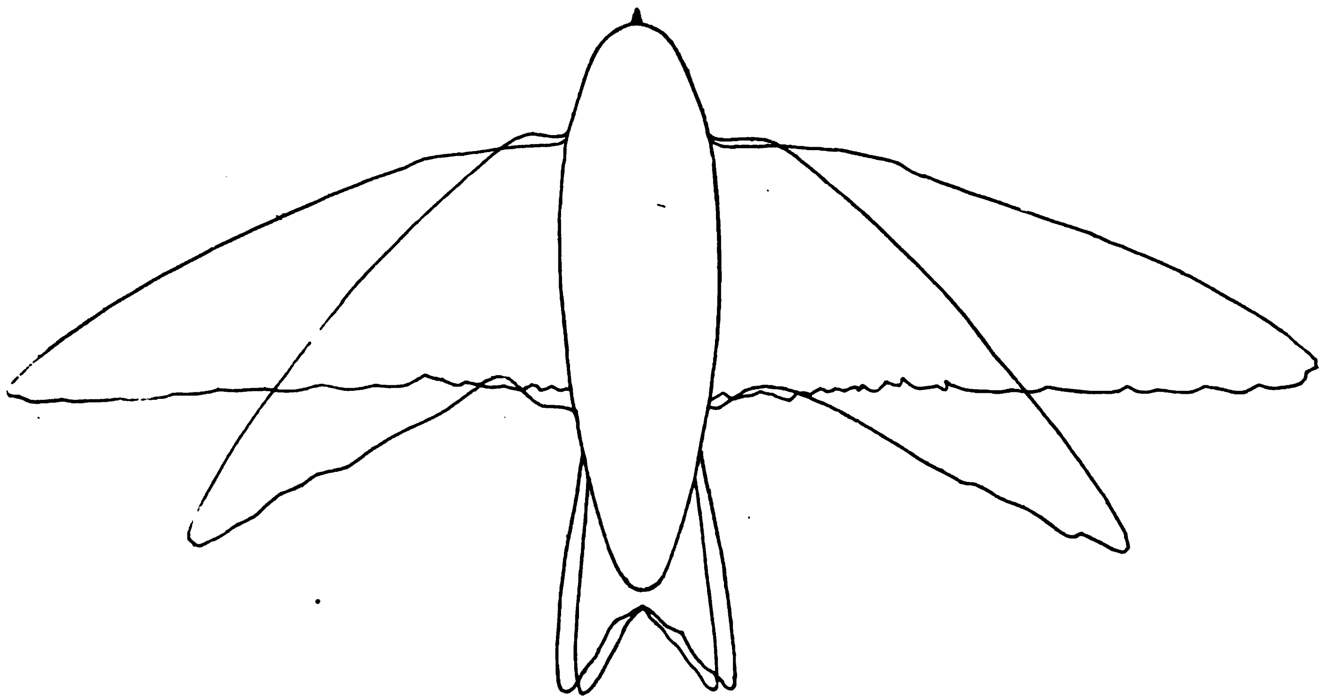


FIG. 66.

by the two extremes, the solid line representing the extent to which the tail is developed under ordinary conditions, and the dotted line showing the full expanse. It may be remarked as a matter of observation that the swift varies its speed of flight

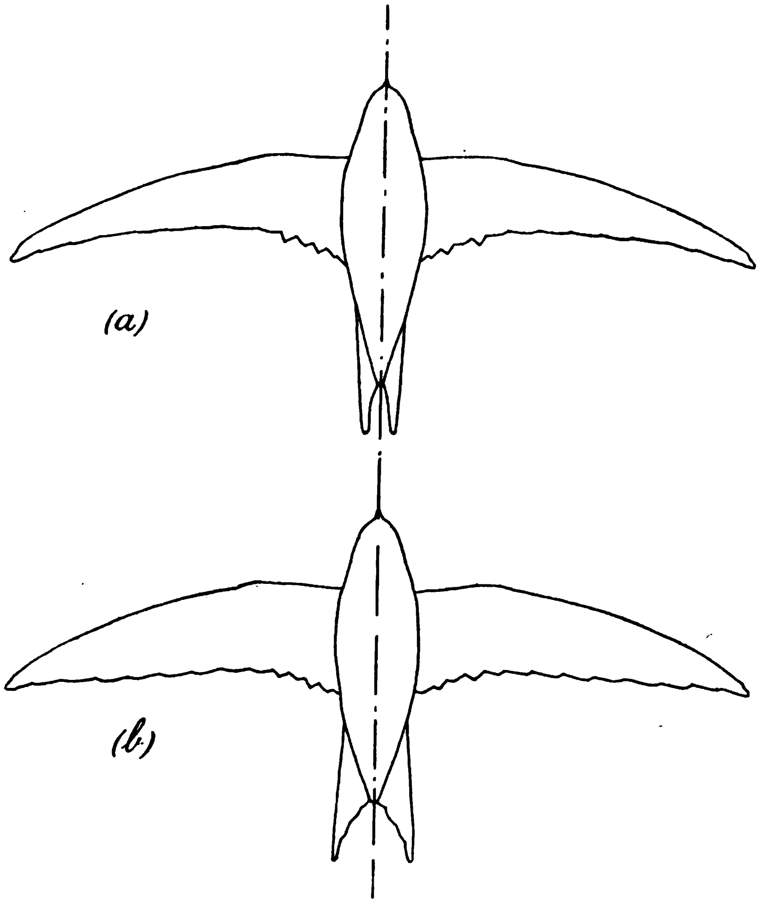


Fig. 67.

within wide limits, probably from about 20 feet per second to something over 50 feet per second,[1] and it does not close its

[1] The speed of flight of the swift, though perhaps at times greater than here given, has been grossly exaggerated by many writers; it has even been stated to exceed 200 miles an hour!

wings in flight, as is common with many of the smaller birds when exceeding their normal speed of flight.[1] The tail area is noticed to be greatest at low velocities, and the wing area may also be observed to undergo a similar variation, the silhouette at high and low flight velocity being represented respectively at (*a*) and (*b*) in Fig. 67.

In the case of the albatros, Fig. 71 ($\frac{1}{4}$ full size), the tail proper is of very small area, but the legs are carried extended, and the area of the webbed feet forms a substantial addition to the effective tail spread.[2]

The conditions under which the tail member or members have

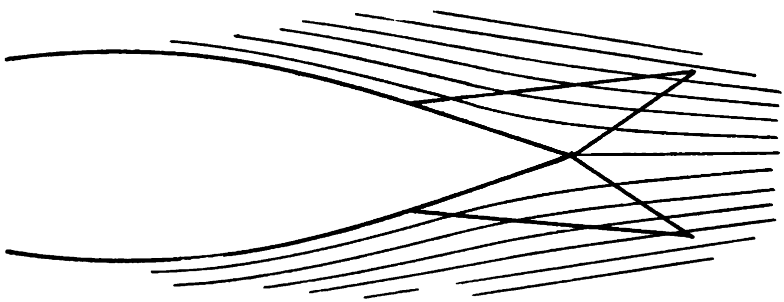


FIG. 68.

to perform their function are illustrated diagrammatically in Fig. 68. Owing to the fact that the region of the tail of a streamline body is one of *increased pressure*,[3] the velocity of the stream (relatively to the body) is less than the velocity of flight; this follows from hydrodynamic principles. Under actual con-

[1] Some of the smaller members of the swallow family partially fold their wings when in rapid flight, the common house martin for example, Fig. 66.

[2] In the official reports of the South Polar Expedition, Mr. E. A. Wilson says: "When on the wing the feet are held folded together at full length under the tail and extending well beyond its longest feathers, giving the effect of a markedly wedge-shaped tail with a white terminal border. This, of course, is not the case, for the tail is bordered by black at the extremity, and the appearance of white beyond the black is due to the whitish feet."

[3] Compare "Aerial Flight," Vol. I., *Aerodynamics*, § 11, also Chap. III.

ditions this difference is added to by the fact that the relative velocity of the stream has been further reduced by the skin-friction, the tail being surrounded by the *frictional wake*.[1] Under these conditions the directive efficiency of a given area must be materially less than would otherwise be the case.

The form of the areas employed by Nature in the tail member is one of comparatively low aerodynamic value, the value of n, the aspect ratio, being generally less than unity.[2] Now for n = 1 we know that the value of the constant $c = 1{\cdot}8$ and probably for the forms actually employed c will have a value considerably less than this amount.[3]

Taking into account both the low value of n and the adverse conditions under which the tail member has to perform its duty, it would appear that in round numbers we may take the effective value of c equal unity without being far from the truth.

§ 77. Stability of Birds in Flight (continued).—The influence of active flight as affecting the conditions of stability is one of which it is very difficult to take account.

Inasmuch as the wings perform their function of support with the same effect, whether they are in motion, as in active flight, or whether they are rigidly extended, as in gliding or soaring, and since the manner in which they act, so far as the support of the load is concerned, is the same in both cases, it is improbable that there is any fundamental difference that will invalidate the application of the theory. It is, however, likely that there may be some quantitative difference, owing to the intermittent nature of the supporting reaction, that would have to be met by a special coefficient.

For the time being it has been deemed inadvisable to attempt to deal with any such addition to the theory. In the calculations that follow it will be assumed that we are studying the conditions

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 17.
[2] "Aerial Flight," Vol. I., *Aerodynamics*, § 150.
[3] "Aerial Flight." Vol. I., *Aerodynamics*, § 159.

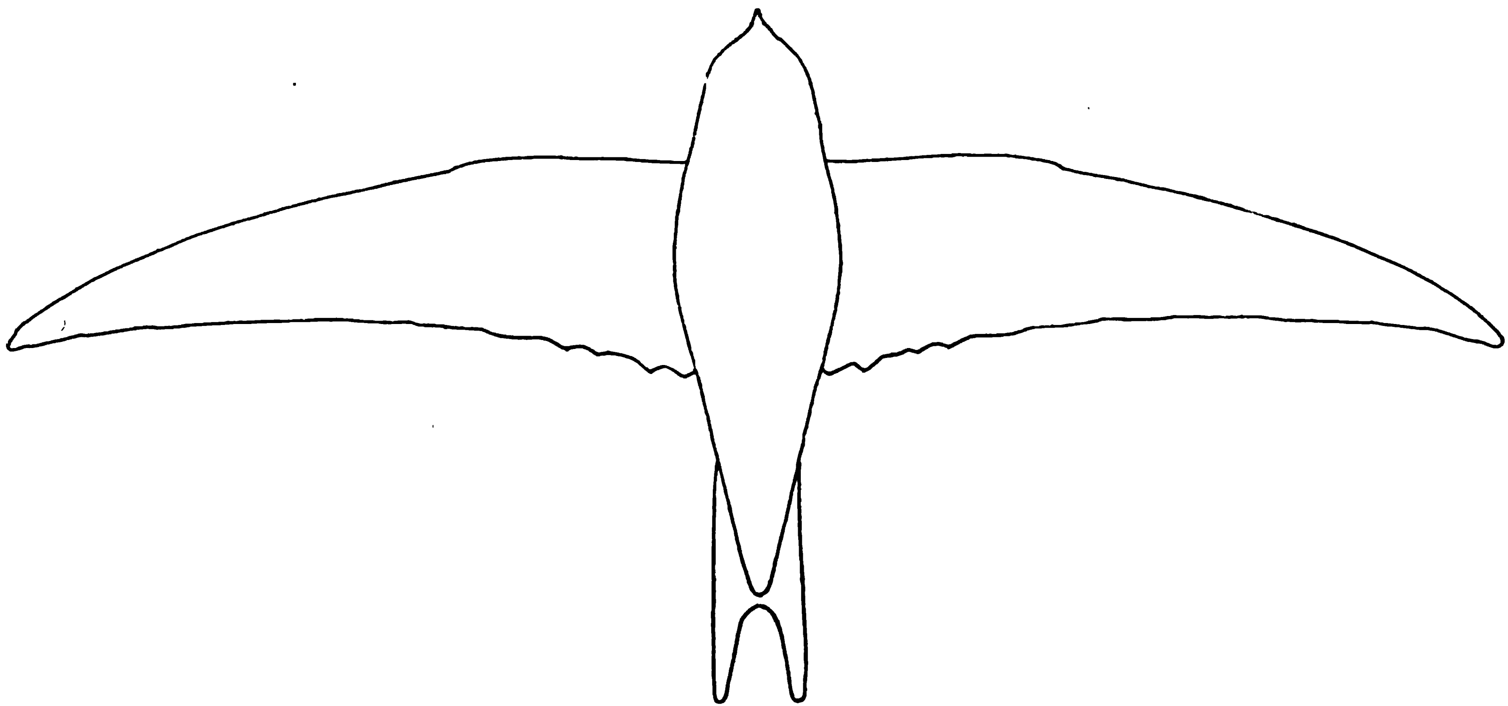


Fig. 69 (half life size)

that obtain when the bird is gliding or soaring, and our results, although strictly speaking limited to these conditions, may be taken as being of the right order of magnitude when the more complex conditions supervene. In the flight of the two birds under consideration, the swift and the albatros, the gliding or soaring mode is very largely employed.

The influence of wing flexion under changes of load is one of which no account has been taken in the Phugoid Theory. From an examination of the conditions it would seem that if this be the only addition imported into the hypothesis of Chapter II., the change in the form of the phugoid curve will be of a *symmetrical character*, and to some extent the form of the curve for a given aerodone will, if the aerofoil be made flexible after the manner of the wings of a bird, undergo a change in the direction of resembling a phugoid of greater H_n value.

It is difficult to follow the matter much further, but it appears certain that an appropriate degree of flexibility may add very appreciably to the stability, both in face of an adverse gust of wind, and as influencing the effective value of the stability coefficient Φ. Thus if it can be shown that a given bird or aerodone possess stability on the assumption that the aerofoil is rigid, then it will probably possess stability in a greater degree if the aerofoil is flexible. We may therefore treat the wing spread as rigid for the purposes of the investigation.

§ 78. Stability of the Hirundo Apus.—Fig. 69 represents, half life size, an example of the *common swift* shot and measured by the author. The data are as follows:—

Mass $= 40{\cdot}6$ grams $= {\cdot}09$ lbs.

$\therefore\ W = 2{\cdot}9$ poundals.

$V_n =$ Natural velocity, taken as $= 32$ ft./sec.

$\therefore\ H_n = 16$ ft.

$K = {\cdot}0028.$

$\beta = {\cdot}15.$

$Tan\ \gamma = {\cdot}16.$

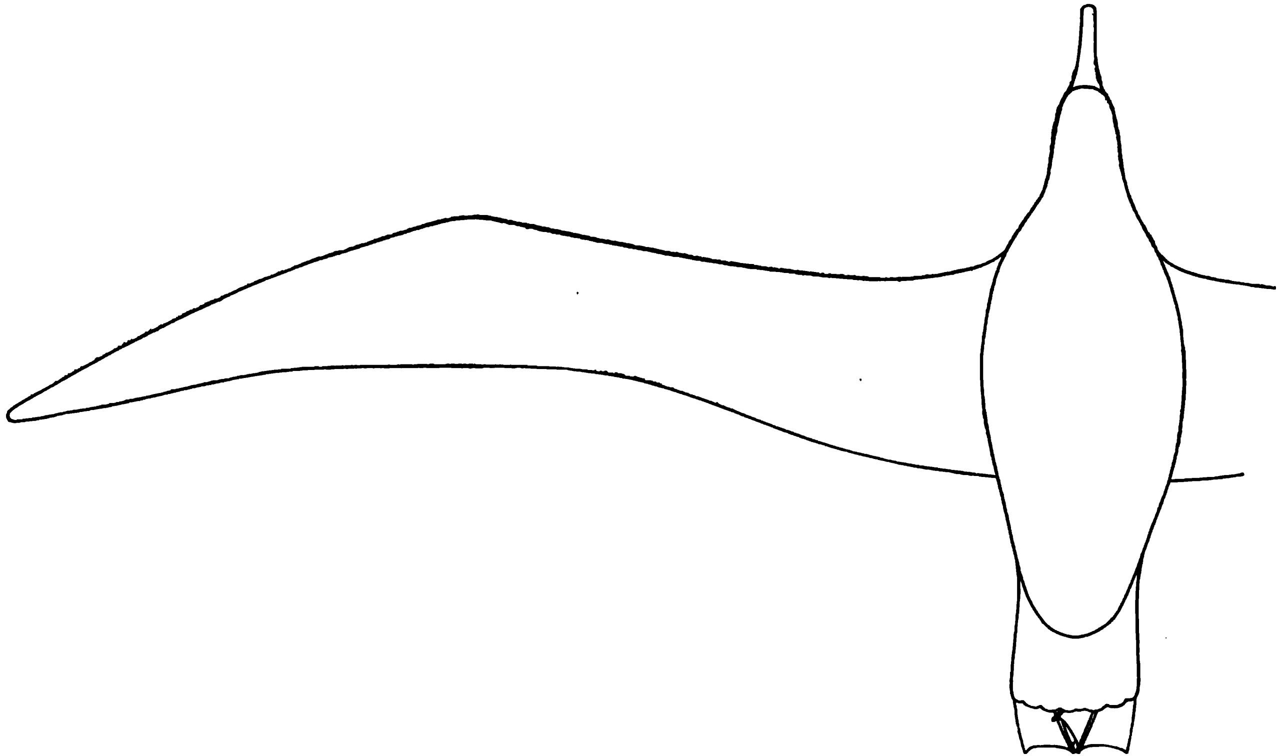


Fig. 70.

Tail data,

$l = \cdot 3.$

$a = \cdot 007.$

Constants,

$\rho = \cdot 078.$

$C = \cdot 66.$

$c = 1 \cdot 0.$

$\epsilon = \cdot 68.$

Moment of inertia,

Radius of gyration = ·11 ft. (measured).[1]

$\mathbf{I} = \cdot 0121 \times \cdot 08 = \cdot 00097$ lb. ft^2.

Calculation of Φ—

$$\Phi = \frac{4 \times \cdot 3 \times 256 \times 16}{\cdot 00097\,(357 + 27{,}500)}$$

$$= \frac{49}{27} = 1 \cdot 8.$$

Thus we have the result that the stability factor is nearly double the critical value. This multiple may be looked upon as the "factor of safety," by which a sufficient rate of damping of the phugoid amplitude is secured.

§ 79. Stability of Diomedea Exulans.—In the preceding example the moment of inertia was determined by actual measurement, in the case now under consideration the author has been under the necessity of taking his measurements from a "set up" specimen of a young bird (Fig. 70), of probably about 14 or 15 lbs. weight when killed. The degree of approximation to which we are at present able to work, renders slight inaccuracies of data unimportant.

[1] Measured by the method of double suspension described in Appendix VI. The influence of the outstretched wings on the moment of inertia is negligible; hence the radius of gyration was measured with the wings removed, and the mass employed in the calculation of the moment of inertia as that of the body of the bird under like conditions.

The body forms of birds capable of flight are of exceedingly uniform design, and it is evident that the relation of the radius of gyration to the length of the body differs but little in different species. The problem is, therefore, similar to that of the moment of inertia of an ellipsoid or other settled geometrical form. Owing to the preponderating influence of the length of the body in the direction of the axis of flight, the radius of gyration may be expressed approximately in terms of this axis; thus,—

$$\text{Radius of gyration} = \frac{\text{Length of streamline axis}}{5}$$

The length of the streamline axis is measured from the point where the body form merges into the tail plumage, to the point where the beak may be estimated to finish, assuming the streamline form to be completed symmetrically, as is actually the case in the example of the preceding section.

Example. Albatros: Figs. 70 and 71. Data as follows:—

Mass = 15 lbs. (assumed value).

$\therefore\ W$ = 480 poundals.

V_n = 50 ft./sec. (= 34 m./h.).

$\therefore\ H_n$ = 39 ft.

$$K = \frac{480}{2{,}500} = \cdot 192.$$

β = ·25 (value assumed from tables).[1]

Tan γ = ·14 (probable value).

Tail data,

l = 1·7 ft.

a = ·32 sq. ft. (including added area due to feet).

Constants,

ρ = ·078.

C = ·66 and c = 1·0 } Tail constants.

ϵ = ·75 Aerofoil.

[1] Aerial Flight, Vol. I., *Aerodynamics*, § 181.

Moment of inertia,

Radius of gyration $= \frac{2 \cdot 5}{5} = \cdot 5$ ft.

$\mathbf{I} = \cdot 25 \times 15 = 3 \cdot 75.$

Calculation of Φ—

$$\Phi = \frac{4 \times 1 \cdot 7 \times 1{,}520 \times \cdot 14}{3 \cdot 75 \times (5 \cdot 2 + 323)}$$

$$= \frac{1{,}450}{1{,}230} = 1 \cdot 18.$$

Thus in the albatros we find a lower "factor of safety" than in the case of the swift. This may plausibly be attributed to the more constant conditions under which this bird finds itself, and the greater difficulty that Nature must have in providing a large stability factor in the case of the heavier bird without an undue increase of speed.

We must at present regard the application of the equation of stability to birds in flight more as an illustration than as even an approximate demonstration; for the data are confessedly inadequate. Thus an error in the computation of the natural velocity of but 10 per cent. will affect the value of Φ in the ratio of 2 : 3.

In the case of the swift where the tail area is a variable quantity, we have a further possibility of error. In the calculation of the preceding section the tail area has been assumed as of approximately minimum spread; the maximum spread is about double the area given, so that the stability factor, Φ, will, when the tail is fully spread, be approximately double that calculated, or say $= 3 \cdot 6$. We may on this basis assess the minimum velocity of flight at which the swift will possess automatic stability; we know that Φ varies as $V_n^{\frac{1}{4}}$, hence the velocity at which with a full spread of tail Φ becomes unity will be

$$\frac{32}{\sqrt[4]{3 \cdot 6}} = 23 \text{ ft./sec.}$$

It is convenient in many ways to employ the equation of stability in a form that will give the critical value of H_n (and thus V_n), from the other known data, both when applying the equation to birds and in some cases to gliding models. This form of the equation is as follows:—

$$H_n = \sqrt{\frac{\mathrm{I}\,\Phi\left(\frac{1}{K} + \frac{1}{c\,C\,\rho\,\epsilon\,a\,\beta}\right)}{4\,l\,tan\,\gamma}}.$$

Thus in the case of the *hirundo apus* we have when $a = {\cdot}014$:—

$$H_n = \sqrt{\frac{{\cdot}00097\ (357 + 13{,}700)}{4 \times {\cdot}3 \times {\cdot}16}}$$

$$= \sqrt{\frac{13{\cdot}6}{{\cdot}192}} = 8{\cdot}4$$

or, $$V_n = 8\ \sqrt{8{\cdot}4} = 23{\cdot}2 \text{ ft./sec.}$$

which is the result already obtained.

A most gratifying feature in the application of theory to the investigation of bird flight is the consistent nature of the results; the agreement between aerodynamic and aerodonetic theory, and the agreement of both with the results of ordinary observation is quite as close as, in the absence of more accurate measurement, can be expected. Thus 20 ft./sec. was the author's own estimate of the lowest velocity of flight employed by the swift, before the theoretical computation had been made; and, in the case of the albatros, the velocity, 50 ft./sec., calculated from the aerodynamic tables (Vol. I., § 187), is in satisfactory agreement with the equation of stability.

As evidence of the soundness of the deductive method applied to the study of flight, a single experience may be cited. The stability of the albatros as first calculated by the author was unsatisfactory, and for a time the reason for this was not clear. It was when casting about for an explanation that the possible employment of the feet as an auxiliary tail area suggested itself, and after reading almost the whole of the albatros literature

extant, the author found confirmation of this view firstly in a photo-print in which the position of the feet could never have been located as a matter of ordinary observation, and later in

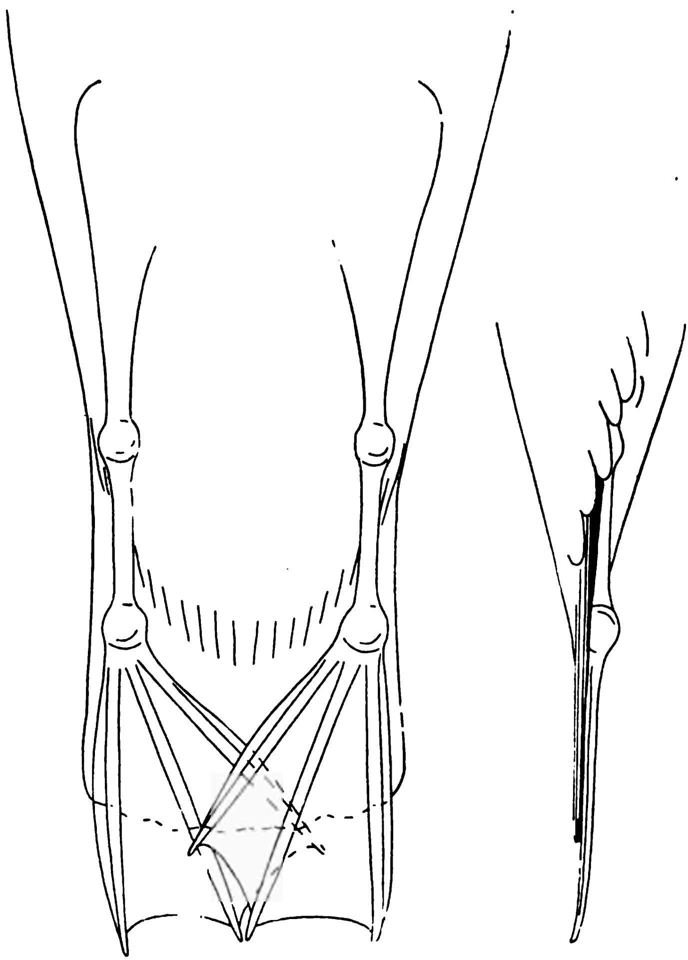


FIG. 71.

the report of the South Polar Expedition by Mr E. A Wilson, to which reference has already been made.[1]

[1] § 76.

§ 80. The Equation of Stability applied to the Author's 1894 Models.—There is considerable interest attached to the retrospective application of the equation of stability, as showing whether the early flight models of the author and others were of stable flight path, or whether their stability was only apparent and due to the comparative shortness of the flights made.

It is evident that, since the loss of equilibrium that takes place is due to the cumulative increase in the amplitude of the path, non-compliance with the equation of stability will not result in an immediate loss of equilibrium, and a flight path of several phase lengths is necessary for an experimental demonstration to be convincing.

Now that the equation of stability has been fully established, we may, in the absence of such prolonged flights, fall back on the equation itself to tell us whether the flight path was stable or otherwise.

Example. 1894 aerodone, Figs. 14, 15, 16, and 17.

Mass $= 1{\cdot}43$ lbs.

$\therefore\ W = 46$ poundals.

$V_n = 70$ ft./sec.

$\therefore\ H_n = 76$ ft.

$$K = \frac{46}{70^2} = {\cdot}0094.$$

$\beta = {\cdot}1$ (mean).

$Tan\,\gamma = {\cdot}25$ (probable value).

Tail data,

$l = {\cdot}39$ ft.

$a = {\cdot}2$ sq. ft.

Constants,

$\rho = {\cdot}078.$

$C = {\cdot}7.$

$c = 2{\cdot}0.$

$\epsilon = {\cdot}75.$

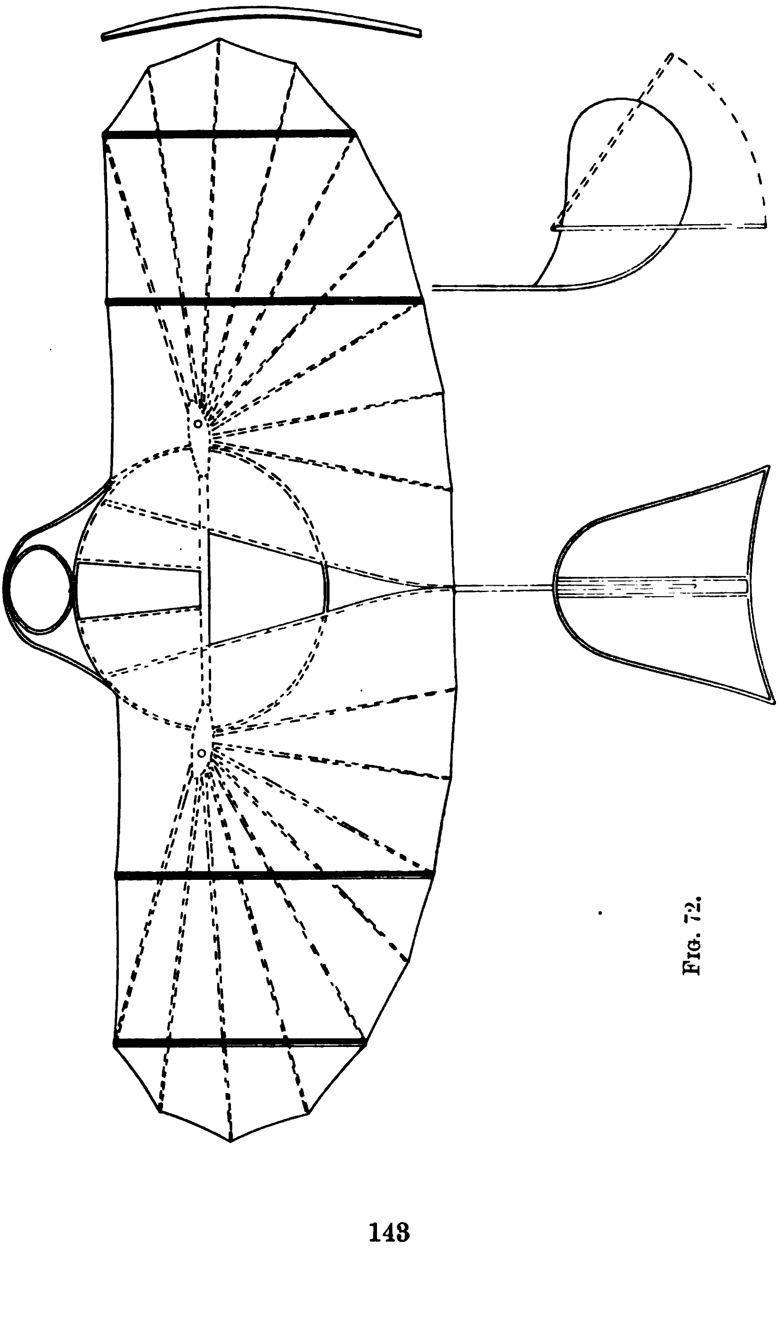


Fig. 72.

Moment of inertia,

$$I = 5 \text{ lb. ft.}^2$$

Calculation of Φ—

$$\Phi = \frac{4 \times 3{\cdot}9 \times 5{,}800 \times {\cdot}25}{5 \times (106 + 601)}$$

$$= \frac{22{,}600}{3{,}535} = 6{\cdot}4.$$

Example. 1894 twin screw aerodrome, Fig. 24.

$$\text{Mass} = 2{\cdot}5 \text{ lbs.}$$

$$\therefore \quad W = 80 \text{ poundals.}$$

$$V_n = 80 \text{ ft./sec.}$$

$$\therefore \quad H_n = 100 \text{ ft.}$$

$$K = \frac{80}{80^2} = {\cdot}0125.$$

$$\beta = {\cdot}1 \text{ (mean).}$$

$$Tan\ \gamma = {\cdot}25.$$

Tail data,

$$l = 3{\cdot}3 \text{ ft.}$$

$$a = {\cdot}25 \text{ sq. ft.}$$

Constants,

$$\rho = {\cdot}078.$$

$$C = {\cdot}7.$$

$$c = 2{\cdot}3$$

$$\epsilon = {\cdot}7.$$

Moment of inertia,

$$I = 13 \text{ lb. ft.}^2$$

Calculation of Φ—

$$\Phi = \frac{4 \times 3{\cdot}3 \times 10{,}000 \times {\cdot}25}{13 \times (80 + 455)}$$

$$= \frac{33{,}000}{7{,}000} = 4{\cdot}7.$$

§ 81. Lilienthal's Machine. Stability Investigated.—The late Herr Lilienthal achieved a considerable number of flights by the aid of an appliance represented in Figs. 72 and 73. In an account of these experiments given in *Nature*,[1] the following passage occurs, and is of interest not only as a statement of the manner in which Lilienthal sought to maintain equilibrium, but also as showing the views that were current at the date in question as to the stability of birds, and on the problem of flight generally.

"Lilienthal depends for the success of his apparatus on himself, trusting to his powers of *instinct* to keep his equilibrium by corresponding movements of his centre of gravity. Man, in this case, is the main flyer, the apparatus being only an adjunct, and it is from the ability of the former that he expects to obtain positive results. His apparatus is simple, cheap, and easily constructed; these are great points, as experiments can be carried on, even at the expense of the loss of a few machines.

"The whole success of aerial flight can be summed up in the word *equilibrium*, and it is here that the difficulty lies. Given a perfectly quiet or very nearly still air, there is no doubt that machines can be constructed so as to soar and travel through the air. This state of atmosphere is very rare; but, on the other hand, there are all sorts of disturbances, currents and wave motions, which render aerial navigation a far greater difficulty than is usually imagined.

"One often envies a bird which, with perfect ease, soars above us; but it must be recollected that it is endowed with a delicate system of nerves, which are always on the alert, to answer to any call made on them to sustain equilibrium. These movements are made quite unconsciously, and with the loss of a minimum amount of energy. To construct an apparatus that would accomplish this in an efficient manner would be simply impossible; but there seems no reason why man should not

[1] *Nature*, December, 1894, p. 177.

approximate to it to a certain extent by the help of an appropriate framework. With perseverance and many trials he should be able to master at least some of the rudiments, and eventually make short flights."

The author has collected the data necessary for the calculation of the stability factor of the Lilienthal machine from various reliable sources, the data for the drawings and most of the particulars having been obtained from an actual Lilienthal machine the property of Mr. T. J. Bennett of Oxford. The total weight with aeronaut, and the velocity are taken as given by Chanute in an article in the "Encyclopædia Britannica." The gliding angle is taken as approximately 10 degrees, which agrees closely with various accounts published, and the moment of inertia has been computed from the weights and disposition of the components.

The quantity β has been calculated aerodynamically, and has also been estimated from the curvature of the fore and aft ribs (Fig. 72), the results being respectively ·35 and ·32; the value ·33 has been taken as probably not far from the truth.

Tabulation of data:—

Mass $= 220$ lbs.

$\therefore \quad W = 7{,}100$ poundals.

$V_n = 23$ m./h. $= 34$ ft./sec.

$\therefore \quad H_n = 18$ ft.

$$K = \frac{7{,}100}{34^2} = 6{\cdot}1.$$

$\beta = {\cdot}33.$

$Tan\ \gamma = {\cdot}18.$

Tail data,

$l = 9$ ft.

$a = 12{\cdot}5$ sq. ft.

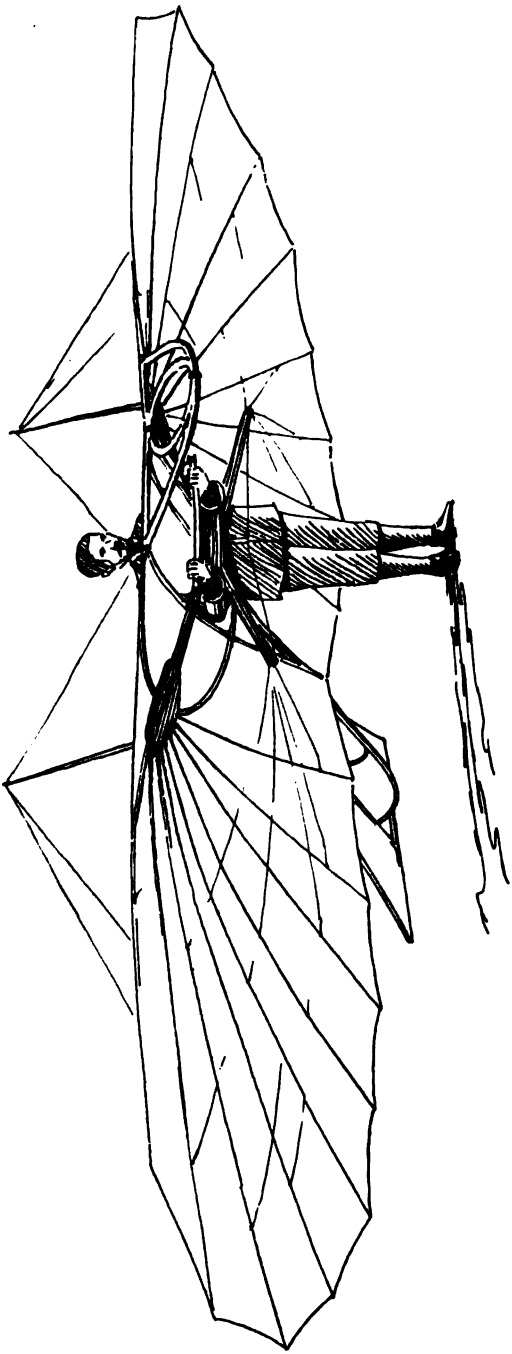


FIG. 73.

Constants,

$\rho = \cdot 078.$
$C = \cdot 66.$
$c = 1{\cdot}8.$
$\epsilon = \cdot 5.$

Moment of inertia,

		Mass.	Rad. of Gyration².	I.
Aerofoil	...	48	× $1{\cdot}75^2$	147
Tail	...	6	× 9^2	486
Man	...	166	× $1{\cdot}33^2$	294
			Total ...	927

Calculation of Φ—

$$\Phi = \frac{4 \times 9 \times 324 \times \cdot 18}{927 \times (\cdot 164 + 5{\cdot}2)}$$

$$= \frac{2{,}100}{4{,}960} = \cdot 424.$$

Thus the Lilienthal machine did not conform to the equation of stability, and it is literally true that the aeronaut trusted to his instinct to maintain his equilibrium. It is a most lamentable circumstance that Herr Lilienthal paid for his temerity with his life.[1]

[1] According to an account contributed to *Nature*, September 3rd, 1896, by Prof. Karl Runge, Herr Lilienthal met with his fatal accident on August 6th, 1896, when experimenting with a machine of the double deck type, that is to say, with an aerofoil consisting of two superposed members, otherwise closely resembling that of the illustration. It would thus appear that the actual machine from which the accident took place was not identical with that to which the above calculation applies. Further than this, according to the account, the loss of equilibrium was due to a sudden gust rather than an accumulated oscillation. In spite of these facts it is probable that a sufficient factor of stability would have been the means of saving Lilienthal's life; to appreciate this fact it is only necessary to experiment with models whose stability factor is less than unity, both indoors in still air and out of doors when there is the slightest movement of the air. The tendency of an aerodone whose Φ value is low (materially less than 1), to indulge in fatal

§ 82. **Resumé.**—The experimental evidence offered in the present chapter fully demonstrates the *Phugoid Theory* with the extension leading up to the *equation of stability* to be established fact. We are entitled to regard the main results achieved as proved beyond dispute.

Before passing on to the next branch of the subject it is desirable to review the position and point out some of the immediate consequences of the preceding work.

In the first place it may be remarked that the Phugoid Theory, besides providing us with a new point of vantage from which to study the phenomenon of flight as presented in various forms by Nature, constitutes *a key*, if not *the key*, to the problem of mechanical flight. By its means we can make all the calculations necessary to ensure the longitudinal stability of an aerodone or aerodrome. Besides being enabled to realise at a glance the essentials as to the general disposition of weights and supporting surface, etc., we can calculate the necessary tail length, tail area, and flight velocity, that will render the machine automatically stable in its flight path. The application of the theory in this respect and its extension to meet the conditions of power propulsion will be discussed in a later chapter.

One of the salient consequences of the phugoid theory is the *necessity of speed*. There is a relation between the size of an aerodone and the minimum velocity at which it is stable, thus the larger the model the higher the minimum velocity of stable flight becomes.

It might be thought that, by adding, as necessary to comply with the equation, to the length and area of the tail, the velocity

plunges, on the least provocation, is such as to impress vividly on the mind the terrible nature of the risk to which Herr Lilienthal exposed himself. Even the fact stated in the account, that the improved type of machine showed itself stable "in ballast," does not save the situation, for the moment of inertia of the man himself is a serious factor in the calculation, and the ballast, which in all probability was concentrated, would be comparatively deficient in this respect, and the stability of the machine would be proportionately improved when flying under ballast.

might be reduced to any desired extent. It may be easily shown, however, that this cannot be carried beyond a certain point, for the addition to the tail either as to area or length, beyond this critical point, results unavoidably in such an increase in the moment of inertia, as to do more harm than good, so that the stability factor will actually be diminished instead of being increased. Under these conditions the only method by which an increase of stability can be brought about is by an increase in the velocity of flight.

An examination of Herr Lilienthal's machine discloses the fact that this critical condition must have been very near at hand, for a large proportion of the moment of inertia is in the tail itself; thus an addition to the tail would have been of but little value; indeed, if not carefully designed it might actually have been detrimental. A few feet a second, however, added to the velocity of flight would have given the machine a sufficient reserve of stability.

It is evident that, the limiting conditions once settled, the velocities appropriate to a series of geometrically similar machines will be determined by the law of corresponding speeds, *i.e.*, the velocities will be proportional to the square roots of the linear dimensions. Now since under the supposed conditions the linear dimension increases as the cube root of the mass (or weight), the minimum safe velocity will require to increase as the sixth root of the mass. Thus, if we take it that Herr Lilienthal's machine (220 lbs.) would have been reasonably safe at 30 m./h., a machine of three tons total weight would require to fly at over 50 m./h., a velocity at which the h.-p. necessary suggests that a practical limit is being approached. We may therefore anticipate that the flying machine will not, on the most sanguine estimate, exceed some few tons in weight, unless some considerable advance is made in the prime mover, and in the near future at least, success may be looked for in the direction of far smaller units, certainly less than a ton, probably about half a ton in weight, constructed to carry but one, or at the most two,

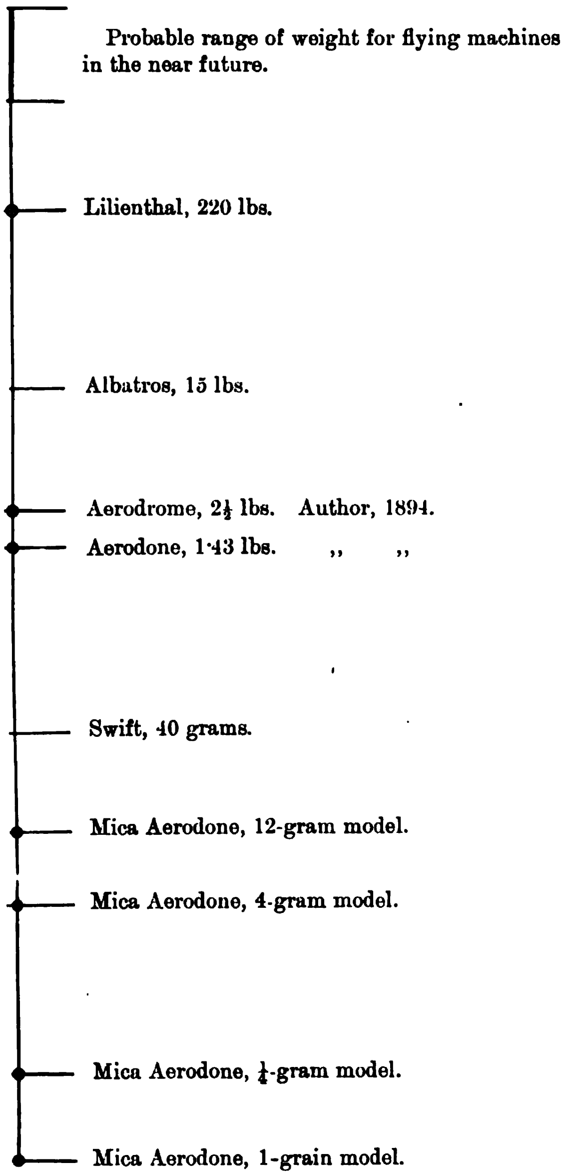

Probable range of weight for flying machines in the near future.
Lilienthal, 220 lbs.
Albatros, 15 lbs.
Aerodrome, 2½ lbs. Author, 1894.
Aerodone, 1·43 lbs. ,, ,,
Swift, 40 grams.
Mica Aerodone, 12-gram model.
Mica Aerodone, 4-gram model.
Mica Aerodone, ¼-gram model.
Mica Aerodone, 1-grain model.

Fig. 74.

aeronauts. This condition may cease to apply if some artificial means be found for adding to the stability beyond that inherent to a rigid aerodone.

The theory does not enable us to predict the conditions of stability for phugoids of considerable amplitude. It applies, strictly speaking, to curves of flight whose amplitude is a vanishing quantity, but evidently from the nature of the conditions and the demonstration it may be taken as applicable to curves of small amplitude such as those numbered 2 and 3 on the phugoid chart. This conclusion is confirmed by the experiments.

When the flight path amplitude becomes considerable we have at present only experience to guide us, and the author has found that an aerodone *just* stable for small amplitude, is distinctly unstable for greater amplitude; it is, however, at present not settled whether this is universally the case. Under all circumstances the stability factor required to ensure equilibrium for all amplitudes of flight path is not very great, a factor = 2 is more than sufficient under still air conditions, and an aerodone designed to such a factor will behave quite satisfactorily under what may be termed "ordinary working conditions," that is to say, in a wind to which its natural velocity is otherwise adapted.[1]

The range over which the Phugoid Theory and laws of stability have been experimentally demonstrated does not, unfortunately, include aerodones of the weight and velocity such as are involved in mechanical aeronautics, although the test as applied to the Lilienthal machine comes near to this point. There is no reason, however, to anticipate any departure from the laws of flight as laid down from theory and established by small scale experiment, for the range of these experiments is in itself sufficient to show that the equations can be considered as perfectly reliable. If we assume for example that the equations apply merely for as great a proportional range above the largest model experimentally

[1] Compare § 40.

investigated,[1] as that over which the experiments extend, we have for the upper limit an aerodone weighing 14,000 lbs., or more than six tons. Or, if we plot the weights of the aerodones investigated on a logarithmic scale (Fig. 74), the range from one half ton to two tons, such as probably constitutes the range of sizes of the flying machine of the more immediate future, is as represented. In such a logarithmic scale equal increments represent equal degrees of proportion, hence Fig. 74 represents graphically the relation of the range of the experimental verification, to the range with which the engineer is more immediately concerned in the problem of mechanical flight.

[1] The 1 lb. 7 oz. aerodone whose flight path is depicted in Fig. 25. This flight path showed by its rate of damping that the value of Φ was considerable, hence we may take the flight of this model as an experimental verification of the equation.

CHAPTER VII

LATERAL AND DIRECTIONAL STABILITY

§ 83. Introductory.—Lateral and directional stability relate to the equilibrium of the aerodone in respect of its three degrees of freedom involved by motions other than those in the phugoid plane.[1] Thus lateral stability is concerned primarily with rotation about the axis of flight; that is, a motion resembling the

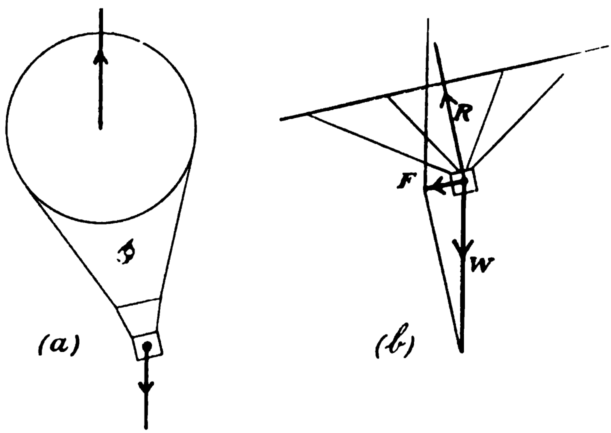


FIG. 75.

rolling of a ship; if such a motion does not give rise to a restoring couple an aerodone may turn over, otherwise "capsize," and so lose its equilibrium. Directional stability involves rotation about a vertical axis and motion of translation at right angles to the phugoid plane, these two kinds of motion being associated in any *change of course*.

[1] The vertical plane containing the flight path: see Chap. II.

In the case of an aerostat or balloon the lateral stability is due to the relative positions of the respective centres of gravity and displacement, the former being vertically beneath the latter, so that if at any instant the conditions of equilibrium are disturbed, a righting couple arises automatically, as shown diagrammatically in Fig. 75 (*a*). It is evident that in the case of an aerodone we cannot obtain stability this way, for the support is derived from the reaction of the aerofoil, which is not constant as to direction, but changes with the movements of the aerodone. Thus in Fig. 75 (*b*) the resultant of the weight of the aerodone W and the supporting reaction R does not constitute a couple as in Fig. 75 (*a*), but a lateral force F that gives rise to an acceleration at right angles to the line of flight. Hence, although only one degree of freedom is *primarily* concerned in the question of lateral stability, motions in two degrees of freedom are actually involved, and it is by the interaction of motions of these two kinds that we must look for the means of giving lateral stability to an aerodone in flight.

§ 84. The Mutual Relationship of Lateral and Directional Stability.—It is shown in the preceding section that each of the kinds of stability that we are about to study involves *two* degrees of freedom, and as there are in all but three degrees of freedom, one of these, *i.e.*, motion of translation at right angles to the phugoid plane, is involved in common. One consequence of this is that the two kinds of stability are very closely associated. To such an extent is this found to be the case that the propriety of treating lateral and directional stability as two separate problems is open to question.

The mutual relationship of the two problems will be made more clear when some consideration has been devoted to them individually, for although the author has under consideration a broader and more comprehensive method of dealing with the present branch of the subject, the only treatment ready for presentation is that which has to some extent stood the test of

experience, and in which each problem is first treated as if the other did not exist, the interaction of the two being taken subsequently.

It is perhaps of some interest to state that the individual problems of lateral and directional stability are comparatively simple, and that the whole of the complication of the present subject arises when they are taken in conjunction. The actual discovery of the connection, simple as it appears when the case is clearly stated, was made owing to the anomalous behaviour of certain of the author's models, the converging spiral flight path, mentioned in § 6, being the disorder that in the first place drew attention to the point.

§ 85. Lateral Stability.—The problem of providing lateral stability consists in securing that the aerodone shall not upset by turning about the axis of flight. It is necessary to so arrange that any departure from the plumb shall automatically call into existence a restoring couple.

It is assumed in the present stage of the discussion that the aerodone is only permitted the two degrees of freedom essentially involved, in other words, it is not allowed to turn about a vertical axis, and its motions in the phugoid plane are supposed to be in equilibrium *inter se* in accordance with the principles demonstrated in Chaps. II. to VI.

We have already examined a case of automatic stability in the behaviour of the ballasted aeroplane. In § 5, referring to Fig. 5, it has been shown that when the plane assumes a laterally inclined position (owing to some initial disturbance), the resulting motion causes the centre of pressure to move laterally, so that the necessary restoring couple arises and the plane returns in due course to its horizontal position. Similarly, in the author's flight models, §§ 10, 11, etc., the vertical fins perform the same function as is performed in the ballasted aeroplane by the change in the position of the centre of pressure; when the aerodone acquires a lateral motion the

pressure that arises on the upwardly projecting fins acting above the mass centre of the aerodone supplies the righting moment.

There is another modification of design by which the same

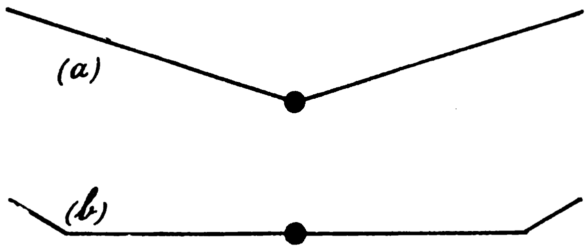


Fig. 76.

effect can be produced. If an upward inclination is given to the two wings of the aerofoil, Fig. 76 (*a*), or to the extremities only, Fig. 76 (*b*), a restoring couple arises as an immediate result of any attempt on the part of the aerodone to capsize. Thus the

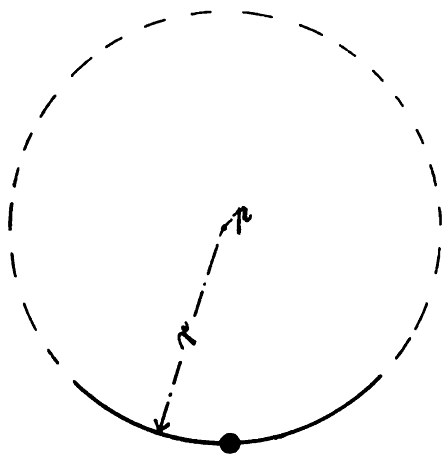


Fig. 77.

upwardly inclined "wings" (*a*), or the alternative form of (*b*) may be looked upon, in effect, as giving the aerofoil the form of the arc of a circle, Fig. 77, in which case it is evident that if oscillating its *trace* in space will be a part of a cylindrical surface

whose axis is parallel to the line of flight; this cylindrical surface is represented in the figure by the dotted circle.

Now if we suppose an aeorodone having an aerofoil of arc form and radius r to suffer disturbance it is evident that it will, as the result of such disturbance, proceed to oscillate in the cylindrical flight path trace, just as if it were suspended from the point p the centre of the dotted circle (Fig. 77), and its period of lateral oscillation will be that of a pendulum of length r. We require to examine the problem from this point to ascertain the conditions under which this oscillation takes place, as affecting the growth or damping of its amplitude.

§ 86. Lateral Stability. Oscillations in the Transverse Plane.— There is a certain similarity between the problem now before us

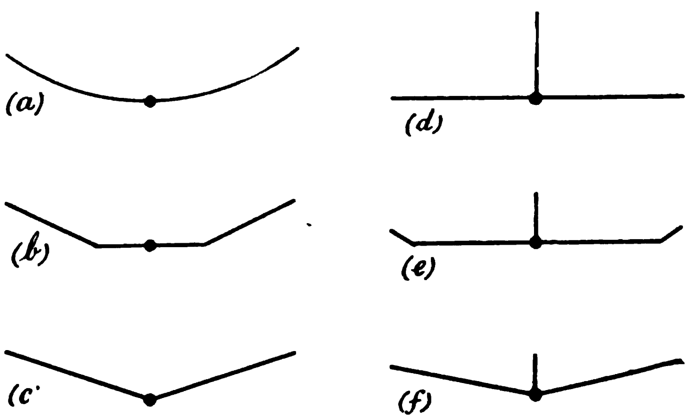


Fig. 78.

and that dealt with in Chap. V., the oscillations in the phugoid plane; there are two influences at work, one tending to an increase and the other to a decrease in the amplitude of motion.

In the first place it may be remarked that any departure from the arc form depicted in Fig. 77, results in a resistance to the oscillations and consequently a diminution of amplitude. Similarly, an artificial damping may be effected by employing

a combination of the fin and inclined extremities, or the fin method of obtaining lateral stability alone constitutes a damper of the most effective kind. Thus, referring to Fig. 78, *a*, *b*, and *c* are forms relatively deficient in damping action, whereas forms *d*, *e*, and *f* may be regarded as highly efficient in this respect.

It is one of the advantages of the finned aerodone adopted by the author that it is a form whose behaviour lends itself readily to calculation; it is a disadvantage of this form that it offers a greater surface to the air for a given degree of stability, and hence its skin friction is greater than is necessary. A comparatively slight "turn up" of the wing extremities will give lateral stability, without adding materially to the frictional resistance.

The above facts fully account for the fin form being unknown in Nature. It is no advantage from the point of view of Nature that her designs should be calculable by mathematical processes, but it is of the greatest importance that the resistance should be reduced to a minimum.

§ 87. Lateral Stability. Oscillations in the Transverse Plane (continued).—Let us now, in the typical case represented in Fig. 77, examine the influence of changes in the position of the centre of pressure as due to an oscillation of small amplitude.

We will, in the first instance, assume that the moment of inertia of the aerodone about the axis of flight is a negligible quantity, that it is in fact of zero magnitude. We will further assume that the aerodone experiences no resistance to its oscillation, a condition that is most nearly approached by the arc form of Fig. 77, it being supposed that skin-friction is absent. It is evident that in practice an aerodone must possess moment of inertia about the axis of the path of flight, and that there will be damping of the oscillation due to skin-friction and other kinds of lateral resistance, but the influence of those factors will be considered subsequently.

In Fig. 79 it is supposed that the aerodone is in the middle of

an oscillation, the direction of its motion being indicated by the arrow. Now the lateral motion will result in the displacement of the centre of pressure of the aerofoil (laterally) in the direction of movement, and in the ordinary way this would give rise to a couple or torque acting on the aerodone about the axis of flight,[1] Fig. 79 (*a*); but under the assumed conditions, owing to

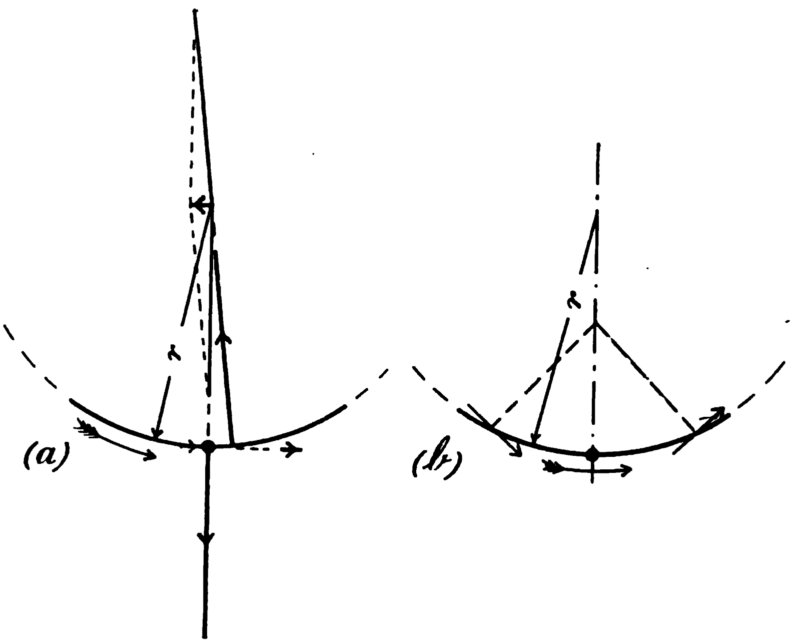


FIG. 79.

the absence of moment of inertia, this is impossible, the resulting torque must be again transmitted to the air immediately, consequently the directions of motion of the extremities of the aerofoil will be deflected as shown in Fig. 79 (*b*), so that the

[1] In Fig. 79 (*a*) the resultant of the two forces, gravity and pressure reaction, passes through the centre of curvature and not through the mass centre; consequently this resultant and the equal and opposite inertia force are not in equilibrium, but form a couple. This couple is indicated by the horizontal forces shown in the figure.

change of position of the centre of pressure due to change of *aspect* will be neutralised by an equal and opposite change in the centre of pressure due to a certain screw-like motion impressed on the aerofoil.

It would thus appear that the radius of the path of oscillation would be less than that due to the radius r by an amount depending upon the change of effective *attitude* of the two wings of the aerofoil as represented by the arrow heads in Fig. 79 (b). We may also regard this shortening of the oscillation radius as due to the superposition of the rotation of the screw-like motion (consequent upon the change in the centre of pressure) on the normal oscillatory movement, the two being always of like sense.

§ 88. Oscillations in the Transverse Plane (continued).—At and about the limits of the oscillation, when the lateral velocity is negligible, the radius will evidently become equal to r, for the screw-like motion vanishes. We may, therefore, represent the path of oscillation as generated from an evolute whose "graph" is given diagrammatically in Fig. 80. Under the assumed conditions the oscillation, once initiated, will be permanent, neither increasing nor decreasing in amplitude.

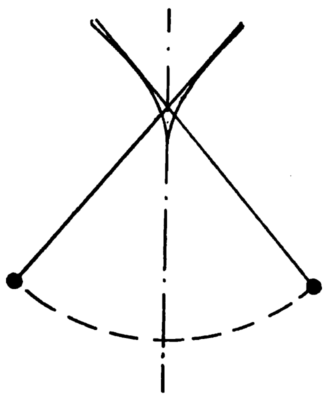


FIG. 80.

Since the mean radius of the path of oscillation is less than that of the aerofoil (assuming the arc form), it is evident that in the case of an aeroplane, where r becomes infinite, there will still be an oscillation about an approximate centre situated some distance above the line of flight. This is illustrated in Fig. 81, it being supposed, as in Fig. 79, that the aerodone is in mid position, the displacement of the centre of pressure due to the lateral component of motion giving rise to a screw-like motion

as indicated. The period of oscillation will evidently be slow in comparison with a form such as illustrated in Fig. 79.

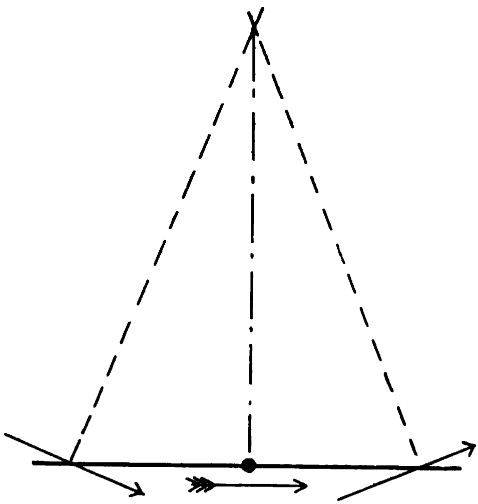


FIG. 81.

The limiting case, the aerofoil whose motion is the equivalent of an infinite pendulum, evidently lies in the region of inverse curvature, that is to say, the form of the aerofoil will be laterally

FIG. 82.

arched as in Fig. 82. There must manifestly be some particular degree of curvature that under given conditions will be neutral in respect of lateral motion, the change in the position of the

FIG. 83.

centre of pressure due to endwise motion being compensated by the direct pressure components acting on the inclined wing

surfaces. An aerodone whose aerofoil conforms to the necessary conditions is incapable of receiving a lateral oscillation.

It is highly probable that we have here the reason of the habit of flight of many birds whose wings have a somewhat downward trend or "droop"; the common swift (Fig. 83) being a notable example.

§ 89. Oscillations in the Transverse Plane. Damping Influences. —The damping influences at work tending to diminish the amplitude of the lateral oscillation are of two main kinds, *i.e.*, skin-frictional and aerodynamic. The former is always present to a greater or less degree, the latter may or may not be present, but may in any case be provided by departure from the simple arc form as in Fig. 76, or by the deliberate fitting of fins, organs somewhat analogous in their present usage to the "bilge keels" of sea-going vessels, as in Fig. 78 (*d*), (*e*), (*f*).

It is probable that birds rely largely on the lateral resistance of their body form, which perhaps gives rise to motion of the discontinuous type when moving obliquely; also the skin friction on the wing surfaces is evidently an important factor. Beyond this, the nicety of adjustment attainable by the living organism doubtless permits of the oscillation period being rendered so slow that but little damping is required.

In the construction of flying machines also it may be found that the "body" can be relied on as a means of damping lateral oscillation, but the author believes that a certain amount of additional fin area will generally be found of service, as used in his model experiments, and in several actual machines that have been built by other investigators.

§ 90. Oscillations in the Transverse Plane. Influence of Moment of Inertia.—The influence of moment of inertia about the axis of flight as affecting the transverse oscillation is a matter of great complexity, and up to the present the author has only been able to arrive at certain elementary conclusions of a qualitative kind.

Let Fig. 84 represent the path of lateral oscillation of an aerodone whose whole mass is supposed concentrated at its centre of gravity, so that the moment of inertia about the axis of flight is zero. It is evident that in the course of its oscillation

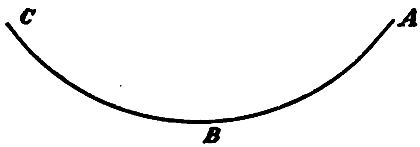


FIG. 84.

the aerodone in passing from A to B acquires clockwise rotation, and in passing from B to C it loses the rotation so acquired.

Let us now investigate the change to be anticipated in the form of the oscillation as the result of supposing the aerodone

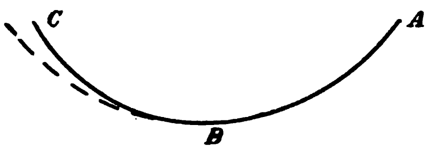


FIG. 85.

to possess moment of inertia, as must in reality be the case. Let us suppose that a change in the moment of inertia from zero to some finite value take place at the point B, then the rotation during the portion of the oscillation B C will tend to

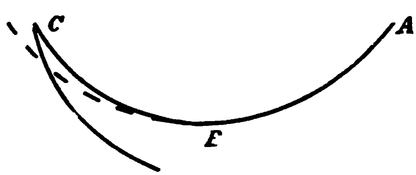


FIG. 86.

persist, and the path of oscillation will be modified accordingly, as shown in Fig. 85.

When the point C is reach the aerodone will still possess some residuary rotation so that its flight path will continue in

the manner illustrated in Fig. 86. If we endeavour to trace the path of oscillation further, we know that on the principle of work, assuming that there is no damping, the aerodone will rise till it touches the datum line A C, say to the point A_1 (Fig. 87), when the conditions will be similar to those already examined

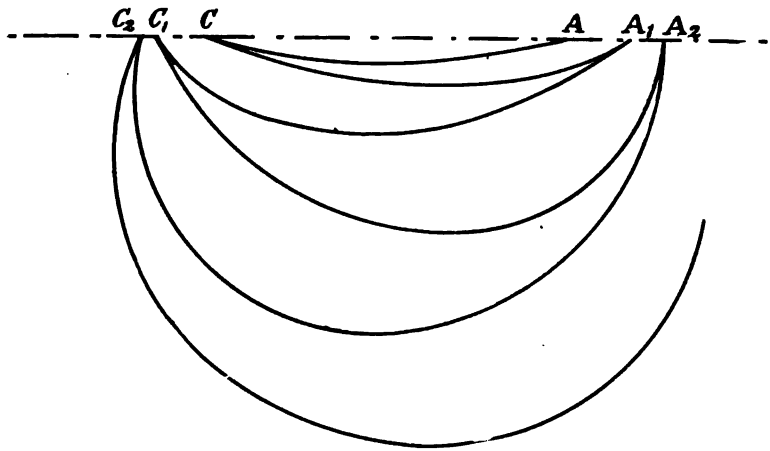


FIG. 87.

at the point C_2, so that we may continue the diagrammatic representation in the manner shown.

§ 91. Oscillations in the Transverse Plane. Form of the Oscillations.—It is probable that the gliding angle is prejudicially affected by the lateral oscillation, so that the greater the amplitude of the latter the steeper the mean gliding angle will become. It has been assumed in the foregoing section that the diagram of the flight path is a projection on a plane at right angles to the mean gliding path, and that the latter is unaffected by the oscillation amplitude. When there is any damping taking place the dissipation of the energy of oscillation will result in the aerodone not rising to the datum line A C (Fig. 87), and the oscillation will consequently terminate at a lower point than would otherwise be the case. When there is no actual damping it is possible

that the same is to some small extent true, owing to the aerofoil not finding itself under the most favourable conditions for economic flight, but this is a point we can afford to ignore; we

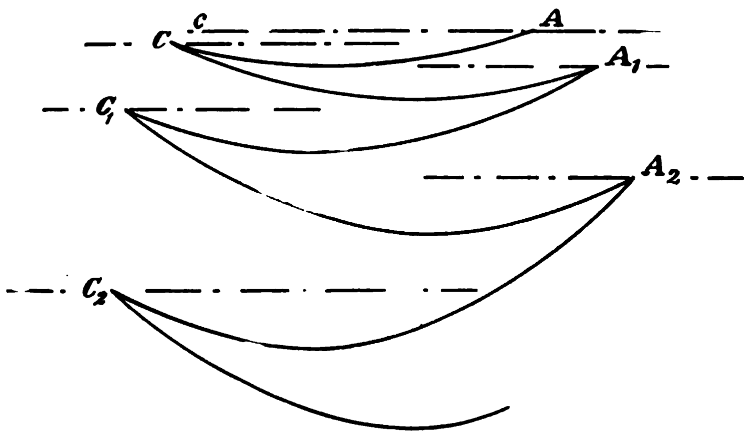


FIG. 88.

may, for instance, suppose that any effect of this kind is in itself a separate kind of damping; we may alternatively presume that the direction of projection is varied according to the changes in

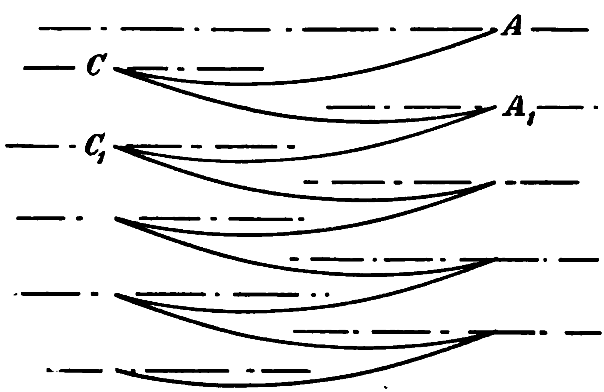


FIG. 89.

the gliding angle, assuming no real damping to take place. Either supposition for the present purpose seems legitimate, the

point, however, might assume greater importance if any means were found of dealing with the problem quantitatively.

Let us now suppose that we have damping influences at work; then, in addition to the moment of inertia effect of the preceding section, by which the amplitude was shown to increase (Fig. 87), we shall have a disappearance of energy at each oscillation that, if of sufficient magnitude, may result in equilibrium or in a definite diminution of amplitude. Thus, in Fig. 88, the aerodone beginning an oscillation at the point A, which in the absence of damping would rise at the end of that oscillation to the point c, will now, owing to the loss of energy, rise to a point C of less

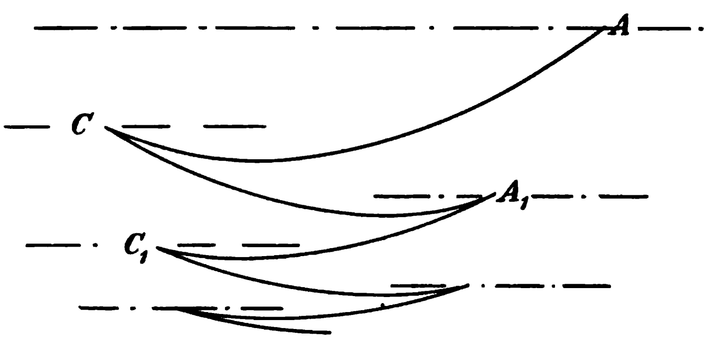


FIG. 90.

altitude, and on the succeeding oscillation it will find itself at a point A_1 of lower level still, and so on.

Under these circumstances it is evident that the path of oscillation may either increase in amplitude as before, but at a slower rate, as in Fig. 88, or it may remain constant as in Fig. 89, or again, it may gradually damp out as illustrated in Fig. 90.

One incidental result of an oscillation in progress of damping will evidently be that the gliding angle proper will have superposed on it a further angle of descent due to the dissipation of the energy of the oscillation. In the Figs. 88, 89 and 90 the direction of projection is supposed to be parallel to the gliding path proper, without this added angle, so that the damping effect makes its appearance as a separate item.

§ 92. Lateral Stability. Oscillations in the Transverse Plane in Practice.—The conclusions of the foregoing sections are fully borne out by actual experience. The form of the observed oscillation path is quite in agreement with that given from theoretical considerations in Figs. 87, 88, 89 and 90. The fact that an aerodone in a state of oscillation has an abnormally steep gliding angle, is also found to be true. Finally the importance of avoiding excessive moment of inertia about the axis of flight is a fact that it is quite easy to verify experimentally.

The author has frequently met with the lateral oscillation in the course of model experiments. From an examination of a number of models that have shown a tendency to develop oscillation it has been found that the weight of the aerofoil should in no case exceed ·5 to ·6 of the total weight, although this is but a rough and ready measure, the moment of inertia does not for a given aerodone depend upon aerofoil weight alone; thus the oscillation will occasionally make its appearance even when the aerofoil weighs considerably less than ·5 of the total.

As illustrating the point in question, it has been frequently noticed that an aerodone that has been quite steady in flight when first made, has developed a habit of "fluttering" when the wings have been repeatedly repaired with paper patches, so that their moment of inertia is increased.

It would thus appear that moment of inertia in any aerodone or flying appliance is bad, whether about the transverse axis or about the axis of flight; the mass should be as concentrated as possible. It is interesting to note how excellent are the forms employed by Nature in both respects.

§ 93. Directional Stability.—In directional stability two degrees of freedom are involved, namely, rotation about a vertical axis, and motion of translation at right angles to the phugoid plane; thus *direction* of flight stands in the same relation to an aerodone or flying machine, as the nautical term *course* to a ship at sea.

In considering the present subject from an aerodonetic

standpoint we are concerned with the equilibrium of the aerodone in respect of its direction of motion for the time being, rather than with the continued maintenance of a given *course;* the latter is a question of navigation, given a machine that is stable; the rest is a matter of holding the machine to the desired direction by appropriate steering mechanism. Automatic steering to a fixed course is a problem that has been very thoroughly worked out in connection with the Whitehead torpedo, and without doubt, if required, the same principle[1] could be applied to an aerodone. This question, however, is reserved for later discussion.

It is essential to the directional stability of an aerodone that any change of *aspect* should give rise to a restoring couple. It is evident that if the aerodone be symmetrical about the phugoid plane, its normal aspect will be one of equilibrium—stable or otherwise. It is also evident that the stability of the equilibrium will depend upon the organs of resistance to lateral motion; thus if the fin area is well abaft the mass centre, as in the feathering of an arrow, the equilibrium will be stable, whereas if the fin area is too far forward instability will result. In the latter case any small departure from the normal will give rise to a couple tending to exaggerate the error.

§ 94. Directional Stability (continued). — Although it is not necessary for our present purpose that an aerodone shall maintain a constant direction of flight, still it is desirable that it shall have some power to resist any disturbance tending to change its course. Thus in the case of an arrow shot from a bow, the directional equilibrium is rendered stable by the feather at the after end of the shaft, but this does not prevent an arrow from obeying implicitly any transverse force, as, for instance, that of gravity; the trajectory is approximately identical with that of any other kind of missile.

The same conditions do not apply in respect of the action of

[1] For a discussion of the gyroscope in its application to problems in stability, and direction maintenance, see Appendix VII.

gravity in the case of an aerodone, for we are no longer concerned with motion in a vertical plane; but considerations of lateral stability and the phugoid theory both require that the aerodone shall preserve its direction with some degree of obstinacy, and thus a tandem system of fins is adopted, as already described and illustrated in § 11.

The simplest case of such a fin combination from a theoretical point of view is one in which the one fin is situated at or about the centre of gravity of the aerodone and acts exclusively as an *abutment*, the function of the other fin, which is arranged some distance from the first, being purely directive, that is to say, it acts as a wind vane, or as the feather of an arrow. It is evident that, other things being equal, the rate of displacement of the aerodone in the face of an applied transverse force will be inversely as the area of the abutment fin,[1] and further, the consequent angular movement, that is the change of course, will be inversely proportional to the distance separating the fins.

§ 95. A Study in Directional Equilibrium and Maintenance.—It is evident from the foregoing that the question of *direction* involves

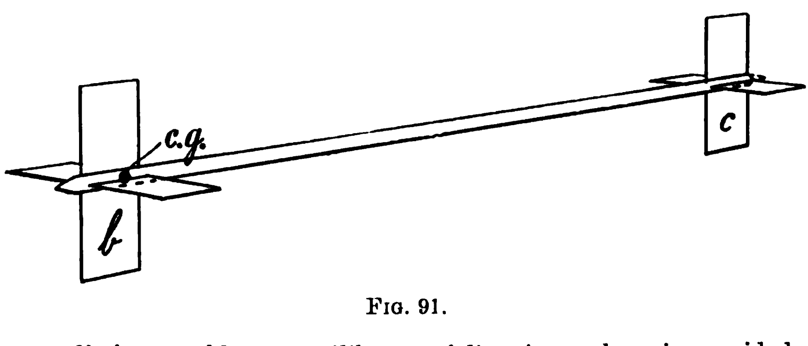


Fig. 91.

two distinct problems, *equilibrium of direction* such as is provided by the feather of an arrow, and *direction maintenance*, that is, the capacity to resist forces tending to divert the aerodone from its line of flight, provision for which takes the form of an *abutment*

[1] Depending on the law of the small angle, $P_\beta \propto \beta$.

fin. The wider problem of securing an absolute sense of direction, which involves some communication between the aerodone and the outer world (either optic or magnetic), or the employment of a gyroscope, is outside our present purview.

Let us take for the purpose of initial study a special kind of arrow illustrated in Fig. 91. Let Λ be the effective[1] distance separating the abutment fin b from the wind vane c, let r be the radius of the path curvature, and let β be the angle made by the

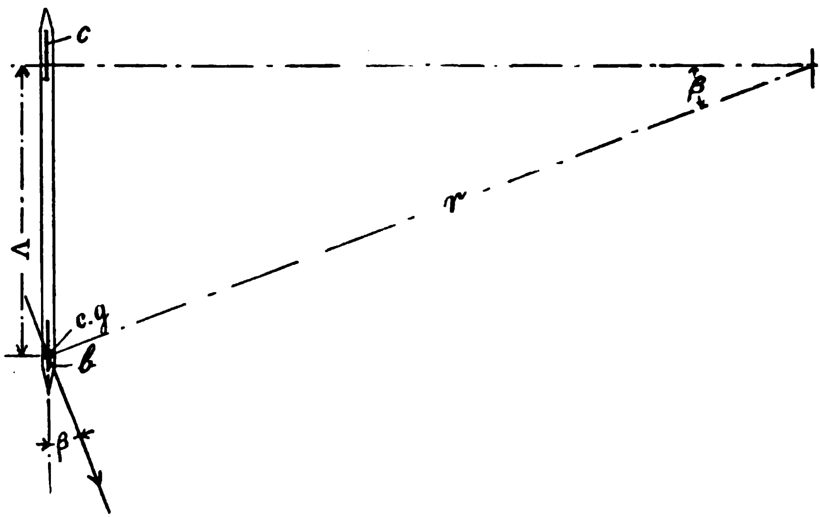


FIG. 92.

abutment plane to its direction of motion through the air (Fig. 92). Then (for small angles) $r = \dfrac{\Lambda}{\beta}$. (1)

Now β depends upon the lateral force, the velocity, and the area and form of the abutment plane, and for any given value of β we have $r \propto \Lambda$.

Let us suppose that Λ be made infinite, then if a force act laterally

[1] Strictly speaking from *centre of pressure* to *centre of pressure*; this, under the assumed conditions, will be from $\frac{1}{4}$ to $\frac{1}{3}$ the width of the fin from its leading edge.

on the mass centre it will give rise to a sideways movement, the lateral velocity eventually becoming sufficient to produce an aerodynamic reaction on the abutment plane just equal to the applied force. Under these circumstances the direction of motion is for the time being modified, but owing to the supposition that Λ is infinite, no rotational movement takes place, and so when the external force ceases to be applied the aerodone or arrow resumes its original direction.

But here we meet with a point of some importance. If the "tail," instead of extending infinitely rearwards, be conceived to extend forwards, that is, if Λ instead of being $+\infty$ be taken as $-\infty$, we find on examining the conditions that no alteration has resulted; the aerodone will behave in precisely the same manner in either case. It is therefore probable that if the tail *extend forward*, provided it is of sufficient length, it will give directional stability. This result, which is somewhat unexpected, calls for further investigation.

§ 96. A Study in Directional Equilibrium (continued).—

Let $\mathbf{a}$ = area of abutment fin.
,, m = mass of aerodone.
,, V = velocity.
,, ρ = density of fluid.
,, r, Λ and β stand as before.

It is now assumed that the guide vane *precedes* the abutment fin (Fig. 93), and Λ will be taken as positive measured forward.

Now in absolute units the centrifugal force of the mass m when the path becomes curvilinear is given by the expression $\frac{m V^2}{r}$, that is, for a given value of m and V centrifugal force varies inversely as r.

But by (1) r varies inversely as β. Hence the centrifugal force varies directly as β.

And we know that for small angles the aerodynamic reaction

on the abutment fin varies directly as β. Therefore the centrifugal force varies directly as the reaction on the abutment fin. And if the centrifugal force is less than the reaction on the abutment fin the conditions are those of stable equilibrium, and conversely if it is in excess the equilibrium is unstable.

Hence the limiting condition of stable equilibrium is given by equating the two, or,

$$\frac{m V^2}{r} = c\,\beta\,\mathbf{a}\,P_{90} = c\,\beta\,\mathbf{a}\,C\,\rho\,V^2$$

but, by (1) $m = \frac{\Lambda}{\beta}$ $\therefore$ $\frac{m\,\beta}{\Lambda} = c\,\beta\,\mathbf{a}\,C\,\rho,$

or $$m = c\,C\,\rho\,\mathbf{a}\,\Lambda.$$

If the mass of the aerodone is in excess of that given by the above expression, the centrifugal force that results from any

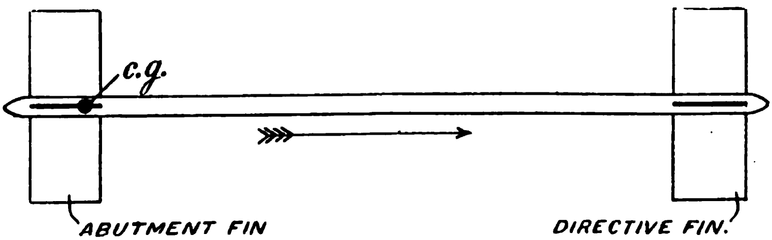


FIG. 93.

slight deviation from the straight line is greater than the restoring force—the reaction on the abutment fin, and the directional equilibrium is unstable; any deviation from the straight, however slight, tends to increase.

If the mass is less than that given by the expression, the restoring force will exceed the centrifugal, and the equilibrium is stable.

On the supposition that the abutment fin is of approximately "square" proportion, the product of the constants[1] $c\,C\,\rho = 2 \times {\cdot}66 \times {\cdot}078 = {\cdot}103$; or, if we suppose the fin to be a plane of $\mathbf{n} = 4$, in pterygoid aspect, this value becomes $2{\cdot}27 \times {\cdot}70 \times {\cdot}078 = {\cdot}124$.

[1] For air.

§ 97. Directional Stability (continued).—The importance of the foregoing investigation is found in the fact that the limiting condition of directional stability lies beyond that at which the aerodone or arrow is neutral. In the actual problem the abutment fin has to deal with an accidental applied force as well as the centrifugal force due to change of direction; this would

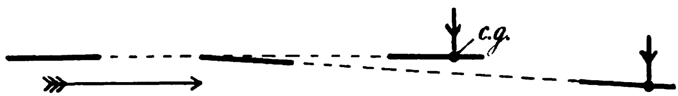


Fig. 94.

naturally lead to the limiting condition being reached sooner, but does not otherwise alter the problem.

Let us examine the two cases in which an aerodone is supposed fitted respectively with a positive and negative tail, we will, for the present purpose, take the rearward tail (Fig. 94) as positive, and the forward tail (Fig. 95) as negative.[1] We will assume that in both cases there is an applied transverse force, as indicated, acting on the mass centre, just as would be the case if the aerodone had assumed a laterally inclined position.

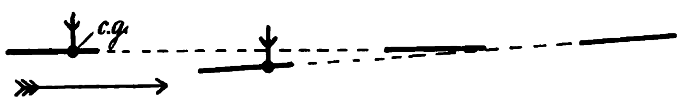


Fig. 95.

Now it will be seen that, in the first case (Fig. 94), the aerodone will undergo a change of *course* in the direction of the applied force. This *change of course* is not due to the direct sideways motion impressed, but is measured by the *change in the angular position;* thus, the instant the applied force is withdrawn, the direct sideways motion rapidly falls to zero, but the change of angular position, that is, the *change of course*, remains. And in Fig. 95, when the tail is negative, it will be seen that the *change*

[1] This seems the most natural convention; the *real tail* is a positive tail.

of course is in the opposite direction to the applied force, in spite of the fact that the lateral displacement directly due to the force during its application is the same as before.

Now it is evident that in the intermediate condition, when the length of the tail is plus or minus infinity, the *change of course* is zero, and if we can simulate this condition we shall have an aerodone whose directional maintenance is perfect in the sense of the present investigation.[1] It is this condition that is above referred to as one in which the aerodone is in *neutral equilibrium*, or *directionally balanced*, and the result that this condition can be reached without imperilling the directional stability is one of considerable importance.

§ 98. Directional Stability. Practical Considerations.—It is evidently not possible, on the lines so far investigated, to construct an aerodone that shall be directionally balanced, owing

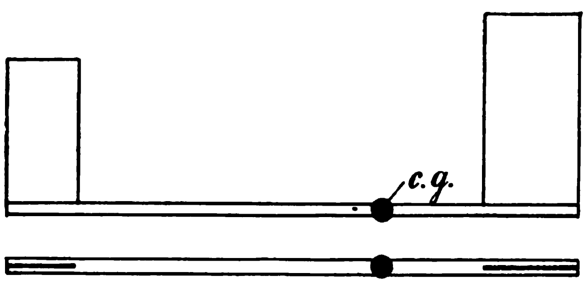


FIG. 96.

to the fact that whether the tail be made positive or negative, its length requires to be infinite. It is further evident that the required conditions cannot be *directly* approximated, for the length of tail necessary would so add to the moment of inertia about the transverse axis as to lead to longitudinal instability.

There is, however, an indirect method. The vane employed

[1] §§ 93, 94.

for directional equilibrium may be also made to do duty as part of the abutment area; thus, in Fig. 96, the two fins are jointly responsible for the abutment and jointly responsible for the directional equilibrium. Any combination of fins of this kind will be found to possess an *equivalent* tail length and abutment area, and this tail length may be made positive, neutral, or negative at will, without the actual distance between the fins exceeding a given desirable value. We may term the process of determining the equivalent value of any particular combination, *fin resolution*, and the object of the designer is to obtain a combination that will render the aerodone directionally balanced, that is, an equivalent tail length $= \infty$, or such other value as he may deem expedient. The resolved abutment area, also, must be sufficient for the requirements of lateral stability, *i.e.*, capable of sustaining any chance applied forces, and of rapidly damping any side oscillation.

§ 99. Fin Resolution.

Let a_1 = area of front fin.

„ a_2 = area of rear fin.

„ Λ = fin tail length (as before).

„ λ = distance from fin to fin (centres of pressure).

„ λ_1 = distance from mass centre to front fin.

„ λ_2 = distance from mass centre to rear fin.

„ $\frac{p_2}{p_1} = y$ = ratio of pressure on rear, to pressure on front fin, due to force acting through mass centre.

„ β_1 = angle made by the front fin to line of flight.

„ v_1 and v_2 = the lateral components of the velocities of the front and rear fins respectively.

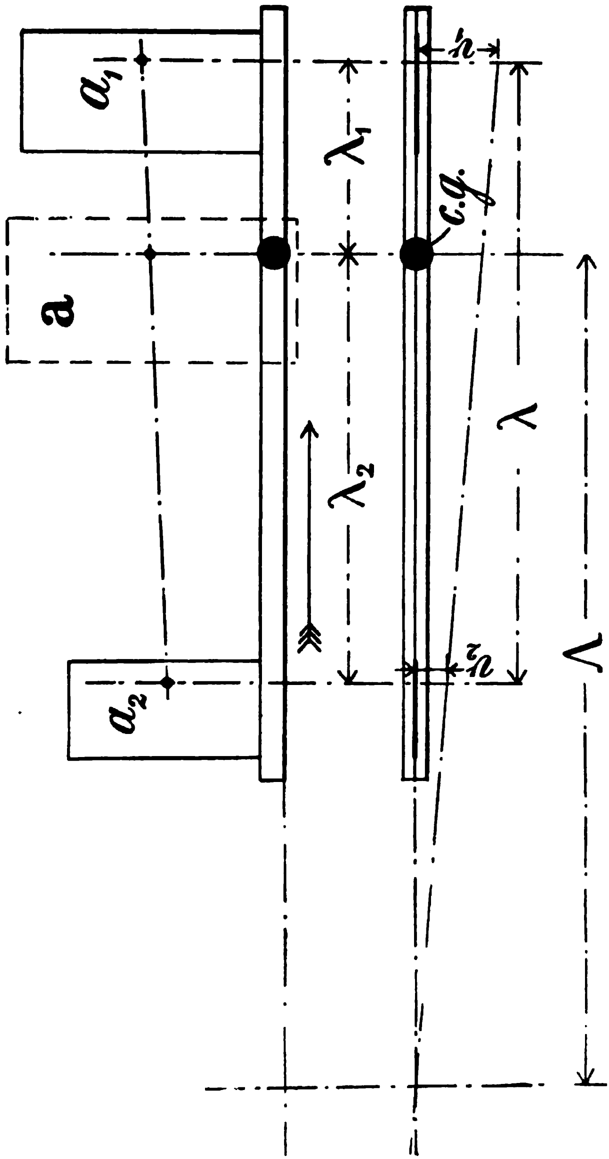


Fig 97.

Referring to Fig. 97, we have, from geometrical considerations,

$$\frac{v_2}{v_1} = \frac{\Lambda - \lambda_2}{\Lambda + \lambda_1},$$

whence, since
$$\lambda = \lambda_1 + \lambda_2$$

$$\Lambda = \lambda \frac{v_1}{v_1 - v_2} - \lambda_1. \tag{1}$$

Now from aerodynamic considerations,

$$p_1 = c_1\, C_1\, \rho\, V^2\, \beta_1$$

or
$$\beta_1 = \frac{p_1}{c_1\, C_1\, \rho\, V^2}$$

but
$$v_1 = V\beta_1, \text{ or } v_1 = \frac{p_1}{c_1\, C_1\, \rho\, V}. \tag{2}$$

The expression for v_2 is more complicated. The rear fin is situated in the wake of the front fin, and is, consequently, influenced by its "wash." We may regard the lateral velocity of the rear fin as made up of two components; the one due to its motion through the fluid, the value of which is computed in the same way as for the front fin, and the velocity of the fluid itself as represented by the motion impressed upon it by the front fin, that is to say, the wash aforesaid.

The first of these will be given by the expression,

$$\frac{p_2}{c_2\, C_2\, \rho\, V}.$$

The second, it follows from aerodynamic considerations, will be $v_1 \times (1 - \epsilon_1)$

$$= (1 - \epsilon_1) \frac{p_1}{c_1\, C_1\, \rho\, V},$$

or
$$v_2 = \frac{(1 - \epsilon_1)\, p_1}{c_1\, C_1\, \rho\, V} + \frac{p_2}{c_2\, C_2\, \rho\, V}.$$

It is convenient at this point to make the assumption that the front and rear fins are of the same *aspect ratio*; it is usually easy to make them so, at least approximately. The only quantities affected are the constants c and C, the effect of the

assumption being that c_1 and c_2, also that C_1 and C_2 become identical. In any case, we know that the variations of C are not worth taking into account, and the changes in c for moderate variations of n are not great. The expression for v_2 thus becomes,

$$v_2 = \frac{(1 - \epsilon)\ p_1 + p_2}{c\ C\ \rho\ V}. \qquad (3)$$

From (2) and (3) we obtain,

$$\frac{v_1}{v_1 - v_2} = \frac{p_1}{p_1 - (1 - \epsilon)\ p_1 - p_2} = \frac{1}{1 - (1 - \epsilon) - y}. \qquad (4)$$

Whence (1) becomes

$$\Lambda = \lambda \frac{1}{1 - (1 - \epsilon) - y} - \lambda_1 \qquad (5)$$

in which $y \left(= \frac{p_2}{p_1} \right)$ will be given by the expression,

$$\frac{\lambda_1\ a_1}{\lambda_2\ a_2}.$$

§ 100. Fin Resolution (continued).—We have thus determined the length of the theoretical tail of which the fin combination is the equivalent; it remains now to show how the area of the theoretical abutment plane, **a**, is related to a_1 and a_2, the actual fin areas employed. We do not require to attack the problem *de novo*, as the ascertained value of Λ may be employed in effecting the solution.

If F' be the force at any instant acting through the mass centre, the component acting on the leading fin is $\frac{\lambda_2}{\lambda} F'$, and the lateral velocity due to this force is $\frac{\Lambda + \lambda_1}{\Lambda}$ times that resulting at the mass centre itself. Therefore the equivalent fin area required is,

$$a_1 \times \frac{\lambda}{\lambda_2} \times \frac{\Lambda + \lambda_1}{\Lambda} = \mathbf{a}$$

The equivalent, or resultant, fin is of the same aspect ratio as those from which it is derived. This is represented in a diagrammatic way in Fig. 97, in which the resultant fin is shown dotted. It is to be understood that this phantom abutment fin together with an imaginary tail of length = Λ, constitutes the equivalent of the two real fins, a_1 and a_2.

The position of the effective centre of pressure of the fin combination is a matter of some interest in view of the function of the fins in securing lateral stability. Calling the distance of the effective centre of pressure above the centre of gravity, b, the value of b may be ascertained by drawing a line through the centres of pressure of the two real fins. Thus, referring again to Fig. 97, the positions of the centres of pressure of the two fins, a_1 and a_2, are first determined, and the centre of pressure of the resultant fin is given by the intersection of the straight line joining them, and the perpendicular set up from the mass centre.

§ 101. Rotative Stability.—There is a peculiar form of instability that sometimes arises as the result of the interaction between the motions involved in the maintenance of lateral and directional equilibrium. In separately solving these two problems, we have, in both cases, invoked the aid of motion of translation at right angles to the phugoid plane, and it is necessary so to harmonise the proportions of an aerodone that there shall be nothing inconsistent in this fact. In other words, we require to study the mutual interaction of the motions involved in order to define the relationship of the various functional parts of the aerodone necessary to the conditions.

The manifestation of instability of this form is that the aerodone, apparently with little or no provocation, loses its equilibrium and comes rapidly to earth with a kind of spiral dive. The model, if carefully constructed, will heel and dive either to the right or to the left, depending upon some accidental disturbance to initially decide which way it will go; a want of

symmetrical accuracy in launching is a sufficient determinant. It is from the nature of the motion involved, that the author has taken the name *rotative* as applying to this form of instability.

In the consideration of the question of lateral stability the effective centre of pressure of the fins was tacitly assumed to be situated vertically above the mass centre of the aerodone, for it was supposed that the *direction* is unaffected by the fin reaction, the function of which was taken as exclusively that of restoring the lateral equilibrium and of damping out lateral oscillations. In the study of directional stability and maintenance it was found that the position of this centre of pressure is one permitting of considerable latitude; the fin-tail length Λ might vary from any positive value, through infinity, to a limiting (minimum) negative value. Now $\Lambda = \pm \infty$ means that the aerodone is *directionally balanced*, and the centre of lateral pressure is perpendicularly over the mass centre, so that in this case the condition assumed in the sections on lateral stability is complied with.

Were it possible to ensure in every case that $\Lambda = \infty$, the form of instability that we have now under consideration would not exist, and the present investigation would be unnecessary; in the case of birds, where the whole machine is under continual surveillance, without doubt this is a point that can be relied upon, and in actuality there is every probability that directional balance is obtained to a high degree of approximation. In artificial flight, however, it is advisable not to approach too closely to a limiting condition, and although it has been shown that the limiting condition in this case lies beyond that of $\Lambda = \infty$, still the margin is small, as shown by the fact that the *negative tail* is a phenomenon difficult or impossible of demonstration in air from the gossamer-like construction that would be necessary.

§ 102. Rotative Stability (continued).—Given that the fin-tail length Λ is positive and finite, that is, the fin combination is the

equivalent of a fin-tail of finite length extending rearward, rotative instability depends upon the fact that any change in lateral *trim* has an effect on the *course*, and the resulting change of *course* reacts on the lateral *trim* and so on with cumulative effect. Let us suppose that an initial "list" be imparted to the aerodone by any accidental cause, the immediate consequence is that the aerodone commences to slide sideways down an imaginary inclined plane of its own making—the approximate plane of the aerofoil; this lateral motion, as we have already seen, brings pressure to bear on the fins, tending to "right" the aerodone. At the same time this pressure, acting on the whole behind the mass centre, results in a *change of course* in the direction of the initial list (port or starboard as the case may be). This change of course involves a curvilinear flight path, such that the one wing travels faster than the other, and therefore experiences a greater pressure reaction; and the wing that travels the faster is that having the upper position by virtue of the initial list, so that this initial list will be increased. This augmented list results in turn in a more rapid change of course, and so on, the ultimate result depending upon whether or not the accumulation approaches a finite limit.

We have therefore to deal with a problem involving a rotary motion of the aerofoil about an axis perpendicular to the flight path, and we have to correlate such motion with the couple to which it gives rise, both as to magnitude and direction. The problem is one of some difficulty.

It would appear that the least compromising method of treatment is to suppose that the whole lifting power of each wing of the aerofoil is concentrated at a particular point, which we may term the *aerodynamic centre*, so that we may base our calculations of lifting effort, and therefore capsizing moment, on the relative velocities of the two aerodynamic wing centres. It is evident that for any given aerofoil such points must exist, although they may require to be determined experimentally for each particular form. We will term the distance of the aerodynamic wing

centre from the mass centre of the aerodone the *aerodynamic radius*.

Similarly the resistance in the line of flight may be conceived to be due to the whole resistance of each wing concentrated at some stated point along its length. We will term this point the *aerodromic wing centre*,[1] and its distance from the mass centre the *aerodromic radius*.

These two quantities, the aerodynamic and aerodromic radii, may be taken as fundamental data belonging to any particular aerodone that must be independently ascertained. Certain theoretical deductions relating to the same will be given at the conclusion of the investigation.

§ 103. Rotative Stability. Basis of Investigation Defined.

Let W_1 be the *apparent* weight, *i.e.*, resultant of actual weight and centrifugal force of flight path.

,, V = velocity of flight, as before.

,, ρ = density of air, as before.

,, Λ = fin-tail length, as before.

,, $\mathbf{a}$ = area of abutment fin, as before.

,, b = fin centre height, as before.

,, r = flight path radius.

,, σ = aerodynamic radius.

,, s = aerodromic radius.

,, v = lateral velocity of mass centre.

,, β = inclination of abutment fin to its direction of motion.

,, τ = torque about axis of flight.

,, Υ = torque or turning moment about vertical axis.

,, ζ = addendum angle of heel, *i.e.*, relatively to the apparent plumb.

,, c and C be the aerodynamic constants of the abutment fin, as before.

[1] This term is employed for want of a better. The quantity is one that, so far as the author is aware, is only of use in connection with the present problem.

It will be supposed in the investigation that follows that the aerodone is gliding in a flight path forming the arc of a circle, and the investigation will have for its object to determine the point at which the radius of curvature is constant, that is to say, to determine the conditions under which the various torques and reactions are balanced. The general solution to include variations of r (the flight path radius) would be needlessly complicated; the particular case in which we are interested for the time being is the straight gliding path, and while the physical supposition will be throughout that the path is of curvilinear form, it will be supposed that the value of r is of a higher order than the other linear quantities concerned, and the usual consequences of a supposition of this kind will be taken advantage of as occasion may arise.

The torque about the vertical axis, due to the variation in the relative wing resistances (in the line of flight), if included in the main investigation causes some tiresome complication. This factor has been eliminated by a preliminary process, that takes the form of a *correction* applied initially to some of the data or to the design of the aerodone.

§ 104. Rotative Stability. Preliminary Investigation. — The velocities of the aerodromic wing centres (Fig. 98) will be given by the expressions[1]:—

$$\frac{r+s}{r} V \quad \text{and} \quad \frac{r-s}{r} V.$$

Now γW_1 = total resistance in the line of flight at velocity V or $\frac{\gamma W_1}{2}$ is resistance due to each wing of the aerofoil for straight path. And since the resistance is proportional to the velocity squared,[2] the resistance of the wings will be respectively:—

$$\frac{\gamma W_1 (r+s)^2}{2 r^2} \quad \text{and} \quad \frac{\gamma W_1 (r-s)^2}{2 r^2}.$$

[1] This statement, as applied to the figure as drawn, is scarcely accurate; the actual conditions, however, are those of the *small angle* where $\cos \beta$ may be taken as unity.

[2] Compare "Aerial Flight," Vol. I., *Aerodynamics*, § 159.

But s is small in comparison to r, therefore s^2 is negligible, and the expressions become :—

$$\frac{\gamma W_1}{2} \times \left(1 + \frac{2s}{r}\right) \quad \text{and} \quad \frac{\gamma W_1}{2} \times \left(1 - \frac{2s}{r}\right)$$

and the difference of these is

$$\gamma W_1 \frac{2s}{r}$$

therefore the turning moment $\Upsilon = \gamma W_1 \dfrac{2s^2}{r}$ (1)

and, (Fig. 98), $$r = \frac{\Lambda}{\beta} = \frac{\Lambda V}{v}$$

$$\therefore \qquad \Upsilon = \gamma W_1 \frac{2s^2 v}{\Lambda V} \tag{2}$$

Now let F (Fig. 98) represent the transverse force sustained by the abutment fin, then $F = \mathbf{a}\, c\, \beta\, C\, \rho\, V^2$

but $\beta = \dfrac{v}{V}$ $\therefore$ $F = \mathbf{a}\, c\, v\, C\, \rho\, V$ (3)

Now in expressions (2) and (3) the only variable in respect of the path curvature is v, the whole of the other quantities are constants whose value is known, and thus we have $\Upsilon \propto F$, or equating the two we have,

$$\Upsilon = F \frac{2\gamma W_1 s^2}{\mathbf{a}\, c\, C\, \rho\, V^2 \Lambda}$$

But where the radius of curvature is great, that is, when the flight path is approximately straight, W_1 is sensibly equal to W the weight of the aerofoil. And W/V^2 is the constant of the aerofoil denoted by K,

$$\therefore \quad \Upsilon = F \frac{2\gamma K s^2}{\mathbf{a}\, c\, C\, \rho\, \Lambda} \tag{4}$$

Now the quantity $\dfrac{2\gamma K s^2}{\mathbf{a}\, c\, C\, \rho\, \Lambda}$ in this equation is a constant of linear dimensions, and represents a distance l at which a force $= F$ must act in order to give the torque Υ. In Fig. 98 this torque is represented by the couple whose constituent members

are the forces F_1 and F_2. Of these two forces F_1 is arranged acting through the mass centre of the aerodone,[1] and is therefore equal and opposite to F, consequently the force F_2 alone remains, and we have the result that *the turning moment due to the difference of the resistances on the two wings of the aerofoil is equivalent*

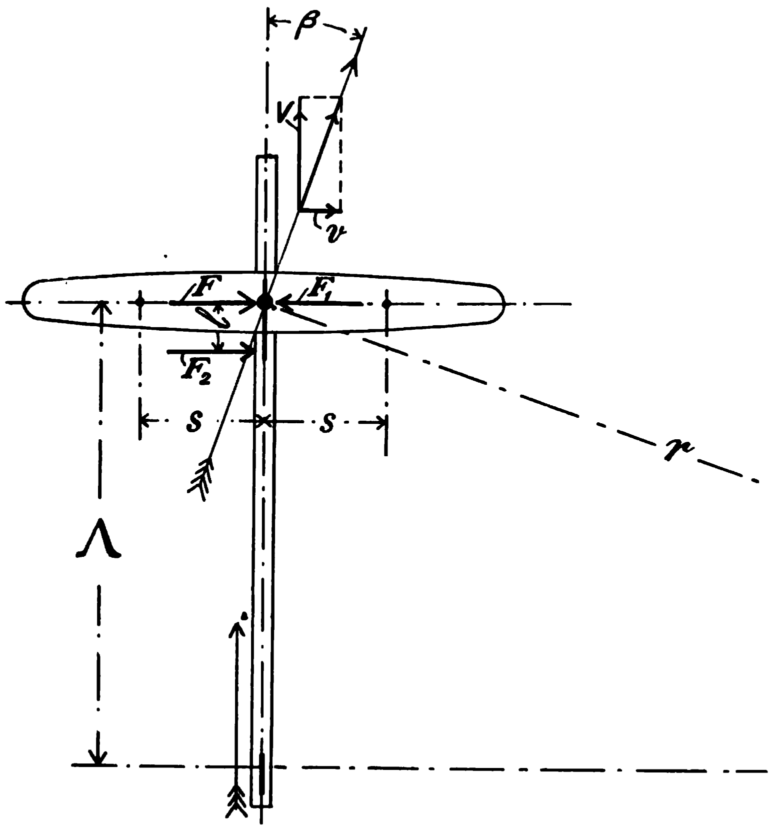


FIG. 98.

to a displacement of the centre of gravity rearwards by an amount $= l$ *where*

$$l = \frac{2\,\gamma\,K\,s^2}{\mathbf{a}\,c\,C\,\rho\,\Lambda}.$$

[1] The usual method of resolution.

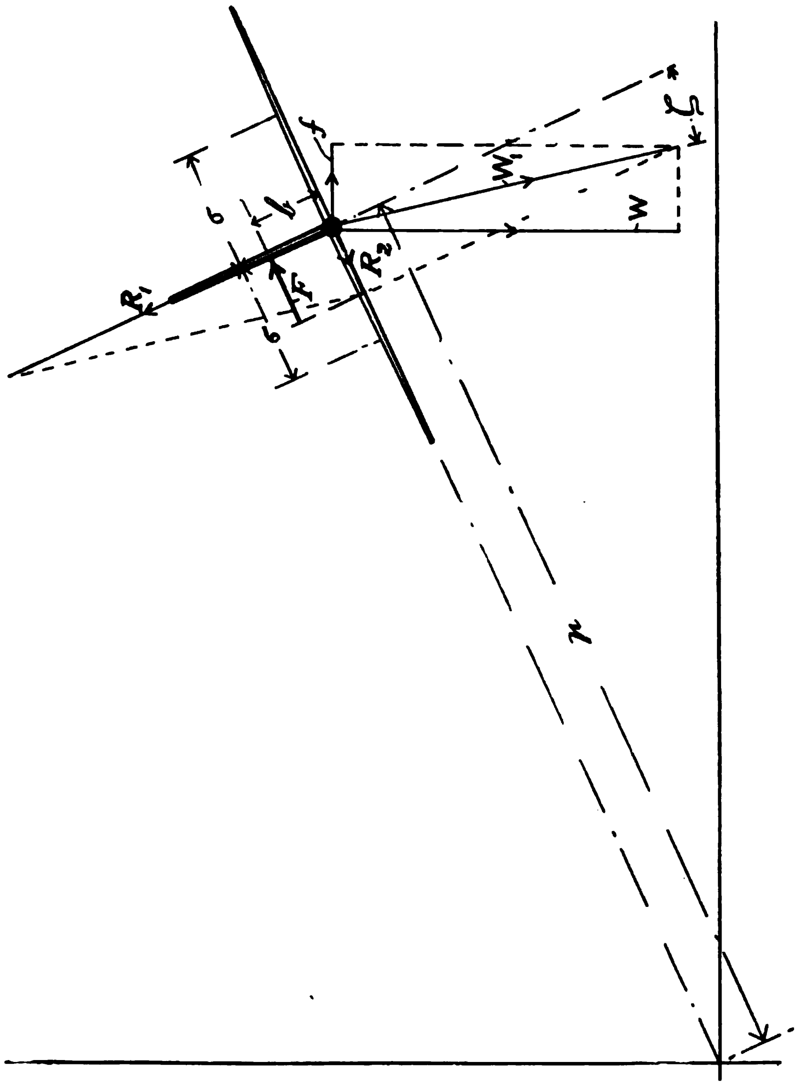


FIG. 99.

There is one point of some subtlety that requires consideration in relation to this *correction*. The quantity Λ is initially calculated from the actual position of the centre of gravity, and the *fin plan*, as in § 99, and the displacement of the centre of gravity in accordance with the present equation will vitiate the value of Λ so obtained. Two courses are open: one is to move the fin-plan back bodily by an amount $= l$, that is to say, *actually move it back* by this amount in the design and in the aerodone; the other is to repeat the calculation with the new value of Λ substituted, and repeat again as many times as may be necessary to arrive at a sufficiently close approximation. In either case the present method constitutes far the simplest solution to this portion of the problem.

§ 105. Rotative Stability. Investigation.—Let Fig. 99 represent diagrammatically an aerodone in flight, the symbols corresponding to those given in § 103.[1]

Then, as in the preceding section, the velocities of the aerodynamic wing centres are respectively—

$$\frac{r+\sigma}{r} V \text{ and } \frac{r-\sigma}{r} V,$$

and since the lifting reaction varies as V^2 the lifting efforts of the wings will be respectively:—

$$\frac{W_1}{2}\frac{(r+\sigma)^2}{r^2} \text{ and } \frac{W_1}{2}\frac{(r-\sigma)^2}{r^2}.$$

But σ being small in comparison to r, σ^2 is negligible, hence difference between lifting efforts is—

$$\frac{W_1}{2}\left(1+\frac{2\,\sigma}{r}\right) - \frac{W_1}{2}\left(1-\frac{2\,\sigma}{r}\right)$$

$$= W_1 \frac{2\,\sigma}{r}$$

[1] In Fig. 99 f is the centrifugal force due to the curvilinear flight path, W_1 is the resultant of W and f, R_1 is the total reaction on the aerofoil, and R_2 is the resultant of R_1 and W_1.

and torque $$\tau = W_1 \frac{2\,\sigma^2}{r}. \qquad (5)$$

This is the torque tending to capsize the aerodone, and the condition of equilibrium is that this shall be balanced by the righting moment due to pressure on the fin area due to the lateral motion.

Now force F acting on abutment fin $= \mathbf{a}\ c\ \beta\ C\ \rho\ V^2$ and $\beta = \frac{v}{V}$

$$\therefore \quad F = \mathbf{a}\ c\ v\ C\ \rho\ V$$

(this is equation (3) of § 104),

and $$r = \frac{\Lambda\ V}{v} \text{ or } v = \frac{\Lambda\ V}{r},$$

$$\therefore \quad F = \frac{\mathbf{a}\ c\ C\ \rho\ V^2\ \Lambda}{r}$$

or righting moment $$= \frac{\mathbf{a}\ b\ c\ C\ \rho\ V^2\ \Lambda}{r}, \qquad (6)$$

and the condition of stability is that this shall be equal to or greater than the torque τ or by (5) and (6),

$$\mathbf{a}\ b\ c\ C\ \rho\ V^2\ \Lambda > 2\ W_1\,\sigma^2,$$

but for the nearly straight flight path $W_1 = W$, $\therefore\ \frac{W_1}{V^2} = K$, hence we may write the above,

$$\frac{\mathbf{a}\ b\ c\ C\ \rho\ \Lambda}{2\ K\ \sigma^2} > 1,$$

which is the equation that must be satisfied if the aerodone is to be rotatively stable.

It is worthy of note that the above expression is independent of the velocity of flight; thus if an aerodone be designed that is *just stable* for any given values of weight and velocity, then it will be just stable, if its geometrical form be the same, for all other appropriate values of W and V. Thus if it be made four times its original weight, so that its velocity be twice as great, its rotative stability will undergo no change.

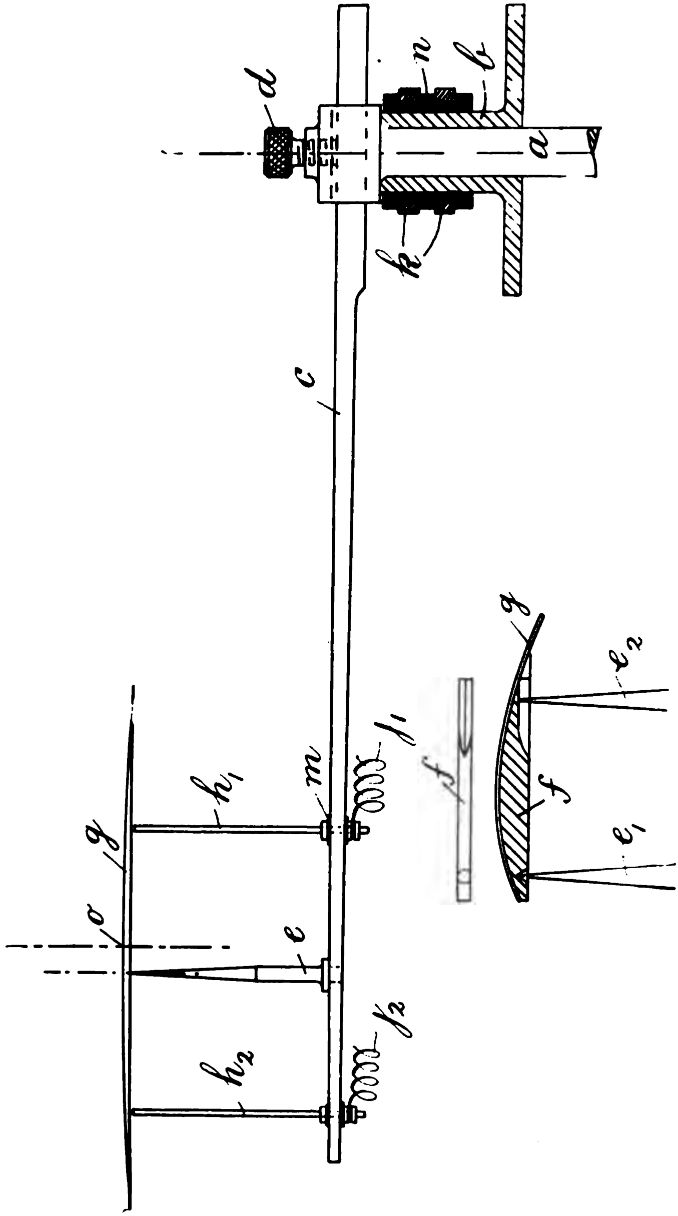


FIG. 100.

§ 106. On the Aerodynamic Radius.—The value of the quantity that we have defined as the *aerodynamic radius*, can, if desired, be determined experimentally. Thus, employing a *whirling table*, if determinations are made of the total lifting effort of any aerofoil under given conditions, and if also the turning moment about the axis of flight, due to the fact that the two wings of the aerofoil are at different radii, be measured, the value of σ is a matter of simple calculation. The disposition of the apparatus

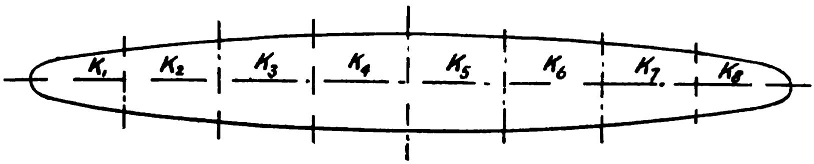


FIG. 101.

required to carry out this method is given diagrammatically in Fig. 100.[1] Probably the best procedure would be to find the

[1] Referring to Fig. 100 the arm of the whirling table c is mounted adjustably on the head of the vertical shaft a arranged to run in a fixed bearing b. The aerofoil g is mounted on a bolster f adapted to pivot on the steel points e and e; the bolster f is arranged a short distance from the geometrical centre of the aerofoil o, so that the reactions on the wings of the aerofoil shall balance, and this distance may, if desired, be made adjustable by suitable construction.

In the arrangement shown in the figure, the radius of the whirling table is made adjustable by means of the screw d, instead of providing means of varying the distance of o; the result is the same.

In order to signal the condition of equilibrium two electrical contacts are provided, h and h, connected by leads j and j with two commutator rings k, mounted on the insulating sleeve n. These two contacts control two alternative circuits, and indicate the position of the aerofoil by a galvanometer, or by electric bells in the well-known manner.

The arrangement as shown is suitable for small scale experiment; for large scale work modifications in the design would probably be required.

It is evident that if the radius of the whirling table be increased the reaction on the inner wing will increase faster than that on the outer one, and *vice versâ*; the adjustment should be made till the reactions balance.

The aerofoil should be initially adjusted to balance gravitationally about its pivot support, in order to eliminate centrifugal force.

position of the point of suspension at which the reactions on the two wings accurately balance; the velocity of flight as a factor being in this way eliminated. In any case the whirling table employed should be of not too great a radius, as the magnitude of the quantity measured will be greater as the radius of the whirling table is less.[1]

The above method may be regarded as particularly appropriate in view of the resemblance that exists between the conditions of experiment and the conditions of flight as presented in the theory.

§ 107. On the Aerodynamic Radius. Theoretically Considered.— Let us assume as in §§ 192 and 205, Vol. I., that each element of the length of the aerofoil sustains a reaction appropriate to its velocity through the air; this assumption being that we may apply to every small increment of the length of the aerofoil the equation, $W = K V^2$ which we know applies to the aerofoil as a whole. Thus let Fig. 101 represent the aerofoil which we will suppose divided into a number of sections whose constants are respectively K_1 K_2 K_3 K_4, etc., then we are assuming that the lifting power or weight supported by these sections individually is given by $K_1 V_1^2$ $K_2 V_2^2$ $K_3 V_3^2$, etc., where V_1, V_2, etc., are the mean velocities of the different sections.

That this assumption is *not* strictly accurate can be easily demonstrated; thus, let us suppose the aerofoil to revolve about one of its terminal elements, say K_8 in the diagram; then if the supposed law hold good there will be no lifting effort whatever on K_8. But we know that K_8, although itself virtually immobile, is preventing the immediate flow of air from the under to the upper side of the section marked K_7, consequently it must itself experience some difference of pressure between its under and upper surfaces.

Dealing with the matter broadly, the tendency of the "field of

[1] At the time of writing the author has not actually employed the method proposed; an estimate on the basis of § 107 having been made to serve.

force "[1] is to spread from the regions of greater intensity to those of less, so that any difference from normal in the distribution of the pressure reaction will be ameliorated to some extent by a redistribution of the field, and the differences and therefore the aerodynamic radius will be less than would be computed on the assumed basis of constant K values. The extent to which these considerations invalidate the results of our assumption we have no means of gauging other than by direct comparison with

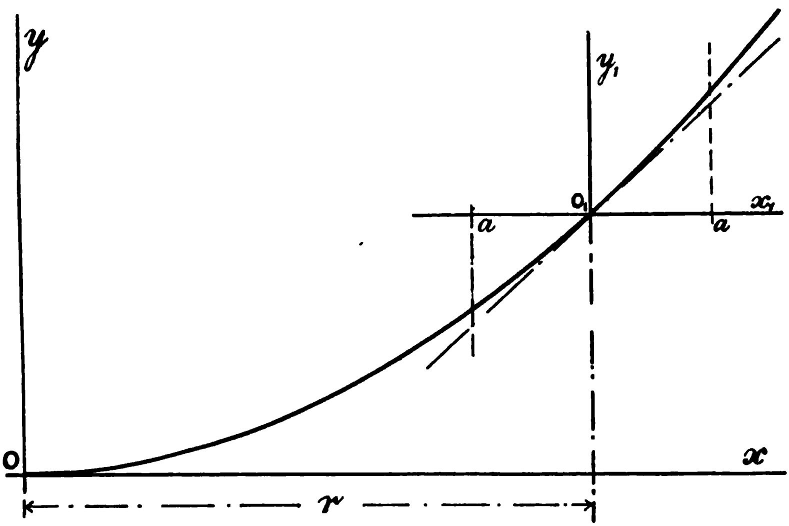


FIG. 102.

experiment, experiments that have not at present been made, but it is evident that the greater the aspect ratio the more completely will the conditions be complied with, and, as stated in Vol. I., § 205, our assumption may be considered a close approximation when the aspect ratio is considerable (say upwards of four or five), and a rough approximation only when the aspect ratio is small. At the worst the final result will be an error on the safe side.

Referring now to Fig. 102, we have plotted with respect to

[1] "Aerial Flight," Vol. I., *Aerodynamics*, Chap. IV.

the origin O and co-ordinates x and y, the values of W per unit K for an aerofoil whose mean path radius is r; abscissæ represent radius values and ordinates $= W/K$. Then since $W/K = V^2$ the curve is of the form $y = n\ x^2$ where n is a constant.

Now let us suppose that the distance $a\ a$ be that occupied by the aerofoil, and let us assume, as in § 105, that we are dealing with the nearly straight flight path, so that r is great compared with $a\ a$, then we may take the form of the curve within the span of the aerofoil to be a straight line. Let x_1 and y_1 be the abscissæ and ordinates about a new origin O_1 chosen on the line of flight: then the equation to the straight line approximation, from the new origin, will be $y_1 = n_1\ x_1$ where n_1 is a constant. But y_1 is the increase in lifting force per unit K, due to the excess of velocity consequent on the circular flight path, and is positive on the one wing of the aerofoil and negative on the other. And the turning moment about o_1 resulting from this increase $= y_1\ x_1 = n_1\ x_1^2$ which is positive whether x is positive or negative, hence it is positive for both wings of the aerofoil. Or, the total turning moment may be represented by the expression $\Sigma\ (K\ n_1\ x_1^2)$ that is to say, the sum of the turning moments of the individual small elements.

It may be recognised at once that the problem of determining the turning moment for any given aerofoil is strictly analogous to that of finding the moment of inertia of a thin bar in which the mass per unit length of the bar is represented by the value of K per unit length in the aerofoil, hence, *the aerodynamic radius of the aerofoil is equal to the radius of gyration of a thin bar whose mass is at every point proportional to the value of* K, *increment for increment, of the aerofoil.*

If the "grading" of the aerofoil be that proposed in Vol. I., § 192, that is to say, parabolic, we know by Routh's rule that the aerodynamic radius σ will be $\dfrac{L}{2\sqrt{5}}$ or, $\sigma = {\cdot}223\ L$, where L is the length of the aerofoil.

§ 108. On the Aerodromic Radius.—The aerodromic radius could evidently be determined experimentally on the lines proposed for the aerodynamic radius in § 106, the axis or pivot on which the aerofoil is mounted being arranged vertically instead of horizontally; the forces to be measured are, however, comparatively small (commonly from one-sixth to one-eighth of the vertical reaction), and the apparatus would require to be appropriately sensitive.

From the theoretical point of view, if the form of the aerofoil were that discussed in § 120 of Vol. I., the problem would be greatly simplified, for the aerodynamic and aerodromic radii would be equal. We know, however, from §§ 190, 192, that such a form does not comply with all the conditions, and that the aerofoil extremities have to be formed to give rise to motion of the discontinuous type. In nature we know that the wing terminations are given much diversity of form to this end, and that in general the sectional form should be flattened so as to become virtually an aeroplane at each extremity.

Under these conditions the value of the aerodromic radius is a quantity that, from a theoretical standpoint, we are unable to estimate with any great degree of accuracy in the existing state of knowledge.[1]

[1] Compare § 114.

CHAPTER VIII

REVIEW OF CHAPTERS I—VII AND GENERAL CONCLUSIONS

§ 109. **Preliminary.**—In the foregoing chapters many results individually of importance have been demonstrated; the main consequence of these results—the definite proof of the automatic stability of a properly designed aerodone—is alone a matter of very great moment. The establishment of the laws of stability in a form immediately available to the designer, cannot fail to have a far-reaching influence on the future of mechanical flight, and on the development of the flying machine for aerial navigation.

In the present chapter we shall first carefully review the theory, especially with regard to its aerodynamic foundations, which will be examined minutely, and subsequently any unaccounted factors and limitations, where such limitations exist, will be discussed. We will then briefly consider the application of the theory to the problem of mechanical flight, dealing with such points as the influence of the mode of propulsion, the rate of damping of the phugoid oscillation, the law of corresponding speed and the conduct of model experiments, departures from the elementary type, etc., etc.; afterwards discussing the experimental verification and employment of the theory of lateral stability of Chapter VII., and in conclusion a brief note will be given of the historical development of the present aerodonetic theory and of its experimental confirmation.

§ 110. **Aerodynamic Basis of the Phugoid Theory.**—From a strictly logical point of view the series of investigations forming the subject-matter of the present volume begin at the commencement

of Chapter II. The strictly logical order is, however, not that in which a subject is always best presented, and the preliminary discussion of Chapter I., with the account of some of the author's earlier experiments, serves as a preparation for the more abstract treatment that follows.

It is a remarkable fact that the "Phugoid Theory" of Chapters II. and III. with which the rigid investigation opens, rests, apart from the specific conditions laid down by hypothesis, implicitly on *one aerodynamic law and one law only; i.e.*, the law that the *reaction varies as the square of the velocity.*[1] Thus the aerofoil is supposed to be *rigid* and to suffer no change in *attitude* or *aspect*;[2] and the form and amplitude of the motion set up by it in the air through which it moves are sensibly constant;[3] it follows directly from aerodynamic principles that the reaction is *point for point* proportional to V^2. Apart from the fact that this law has been established experimentally, not only for direct resistance but also for oblique reactions,[4] we know that the limits of its approximate application, as due to viscosity on the one hand, and to elasticity on the other, are far removed from the values with which we are concerned.[5]

It is a most important and striking fact that so great a result from the aerodonetic point of view, as that of the phugoid flight path, can be deduced from one single aerodynamic law. One prime consequence of this fact is that the experimental

[1] In addition to the ordinary laws of motion.

[2] For the specialised meaning of the terms *aspect* and *attitude* see Glossary.

[3] When viscosity is excluded, and the V^2 law applies, the motion of the fluid at different velocities is, in effect, homomorphous.

[4] Proved more especially by the experiments of Dines, in which pressure reactions are balanced by the contrifugal force of an adjustable mass; see "Aerial Flight," Vol. I, *Aerodynamics*, §§ 223 *et seq.*

[5] The influence of viscosity makes itself felt only for small bodies at low velocities where the Newtonian Law is about to give place to that of Allen, "Aerial Flight," Vol. I., *Aerodynamics*, §§ 50 *et seq.* The influence of elasticity is probably not sensible for low flight velocities, but at high velocities a correction is required; compare "Aerial Flight," Vol. I., *Aerodynamics*, Appendix I.

confirmation of the path form and period, incidentally provides an independent proof of the law on which the theory is founded, as well as proving the soundness of the theory itself.

§ 111. Basis of the Equation of Stability.—The aerodynamic basis of the extension of the Phugoid Theory dealt with in Chapter V., culminating in the equation of stability, is constituted by the following:—

(1) Law of pressure on the normal plane, $P_{90} = C \rho V^2$, also the numerical value of the constant C, established experimentally.

(2) Law of pressure on planes at small angles, $P_\beta = c \beta P_{90}$ definitely established for planes of square proportion by the experiments of Duchemin, Dines, and Langley, as obtaining over a very considerable angle of inclination; also found to be generally true for planes of other proportions but over a less angular range. It probably does not apply even approximately for planes of extreme ratio in apteroid aspect; planes of such form are consequently to be avoided.[1]

(3) The law of resistance to flight, R varies as V^2. This law is not the same as that which obtains for $W = constant$,[2] neither is it the law of *least resistance*.[3] In the problem as presented in the Phugoid Theory the load, *i.e.*, the mass of the aerodone, it is true does not change, but the *effective load does change;* it is made up of the component of the weight *plus* the centrifugal force due to the curvilinear flight path, measured plus or minus according as it acts in the one direction or the other (compare Chapter II., § 20). Thus this law is merely a direct consequence of the law on which the Phugoid Theory itself is founded (§ 110), that the reaction on the aerofoil is point for point proportional to V^2; the total reaction can

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, § 151. The pheasant's tail may be cited as an example.

[2] Compare "Aerial Flight," Vol. I., *Aerodynamics*, § 159.

[3] "Aerial Flight," Vol. I., *Aerodynamics*, § 166.

manifestly be divided into its horizontal and vertical components[1] to each of which component reactions the same law must apply; but the horizontal component (or rather line of flight component) *is* the resistance to flight, hence the present law is corollary to that of the preceding section.

§ 112. The Equation of Stability. Unaccounted Factors.—The extent to which the equation of stability has received experimental confirmation shows that any factors omitted in the theoretical investigation can at present be regarded as unimportant. The fact, however, is not without interest that the theory as presented *does* omit items whose influence may in the future, when experimental methods have undergone some refinement, be found to be of sensible magnitude.

In the first place it may be pointed out that the primary influence of the changes in the *attitude*[2] of the tail as affecting the value of β may not, as has been assumed, cause an exactly proportional change in the reaction. Before the validity of this assumption can be discussed, we require to know more about the form of the aerofoil employed. If this be an aeroplane, and if β be a decidedly small angle, then we know that the assumption is justified by direct experiment. If, on the other hand, the aerofoil be one of pterygoid form, it would appear that the angle a, and the curvature of the section, ought, strictly speaking, to undergo simultaneous variation.

The above factor as bearing on the foundation of the equation of stability probably results in an error of a lower order of magnitude than the quantities with which the equation deals, owing to the fact that the demonstration only applies strictly to the phugoid of evanescent amplitude; that is, to the stability of the straight gliding path. The factors in question doubtless

[1] It is convenient to use the term horizontal for the direction of flight, and vertical for the normal thereto. The supposition here is that the flight path is horizontal.

[2] The term *attitude* is defined as the position of the plane or other body about the horizontal transverse axis in relation to the line of flight.

have some sensible influence on the conditions of stability for flight paths of greater amplitude, but this is a wider problem, of which, so far, no attempt has been made to give a theoretical solution.

In the equation of stability as first presented the influence of the "wash" from the aerofoil as affecting the motion of the tail plane is not taken into account, but an approximate method of dealing with this factor is suggested and embodied in the final form of the equation (§ 63). The experimental evidence of the validity of this correction, although at present inconclusive, is on the whole satisfactory.

From the nature of the assumption on which the correction is founded, *i.e.*, that the tail is, in effect, situated outside the peripteral region, the correction is evidently only an approximation whose accuracy depends upon the extent to which the assumption is justified in practice. Thus if the tail length is considerable, so that the tail plane is remote from the aerofoil, the error should be small, whereas if the tail be short, so that the tail plane is close to the aerofoil and so involved in its periptery, the error will be considerable.

The method employed to deal with the "wash" of the aerofoil is also employed in the investigation of directional stability (§ 99).

§ 113. Basis of the Theory of Lateral and Directional Stability.— In the portion of Chapter II. relating to lateral and directional stability, there is very little new importation of aerodynamic theory or data. The qualitative behaviour of the centre of pressure of an inclined plane under changes of angle and aspect, as known by experiment, is assumed in the explanation of the behaviour of an aerodone under lateral disturbance, and the law of pressure at small angles is again used to a considerable extent. In § 99 the method of allowing for the effect of the "wash" of the leading fin on the following fin is similar to the method of making an analogous allowance in the case of the tail plane, and involves an application of aerodynamic

theory not otherwise employed, the residuary current from the leading fin being taken as equal to the lateral velocity component of the leading fin multiplied by $(1 - \epsilon)$. Now ϵ represents the velocity of the upcurrent in terms of the velocity of downward discharge of an aerofoil or aeroplane, the upward direction being, of course, taken as that of the pressure reaction; and since the loss of momentum of the air in *recess* is equal to that imparted in *approach*,[1] the residuary current has a velocity $(1 - \epsilon)$ times that possessed by the air at the moment it quits the after edge of the plane in motion, hence the rule given. It will be seen that in practice the "wash" will, if the present line of reasoning is correct, have a greater velocity than that stated, for the following plane is not an infinite distance away, so that the air will not have finished parting with its momentum and the residuary velocity will be greater than that calculated. In spite of this fact, unless the one fin follows the other very closely, the error will not be worth consideration, for by far the greater part of the dynamic action is quite local and takes place within a radius of one or two times the width of the plane or aerofoil.

For the understanding of the foregoing argument, a full knowledge of the author's treatment of the dynamics of the periptery is essential; reference should be made to Vol. I., Chaps. IV. and VIII.

§ 114. Aerodynamic Basis of the Investigation on Rotative Stability.—Beyond the free employment of the aerodynamic laws and data already assumed, the only additional data required in this investigation are the aerodynamic and aerodromic radii. These quantities are fully dealt with in §§ 106, 107, and 108, and may be considered as involving a direct appeal to experiment. As independent experimentally ascertained values, no exception can be taken to their introduction into aerodonetic theory or into the manner of their employment. At present,

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, §§ 115 and 179.

however, no experimental determinations have been made, and the only knowledge on the subject that is available is that deduced in § 107 from the theoretical considerations.

The author believes that in practice, in a well designed aerofoil, the aerodynamic wing centre lies between ·21 and ·23 of the length of the aerofoil from the line of flight, and that the corresponding distance for the aerodromic wing centre is from ·23 to ·25. That is to say,

$$\text{Aerodynamic radius, } > \cdot 21\, L < \cdot 23\, L.$$
$$\text{Aerodromic radius, } > \cdot 23\, L < \cdot 25\, L.$$

§ 115. Limitations and Unaccounted Factors.—No account has been taken in the present theory of the changes that take place in the energy of the periptery, as the aerodone in its flight path undergoes changes in its velocity.[1] It would appear possible that, under certain circumstances, the energy of the periptery may become a factor of sensible magnitude. Regarding the said energy as accountable on the basis of a *concealed mass* of some definite value, then, the peripteral energy for a given aerodone being taken as proportional to its velocity squared, we may regard the force of gravity as in part acting on the mass of the aerodone and in part on the *concealed mass* of the periptery. According to this view only part (a constant proportion) of the potential energy of the aerodone appears visibly in the kinetic energy of its descent, the other part (also a constant proportion) being absorbed in the peripteral system. Thus the motions of the aerodone will be precisely as if the force of gravity were diminished by a certain proportional amount, that is to say, the whole of the relations given in § 36 require to be construed with a value of g below its true value in the constant proportion in which the kinetic energy of the aerodone is at every instant less than the total kinetic energy of the aerodone and its periptery. It seems possible that this quantity is not the

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, §§ 81, 82, 84, 85 and 123.

same for different cases, but we are at present in the dark as to the extent of the correction involved; the experimental verification of § 69 would suggest that under ordinary conditions the error from the cause suggested is negligible.

§ 116. Limitations and Unaccounted Factors (continued).—In addition to the point raised in the preceding section, on which some doubt must, for the time being, exist, the question of the *elasticity* of the aerodone or of its parts, in particular elasticity of the aerofoil itself, is one which may place a limit on the applicability of the theory.

In the theoretical and experimental foundation of the law of the reaction variation (reaction varies as square of velocity), the rigidity of the aerofoil is a definite part of the hypothesis. This condition is fulfilled with a sufficient degree of approximation in the case of the mica models employed by the author for the purposes of verification, and it may be taken as complied with by the generality of artificial flight models used by other experimenters. In the case of birds and other living things capable of flight, the circumstances are different; in general, the wings are flexible in a high degree, so that their form changes with every change in the pressure reaction. Under these conditions the phugoid equation probably only applies as a rough approximation; changes in the sectional form of the aerofoil mean changes in the value of H_n, and although the consequences of such changes are in every likelihood quite amenable to theoretical treatment nothing of the sort has been at present attempted.

The kind of deflection of the wing under load must depend upon the wing structure. In the wing structure of birds any addition to the pressure takes effect principally by straightening, or bending backward, the secondary remiges, so that the camber of the arched section is diminished; this is illustrated by Fig. 103, which is a reproduction of Fig. 57 of Vol. I. In the case of an aerodone or flying machine, in which the aerofoil

consists of a canvas sheet stretched between an anterior and a posterior rail, or arranged like the sail of a ship, the reverse takes place, the greater the pressure the more the canvas will "belly." In the case of the pterodactyles of old, as also in the bat, and in gliding or flying machines modelled on similar lines, the effect of increased pressure must be considered doubtful; it depends upon the directions and degrees of the tensions by which the wing membrane is restrained.

Applied to aerodones or aerodromes whose aerofoil is flexible in any of the manners suggested, the Phugoid Theory must have its limitations, the form of the departures from theory evidently

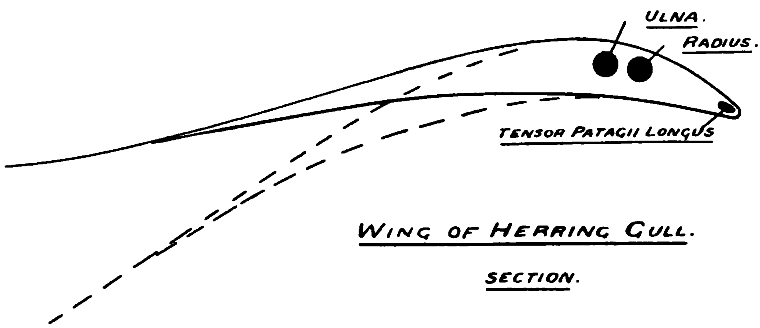


FIG. 103.

depending upon the form of flexibility that the aerofoil exhibits under stress. The application of the extension of the theory in the case of the swift and the albatros suggests, however, that even under these doubtful conditions the results are of value.

§ 117. Limitations and Unaccounted Factors (continued).—When the aspect ratio **n** of the aerofoil is low, so that its fore and aft dimension becomes considerable, the changes in the position of its centre of pressure, consequent on changes of *attitude*, may add materially to the longitudinal stability of the aerodone; the behaviour of the aerofoil becoming comparable to that of a ballasted aeroplane, where stability is provided without any tail

organ at all. This addition to the stability is one for which it is difficult to make proper allowance: the conditions are too indefinite.

A further difficulty is experienced in the application of the equation of longitudinal stability in cases where the tail plane, instead of being of square proportion or of elongate form in pterygoid aspect, is of elongate form in apteroid aspect. The law of pressure on such a plane approximates to the sine square law of Newton,[1] and we can no longer regard the small angle law, which forms part of the basis of the theory of stability, as holding good; consequently, in such a case, the equation cannot be applied. Fortunately, planes in apteroid aspect possess other disadvantages from an aerodynamic point of view that render their employment in artificial flight out of the question, so that the present limitation to the theory is of but little importance. Perhaps the best example in nature of a tail in apteroid aspect is found in the pheasant; this bird, if one may judge by the form of its tail, has some special private objection to its stability being investigated.

The most serious difficulty in the employment of the theory of lateral and rotative stability is due to the fact that the quantities dealt with in the equations can in practice only be inferred. In nature the vertical fins on which this theory is based do not exist, and we have practically no means of applying our knowledge to this branch of the subject. In artificial flight models the employment of fins undoubtedly adds unnecessarily to the skin-frictional resistance, but the extent of this addition is not by any means an intolerable burden, and the author, as will be gathered from the experiments cited, commonly employs models so fitted.

In the case of an actual flying machine the question of power expenditure becomes a matter of first importance, and probably the fin area will have to be reduced to a minimum. In the earlier or experimental stages, however, there seems to be no reason why they should not be retained, at least, as an auxiliary.

[1] "Aerial Flight," Vol. I., *Aerodynamics*, §§ 150, 151.

It has been shown that a large portion of the duty of the fins can be performed by the ends of the aerofoil itself, these being arranged to slope upward at an angle; the question of the equivalent of such upturned ends expressed in fin area will be discussed in a subsequent section.

§ 118. The Influence of the Mode of Propulsion.—In the application of the various branches of aerodonetic theory to the solution of the problem of stability in the construction of an aerodrome or flying machine, the *mode of propulsion* becomes a matter of importance.

In the basis of the investigation of Chapter V., the propulsion was assumed as applied in the simplest possible manner, *i.e.*, as a component of gravity; so that the propulsive force, for curves of small amplitude, could be taken as constant.

In an aerodrome or flying machine the propulsion is not so derived, it is commonly due to a thrust supplied by a screw propeller, or pair of propellers, caused to rotate by some form of prime mover. Now, in such a combination the thrust, for a small variation of velocity, such as contemplated by the theory, may be sensibly constant, or it may increase when the velocity increases, or, as is more generally the case, it may increase when the velocity decreases. Beyond this, when the motor is fitted with a fly-wheel the thrust is also a function of the *change of velocity*, that is to say, other things being equal, when the fly-wheel speed is *increasing* the torque acting on the propeller is less than when the fly-wheel speed is *decreasing*, and the thrust consequently is also less. It is evident, therefore, that factors are introduced into the problem foreign to the initial investigation, to an extent that cannot fail to complicate the subject.

Before we can discuss the influence of propulsion we must devote some consideration to the special circumstances surrounding the employment of motive power in connection with flight, both as touching the motor itself and *the means of propulsion*.

Assuming, firstly, that the latter take the form of a screw

propeller, an assumption that the author believes the future will fully endorse, we know that the relation between the torque and thrust is approximately constant for variations of velocity of small magnitude.[1] Thus instead of being concerned with the variations of the propeller thrust, we can go direct to examine the law of variation of the torque of the prime mover.

In Fig. 104 we have plotted two curves; in the lower diagram abscissæ represent revolution-speed and ordinates horse-power;

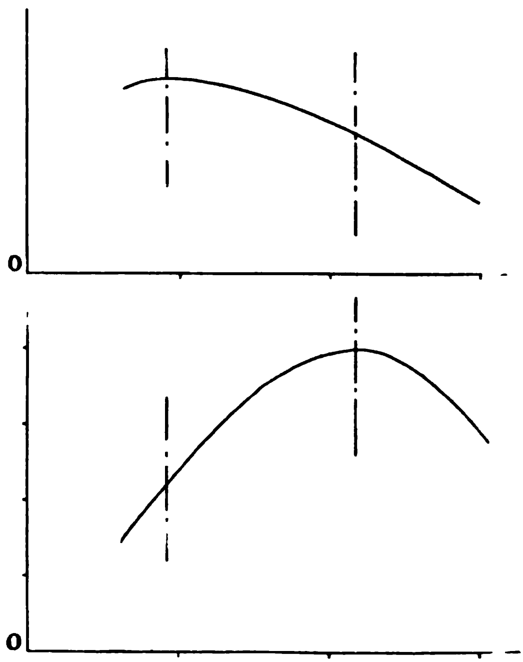


FIG. 104.

in the upper diagram abscissæ again represent revolutions, but ordinates represent torque. Now, referring to the upper curve, it is evident that the law of variation of torque as a function of speed

[1] It is presumed that the propeller is approximately a design of best efficiency; if this condition is infringed the fact stated may not hold good, the essential is that the blade should be properly proportioned for *least gliding angle*. Compare "Aerial Flight," Vol. I., *Aerodynamics*, §§ 202 *et seq.*

of revolution might be approximated by an empirical equation of suitable form,[1] but as in our equation of stability we do not require to deal with the whole curve, it is sufficient to approximate the conditions by a tangent at the point with which we have to deal, that is, the point on the curve at which the abscissa represents the velocity of the motor proper to the natural velocity of the aerodrome. This point for a given motor and for a given value of V_n will depend upon the pitch of the pro-

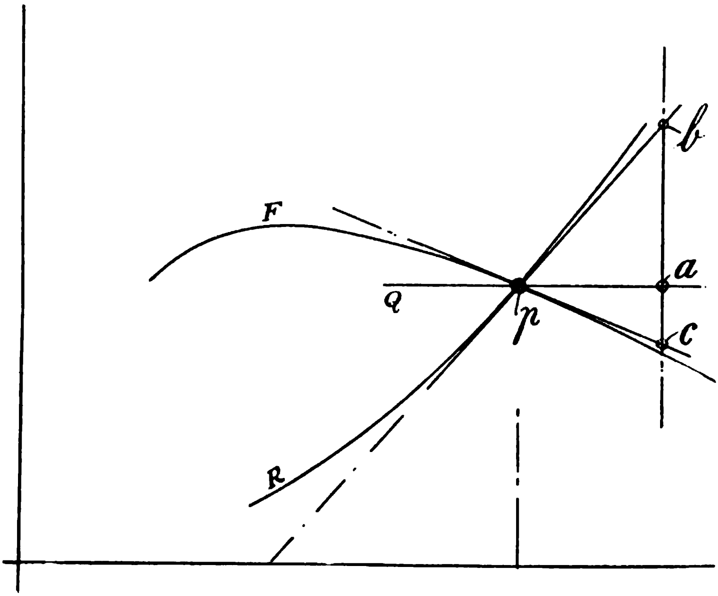


FIG. 105.

peller and the ratio of the gear transmission, and in the abstract we have nothing in the conditions to determine these quantities.

The diagrams given in Fig. 104 are rough plottings from the brake tests of a petrol motor, but this *type* of diagram is common to other prime movers. The point on the curve chosen for engines of fixed speed, or the portion of the curve utilised when

[1] This is the same law for given conditions as that of thrust expressed as a function of velocity V.

the speed is variable, is commonly that in the neighbourhood either of the speed of greatest torque or that of greatest horse-power. Thus in the case of a stationary gas engine, where fuel economy is a matter of first importance, the point on the curve chosen is preferably that where the speed is not greatly in excess of that of maximum torque. When, as is usual in the case of the flying machine, the weight of the prime mover is the most important consideration, the point on the curve chosen should be as nearly as possible that of maximum horse-power.

§ 119. The Influence of the Mode of Propulsion (continued).— In Fig. 105 R is the curve of resistance, and F the curve of

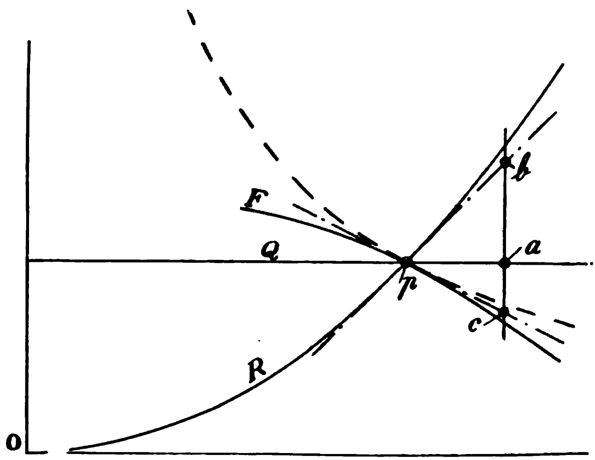


FIG. 106.

propulsion; the ordinates of the curves R and F represent respectively the resistances and propulsive forces proper to different values of V as given by abscissæ. It is assumed that the forces of propulsion and resistance are equal when the aerodrome is travelling at its natural velocity, that is, when $V = V_n$ the point p in the figure.

Let the ordinate of the line $p\ a$ represent the constant force of propulsion Q assumed in §§ 50, 51 and 57; then drawing

tangents to the two curves at the point p, and a line $c\ b$ perpendicular to $p\ a$, we have, for a given small change in V, the difference between the force of propulsion and the resistance represented by the line $c\ b$ as compared with an amount $a\ b$ under the conditions of a constant propulsive force.

Let us denote the quantity $\frac{c\ b}{a\ b}$ by the symbol Ψ.

When the motor is working at maximum horse-power, we have for small variations, $V\ F = constant$, or

$$\frac{dF}{dV} = -\frac{F}{V}. \tag{1}$$

But $R \propto V^2$ hence

$$\frac{dR}{dV} = \frac{2\ R}{V}, \tag{2}$$

or by (1) and 2),

$$\frac{dF}{dR} = -\frac{F}{2\ R}.$$

When $V = V_n$, that is at point p (Figs. 105 and 106), F (the thrust) is equal to R (the resistance), so we have[1]

$$\frac{dF}{dR} = -\frac{1}{2},$$

thus referring to Fig. 106,[2]

$$\frac{c\ a}{a\ b} = -\frac{1}{2}$$

or

$$\Psi = \frac{c\ b}{a\ b} = \frac{c\ a + a\ b}{a\ b} = 1{\cdot}5.$$

§ 120. The Influence of the Mode of Propulsion (continued).—Applying the foregoing result to the investigation of Chapter V.,

[1] The minus sign merely means that $a\ c$ is measured in the opposite sense to $a\ b$.

[2] In Fig. 106 a curve of the form $V\ F = constant$ is indicated by a dotted line. This curve is tangent to the torque curve at the point where horsepower is maximum.

we have, when the force of propulsion is constant (§ 61), the expression—

$$\frac{dH}{dL} = \frac{h_1}{H_n} \tan \gamma,$$

where h_1 is the maximum half amplitude of the flight path.

But by the preceding section (§ 119), under actual conditions $\frac{dH}{dL}$ is Ψ times as great as when the propulsive force is constant, therefore under these conditions,

$$\frac{dH}{dL} = \Psi \frac{h_1}{H_n} \tan \gamma.$$

As in § 61 this becomes

$$\frac{dH}{dL} = \Psi \times \sqrt{2}\, \Theta_1 \tan \gamma,$$

where Θ_1 represents the maximum inclination of the flight path to the line of mean gliding.

On carrying the correction through, as in §§ 62, 63, we obtain the final expression

$$\Psi \Phi > 1$$

as representing the conditions of stability, where Φ, as in § 63, is given by the expression

$$\frac{4\, l\, H_n^2 \tan \gamma}{\mathrm{I}\left(\frac{1}{K} + \frac{1}{c\, C\, \rho\, \epsilon\, a\, \beta}\right)}.$$

§ 121. Rate of Damping of the Phugoid Oscillation.—In Chapter V. we dealt with the damping effect of resistance quantitatively, but only as balanced against an opposed influence; we will now discuss the damping of the phugoid oscillation as a progressive phenomenon, as a measure of the degree of stability. We have first to examine and define the conditions on which the damping rate depends, and decide in what form we desire the solution presented.

The quantitative verification of the theory and equation of

stability depends upon our being able to recognise in the flight of an aerodone or aerodrome the particular case when for small oscillations the amplitude does not vary. Failing an actual photographic record, it is necessary to rely on visual observation; and although by this means any rapid change of amplitude can be detected with certainty, and a drop of 30 per cent. in five or six oscillations is quite appreciable, we do not know, without making the investigation on which we are about to embark, to what extent any given error of observation may affect the experimental estimate of the stability coefficient Φ. It is evident that we require to correlate the loss of amplitude (from one oscillation to the next) with the stability coefficient and with the other known data.

Employing the sine curve approximation and the symbols of Chapter V., we know that the variations of h depend both upon the phugoid path form and upon the damping effect of resistance where the phugoid variation is zero, as where h is maximum and its value is represented by h_1, $\frac{dh}{dL}$ is proportional to h itself (§ 57); let us go over this ground once more.

Ignoring for the present the moment of inertia effect, and taking a point on the flight path when h is maximum, that is, $\frac{dH}{dL}$ *as due to the phugoid equation* is zero, we may write down the following direct from the conditions:—

$W\frac{dH}{dL}$ = energy lost by aerodone per unit length of flight path. (1)

$W\gamma$ = energy supplied by constant force of propulsion per unit length of flight path. (2)

$\frac{H}{H_n}W\gamma$ = energy expended against resistance per unit length of flight path. (3)

By (2) and (3) energy lost by aerodone per unit length of flight path,

$$= \frac{H}{H_n}W\gamma - W\gamma = W\gamma\,\frac{H - H_n}{H_n}.$$

But $H - H_n = h_1$ and $dH = dh_1$, therefore by (1),

$$W \frac{dh_1}{dL} = W \gamma \frac{h_1}{H_n}$$

or
$$\frac{dh}{dL} = \gamma \frac{h_1}{H_n}, \tag{4}$$

which is the differential equation to the rate of damping (in terms of flight path) at the time when $h = h_1$ that is, when the flight path is at a point of maximum amplitude.

Let us for the time being ignore the phugoid oscillation, and suppose h to be influenced by damping alone, so that the flight path becomes a curve of the form given by equation (4), h being substituted for h_1, that is,

$$dL = \frac{H_n \, dh}{\gamma \, h}$$

Integrating this expression we have—

$$log_e \, h = \frac{\gamma}{H_n} L. \tag{5}$$

This gives the form of the resulting curve under the artificial conditions supposed; the amplitude damps out according to a logarithmic law.

Let us now introduce factors representing the influence of moment of inertia and propulsion. From an examination of the construction of the equation of stability (comp. §§ 62, 63), on the basis $\Psi = 1$, it is evident that the proportion of the damping factor neutralised by moment of inertia will be $= \frac{1}{\Phi}$ so that the proportion remaining will be $1 - \frac{1}{\Phi}$, which becomes when we take account of the propulsion coefficient,

$$\Psi - \frac{1}{\Phi}, \tag{6}$$

and equation (4) becomes

$$\frac{dh}{dL} = \left(\Psi - \frac{1}{\Phi} \right) \gamma \frac{h_1}{H_n}, \tag{7}$$

or
$$dL = -\frac{H_n}{\gamma\left(\Psi - \frac{1}{\Phi}\right)}\frac{dh}{h}.$$

Integrating as in (5) we have,

$$log_e\, h = \frac{\gamma\left(\Psi - \frac{1}{\Phi}\right)L}{H_n}. \tag{8}$$

So far the solution is on quite an artificial basis. The oscillation has been supposed suspended for the time being in order that we may examine the form of the damping curve; we now see that this is logarithmic. A curve in accordance with the equation is given as an illustration in Fig. 107.

If we could suppose the oscillation to result in a flight path of castellated form, we could interpret this curve as in Fig. 108;

FIG. 107.

this is of course impossible, but it is not a great step to take the integration of the sine curve as a basis, including the resulting factor $\frac{2}{\pi}$ in equation (7) when the integration becomes

$$log_e\, h = \frac{2\,\gamma\left(\Psi - \frac{1}{\Phi}\right)L}{\pi\, H_n}, \tag{9}$$

in which values of h are only comparable for values of L differing by multiples of one half phase length, *i.e.*, by a multiple of the quantity $\frac{L_1}{2}$.

In the examples that follow, L is taken at two successive maxima (or minima), the two values chosen being $L = -L_1$ $L = 0$ respectively. The values so obtained give a ratio that

holds good, in accordance with the well-known property of the logarithmic law, over every subsequent phase length.

Owing to the numerical relation that obtains (for phugoids of small amplitude) between H_n and L_1, equation (9) is susceptible of some simplification, thus $L_1 = 2\pi\sqrt{2}\,H_n$, hence, where

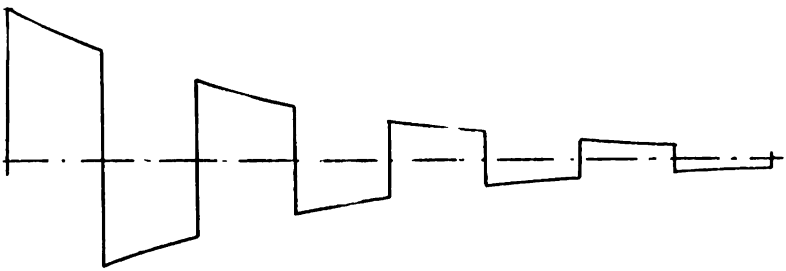


FIG. 108.

we are dealing with whole periods the equation becomes, for $L = \pm L_1$

$$log_e\, h = \pm 4\sqrt{2}\,\gamma\left(\Psi - \frac{1}{\Phi}\right)$$

or in common logs,

$$log\, h = \pm 2{\cdot}46 \times \gamma\left(\Psi - \frac{1}{\Phi}\right). \tag{10}$$

§ 122. Damping of the Phugoid Oscillation. Examples.—The foregoing investigation may be illustrated by a few examples. Firstly we will take the case of a simple aerodone presumed to be without moment of inertia and gliding under the influence of a constant force of propulsion.

Example I.

$$\gamma = {\cdot}2.$$

By the conditions $\Phi = \infty$, $\Psi =$ unity,

hence $$\Psi - \frac{1}{\Phi} = 1 - 0 = 1$$

$$\therefore\ log\, h = \pm 2{\cdot}46 \times {\cdot}2 = {\cdot}492 \text{ or } \bar{1}{\cdot}508,$$

giving, when L is respectively $+ L$ or $- L$, values,

$$h = 3{\cdot}1046$$
$$\text{and } h = {\cdot}3221.$$

Now when $L = 0$; $log\ h = 0$, or $h = 1$, so that for three successive phases we have amplitude values,

$$h = 3{\cdot}104$$
$$h = 1{\cdot}000$$
$$h = {\cdot}322$$

and since we know that a logarithmic series of this kind constitutes a geometrical progression we may continue the series of values by simple proportion thus:—

$$h = {\cdot}1035$$
$$h = {\cdot}0334, \text{ etc., etc.}$$

In tabulating these series for the different examples given we shall take $L = 0$ as the starting point, hence the initial amplitude will be represented as unity.

The units in which h is measured do not matter, the conditions of the investigation are that the amplitude is *small*, but the law probably applies as an approximation even when the amplitude is considerable. It is evident that since the series is a geometrical progression, any given value may be taken as the initial amplitude and the succeeding values calculated by simple proportion.

In the tabulated figures given in respect of each of the examples that follow, the number of completed phases of the curve is given against each of the tabulated values of h, beginning at the instant of launching, as zero, thus,

Phase.	Amplitude.
0	1·000
1	·322
2	·1035
3	·0334, etc.

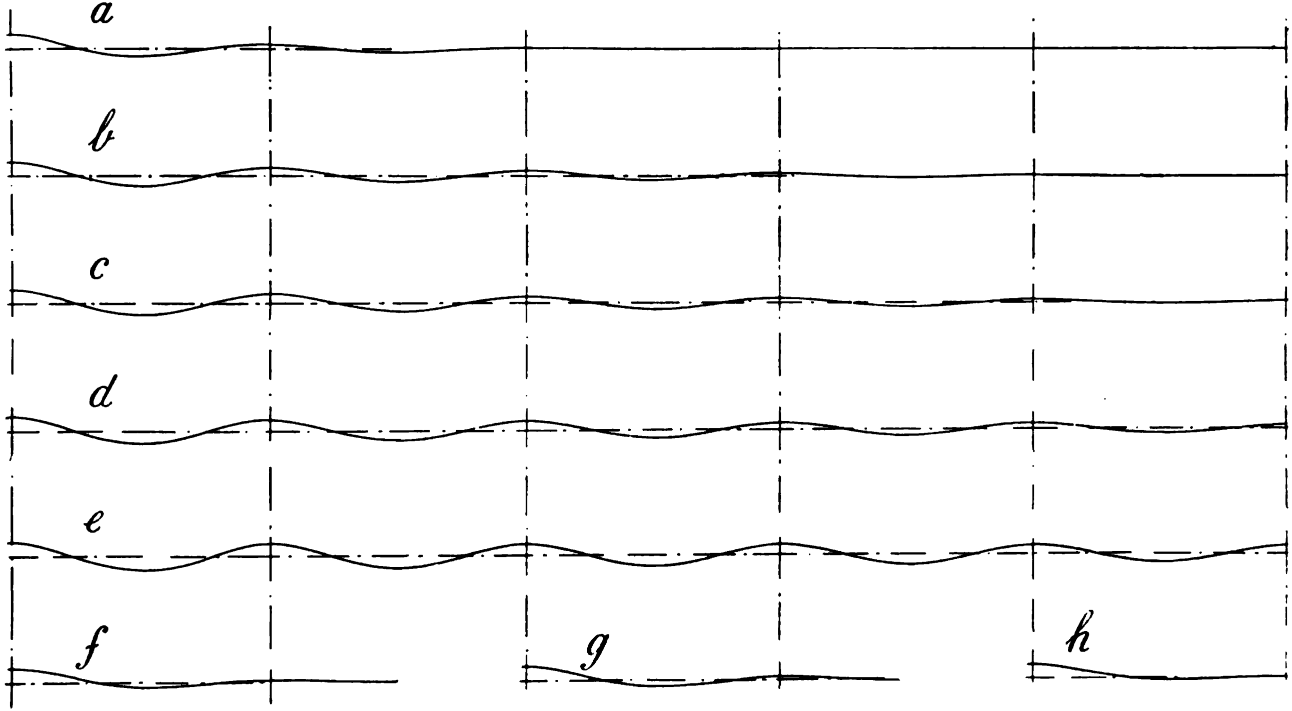


FIG. 109.

It will be noted that the amplitude falls to about one-tenth of its initial value in two oscillations; in this and the examples that follow the number of oscillations required to effect this reduction is taken as a handy criterion of the rapidity of the damping.

The flight path as here deduced for the supposed case is given diagrammatically in Fig. 109 (*a*), the initial amplitude being shown as $= \frac{1}{2} H_n$. The curves given in Fig. 109 are no more than rough representations of the form of the flight paths drawn by hand through the calculated maxima and minima.

§ 123. Damping of the Phugoid Oscillation. Examples (continued).

Example II.

$$\gamma = \cdot 2 \quad \text{(as before)}$$

$$\Phi = 2$$

$$\Psi = 1 \quad \text{(constant force of propulsion)}$$

$$\Psi - \frac{1}{\Phi} = 1 - \frac{1}{2} = \cdot 5$$

$$\log h = -(2{\cdot}46 \times \cdot 2 \times \cdot 5)$$

$$= -\cdot 246 = \bar{1}{\cdot}754,$$

or $$h = \cdot 5675,$$

and series becomes,

Phase.	Amplitude.
0	1·000
1	·567
2	·321
3	·182
4	·103

Example III.

$$\gamma \text{ and } \Psi \text{ as before ; } \Phi = 1{\cdot}5.$$

$$1 - \frac{1}{1{\cdot}5} = {\cdot}333,$$

$$\log h = -\ (2{\cdot}46 \times {\cdot}2 \times {\cdot}333)$$

$$= -\ {\cdot}164 = \bar{1}{\cdot}836$$

or $$h = {\cdot}685,$$

and series becomes,

Phase.	Amplitude.
0	1·000
1	·685
2	·469
3	·321
4	·220
5	·150
6	·102

The flight paths in the preceding examples (II. and III.) are depicted in Fig. 109 (*b*) and (*c*).

§ 124. Damping of the Phugoid Oscillation. Examples (continued).—It is evident that, in the examples so far considered, the rate of damping is such as to render quite obvious the fact that $\Phi >$ unity. We will now take two further cases, in which Φ is taken at values 1·25 and 1·111 respectively.

Example IV.

$$\gamma \text{ and } \Psi \text{ as before ; } \Phi = 1{\cdot}25$$

$$1 - \frac{1}{1{\cdot}25} = {\cdot}2$$

$$\log h = -\ (2{\cdot}46 \times {\cdot}2 \times {\cdot}2)$$

$$= -\ {\cdot}0984 = \bar{1}{\cdot}9016$$

$$h = {\cdot}797,$$

and series becomes

Phase.	Amplitude.
0	1·000
1	·797
2	·635
3	·506
4	·402
5	·320
	Limit of plotting (*d*) Fig. 109.
10	·102

Example V.

$$\gamma \text{ and } \Psi \text{ as before; } \Phi = 1{\cdot}111$$

$$1 - \frac{1}{1{\cdot}111} = {\cdot}1$$

$$\log h = -(2{\cdot}46 \times {\cdot}2 \times {\cdot}1)$$

$$= -{\cdot}0492 = \bar{1}{\cdot}9508$$

$$\therefore \quad h = {\cdot}8929,$$

and series becomes

Phase.	Amplitude.
0	1·000
1	·893
2	·798
3	·713
4	·636
5	·568
	Limit of plotting (*e*) Fig. 109.
20	·103

It is evident from the foregoing examples that when as many as five or six oscillations are under observation, the error of determining the condition $\Phi = 1$ is less than 10 per cent. where the conditions for observation are favourable; thus in the

case last considered, in which $\Phi = 1{\cdot}111$, the extent of damping in six oscillations is sufficient to approximately halve the initial amplitude, an amount that is quite obvious.

In the present state of knowledge it is evident, therefore, that an ordinary visual estimate of the rate of damping is sufficient for practical purposes, either for scale model experiments or for the verification of the equation of stability.

§ 125. Damping of the Phugoid Oscillation. Examples (continued).—We will now consider a case of an aerodrome propelled by a motor under maximum h.-p. conditions.

Example VI.

$\gamma = {\cdot}2$ as before; Φ taken = unity; Ψ from the conditions $= 1{\cdot}5$

$$1{\cdot}5 - 1 = {\cdot}5$$

$$\log h = -(2{\cdot}46 \times {\cdot}2 \times {\cdot}5),$$

which is the same as in Example II. (Fig. 109 (*b*)), hence an aerodrome whose flight path is on the point of instability without propulsion, enjoys a stability equivalent to $\Phi = 2$ when propelled by a motor working at maximum h.-p.

Example VII.

Data as before, but $\Phi = 2$

$$1{\cdot}5 - \frac{1}{2} = 1$$

$$\log h = -(2{\cdot}46 \times {\cdot}2),$$

which is the same as in Example I. (Fig. 109 (*a*)). Here the propulsion effect alone is just sufficient to neutralise the influence of moment of inertia, and the damping rate is the same as that for an aerodone without moment of inertia gliding under uniform propulsive force.

We will now take a few cases in which the flight efficiency is bad, *i.e.*, in which the gliding angle is considerably greater than in the foregoing examples.

Example VIII.

$$\gamma = \cdot 333\ ;\ \Psi = 1\ ;\ \Phi = \infty.$$

$$1 - \frac{1}{\infty} = 1$$

$$\log h = -\ (2\cdot 46 \times \cdot 333)$$

$$= -\ \cdot 82 = \bar{1}\cdot 18$$

$$h = \cdot 151,$$

and series becomes

Phase.	Amplitude.
0	1·000
1	·151
2	·023

and flight path (Fig. 109 (f)), becomes nearly dead-beat.

Example IX.

As before but $\Phi = 5$

$$1 - \frac{1}{5} = \cdot 8$$

$$\log h = -\ (2\cdot 46 \times \cdot 333 \times \cdot 8)$$

$$= -\ \cdot 656 = \bar{1}\cdot 344$$

$$h = \cdot 2208,$$

and series becomes

Phase.	Amplitude.
0	1·000
1	·221
2	·048

and again flight path (Fig. 109 (g)), is nearly dead-beat.

Finally, let us take a case of extreme inefficiency such as one

of the models of M. Pline (compare § 67); we will assume $\gamma = \cdot 5$ and moment of inertia to be absent.[1]

Example X.

$$\gamma = \cdot 5,\ \Psi = 1,\ \Phi = \infty,$$

$$1 - \frac{1}{\infty} = 1.$$

$$\begin{aligned} \log h &= -\ (2\cdot 46 \times \cdot 5) \\ &= -\ 1\cdot 23 = \bar{2}\cdot 77 \\ h &= \cdot 05888, \end{aligned}$$

and series becomes,

Phase.	Amplitude.
0	1·000
1	·059
2	·003

In this flight path, depicted in Fig. 109 (h), the phugoid oscillation appears to die away almost instantly, as actually happens with models of very low efficiency.

§ 126. The Law of Corresponding Speed.—It is not the least of the practical consequences of the Phugoid Theory that the *law of corresponding speeds*, first enunciated by M. Reech for ships, and independently discovered by Mr. W. Froude, holds good also for aeronautical machines. This law, which is the basis of all model experiments in naval architecture, and with which the name of Froude is inseparably associated, was stated by him as follows: —"If the ship be D times the dimension of the model, and if at the speeds $V_1\ V_2\ V_3$ . . . . the measured resistances of the model are R_1, R_2, R_3 . . . . then for speeds $V_1 \sqrt{D}$, $V_2 \sqrt{D}$,

[1] When the value of γ is high, the flight path is not as a rule greatly affected by moment of inertia. Compare Examples VIII. and IX.; Φ is ordinarily of considerable magnitude when γ is great.

$V_3\sqrt{D}$, ... of the ship, the resistances will be D^3R_1, D^3R_2, D^3R_3 . . . . To the speeds of the model and ship thus related it is convenient to apply the term *corresponding speeds*."

The above law may be otherwise expressed by saying that when the speed of one model or vessel is to that of another similar model or vessel as the square roots of their linear measurements, their resistances are as their respective masses, or as the cube of their linear measurements.[1]

The above law of corresponding speed rests upon the negligibility of viscosity as a physical quantity on which the resistance depends. This is not the same as the neglect of resistance due to skin friction, but rather the assumption that such resistance follows implicitly the V^2 law; thus, according to the law of corresponding speed as enunciated, skin frictional resistance must obey the law of total resistance, *i.e.*, it will be proportional to D^3; for the exposed surface varies as D^2, hence the resistance per unit surface varies as D, and since the velocity varies as $\sqrt{D}$ the skin-frictional resistance varies as V^2.

We are acquainted with the fact that resistances due to skin-friction and discontinuity, although definitely depending upon the fluid possessing viscosity, are substantially independent of the value of viscosity itself.[2] We know from the work of Allen that this fact is not strictly true, and further we know that the error in the case of skin friction is greater than for other forms of resistance in the production of which viscosity is involved. This difficulty is usually dealt with by the method proposed by Froude, as a correction to the experimental curve; this method consists in the separate computation of the skin-frictional resistances of the model and the vessel, and the deduction from the experimental curve of resistance of the difference so ascertained.

[1] For experiments with model vessels in a fluid of stated density, the mass must vary strictly as the cube of the linear measurement, otherwise the displacement would not be proportional.

[2] "Aerial Flight," Vol. I., *Aerodynamics*, Chap. II.

§ 127. Theory of Corresponding Speed.—In the case of a body that is totally submerged, and whose density is equal to the surrounding fluid (or where gravity is supposed inoperative), the resistance of a body of stated geometrical form is a function of five physical quantities, namely, linear size of the body, velocity of motion, density of the fluid, viscosity, and elasticity.

When we have to deal with the action of gravity, as when the body is in the region of a free boundary service (as in the case of a ship), or where the body is of greater density than the fluid (as in the case of an aerodrome), we have also to take account of the gravity *as an acceleration*, as one of the physical quantities of which the resistance is a function.

On the basis of ignoring elasticity (as being only a sensible influence at high velocities), the author has shown (Vol. I., § 39) that so long as gravity is without effect, the law of corresponding speed is given by a certain equation due to Osborne Reynolds. Further, in the section cited, a law is deduced (probably of no great practical utility), which takes cognisance of both viscosity and gravity as factors in the problem.

If now we ignore both viscosity and elasticity (a course that when gravity is inoperative gives us the V^2 law) there remain four physical quantities of which the resistance and the form of the fluid motion are functions, *i.e.*, linear size, velocity, density, and acceleration.

§ 128. The Theory of Corresponding Speed (continued). — The form of the fluid motion generated by a body under the present hypothesis depends upon the geometrical form of the body, and the quantities ρ l and V, and the acceleration of gravity which we will represent by f. It is customary to employ the symbol g for gravity, but this suggests gravity as a constant, or approximately so, whereas in the generalisation that follows we take gravity as a variable, just as if we required to apply the resulting law to widely different altitudes or to experiments conducted in planets other than our own.

We will proceed to demonstrate the law of corresponding speeds by the *method of dimensions*.[1] Now since we have to deal with models geometrically similar, *the form of the fluid motion* is a function of ρ, l, f and V, and of these quantities only. And the condition of corresponding speed is that the fluid motion is homomorphous in different cases; that is, the geometrical form of the fluid motion is constant. We therefore write—

$$l^p f^q V^r \rho^s = \text{constant},$$

or,
$$L^p \frac{L^q}{T^{2q}} \frac{L^r}{T^r} \frac{M^s}{L^{3s}} = \text{constant}.$$

Hence,
$$p + q + r - 3s = 0$$
$$2q + r = 0$$
$$s = 0$$

$\therefore$
$$p + q - 2q = 0$$

or,
$$p = q$$

and
$$r = -2q$$

$\therefore$
$$l^q f^q V^{-2q} = \text{constant},$$

or
$$\frac{lf}{V^2} = \text{constant}. \tag{1}$$

This is the law of corresponding speed, including possible changes in the acceleration of gravity, and would be applicable even if we suppose model experiments to be conducted on the planet Mars for vessels or aerodromes to be built on the earth. We are happily under no such obligation, so we substitute for the variable f the constant g, and the law becomes—

$$\frac{l}{V^2} = \text{constant, or } V \propto \sqrt{l}, \tag{2}$$

which is the law of corresponding speed of Reech and Froude.

§ 129. The Law of Corresponding Speed in its relation to the Phugoid Theory.—Although the abstract proof of the law of corresponding speed as generally applicable, under the conditions

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, Chap. II.

of hypothesis, is in itself sufficient, it is of interest to discuss its application to an aerodone or aerodrome in the light of the preceding flight path and stability investigations, more especially in relation to the results of Chapters II., III., and V.

It is at once evident that, since the phugoid chart represents the possible flight paths of an aerodone to a scale that varies as the square of the velocity, the value of H_n, the determining factor of the scale of the chart, will always be proportional to the linear size of the aerodone as given by the law of corresponding speed. This is in perfect harmony with the hypothesis, for it is evident that if the size of the aerodone did not vary directly as the scale of the flight path, the motion of the air could not be exactly homomorphous in different cases, and the conditions of corresponding speed could not be complied with. Thus the law of corresponding speed is fully consistent with the results of Chapters II. and III.

In the theory of stability of Chapter V., the equation,

$$\Phi = \frac{4\, l\, H_n^2 \tan\gamma}{\mathbf{I}\left(\frac{1}{K} + \frac{1}{c\, C\, \rho\, \epsilon\, a\, \beta}\right)}$$

contains quantities which vary when the size of the aerodone varies, these are :—

$H_n^2\, l \propto L^3$

$\mathbf{I} \propto L^2 \times$ mass (assuming the proportionate distribution of the latter).

$K \propto L^2$

$A \propto L^2$

But for similar aerodones the mass is proportional to L^3, for the volumes part for part are as L^3, and the densities part for part are constant;[1] hence $\mathbf{I} \propto L^5$. Therefore for similar aerodones at corresponding speeds—

$$\Phi \propto \frac{L^3}{L^5 \times \frac{1}{L^2}} \text{ which is constant,}$$

[1] The law of similarity must in effect apply to *density* as well as to geometrical form.

that is to say the law of corresponding speeds applies also in respect of the stability equation, and if any given model be proved stable to some stated degree, any geometrically[1] similar model at its *corresponding speed* will be stable in like degree.

§ 130. The Theory of Corresponding Speed (continued).—An examination of the manner in which the law of corresponding speed is dependent upon gravity, both in the case of the steamship or sailing vessel, and in the case of the aerodone is instructive. In the latter case we have seen how the law, as established from dimensional theory, results in a constant relation between the linear size of the aerodone or aerodrome and the linear size of the flight path, that is the scale of the phugoid chart. If we suppose, as in § 128 (1), that gravity be variable, then the scale of the chart will vary inversely as gravity, for H_n will vary inversely as gravity for a given value of V_n, and H_n gives the scale value to the chart. This result is reflected in the equation in the fact that the product of the linear size of the aerodone l, and the variable force of gravity f, is constant for given value of V_n.

In naval architecture the factor analogous to the flight path, as controlled by the acceleration of gravity, is the wave motion impressed on the water by the motion of the vessel. If it were not for wave motion and wave-making resistance the law of corresponding speed would be of no value to the marine engineer; the law of corresponding speed ensures that the comparison between the size and shape of the wave system to which the vessel gives rise and the wave system developed by the scale model holds good, so that the motion of the water is of exactly the same geometrical form in both cases.

Thus the relationship between the size of the waves (wave length) and their velocity is analogous to that between the size of the phugoid curve (phase length) and the velocity of flight.

[1] *Geometrically* used in its widest sense. Compare preceding footnote.

§ 131. Scale Model Experiments.—In practical aerodromics, *i.e.*, in the design and construction of flying machines, it is difficult to overrate the importance of the theory of corresponding speed and its application in experiments with scale models. In naval architecture this method of experiment has done enormous service in the development of the modern sea-going vessel, enabling an exact account to be made out in advance of the resistance and power expenditure for any particular form of hull at stated speed. In aerial navigation we may expect the method of the scale model to be of still greater service, for here it is not only a matter of estimating the power required, but also of ascertaining definitely that proper provision has been made for the necessary stability, both longitudinal and otherwise, and in the investigation of forms and types of which theory may not be able to render an exact account.

In other words, the information furnished by the law of corresponding speed and model experiment, in the case of the marine vessel, is of service as securing a more precise knowledge of the performance of a vessel than would otherwise be obtainable, and as enabling the naval architect to secure a given result at least cost. In the construction of flying machines we may look to the scale model to do all that of which it has hitherto shown itself capable in connection with naval architecture, and in addition to render the experimental and early stages of development free from the danger that at present exists, by placing in the hands of the aeronaut or designer an absolutely reliable means of testing his work without exposing himself to the smallest danger, and otherwise without risk of disaster. A few pence spent in small scale experiment will give as much information as an equal number of pounds expended in a full scale machine.

§ 132. Scale Model Experiments (continued).—Owing to the fact that *density* is one of the physical quantities involved in the dimensional equation, the density of the component parts of a model or machine is not a matter of indifference. As a factor in

the equation of corresponding speed it was shown that density vanishes from the variables, but this does not mean that it ceases to be a factor of importance.

It must be clearly understood that the *method of dimensions* in its present application, and in other cases of the same kind, depends definitely upon the condition of similarity, and the disappearance of density from the equation means that if a proportionate change of density takes place *throughout the system* such change is without effect. Thus in the case of model vessels the difference of density between sea water and fresh water does not matter so long as the density of the model is varied in like ratio; in this case the need for this variation is obvious, for otherwise the extent of immersion of the model would be affected. Beyond this, if the model experiments were extended to deal with rolling and pitching, the distribution of the density in the different parts of the model would require to simulate the actual differences in the vessel. When the experiments are confined to resistance this condition only needs to be complied with to the extent of keeping the centre of gravity in its correct position.

In the case of an aerodone or aerodrome the questions of rolling and pitching, or the equivalent, are actually a most important part of the investigation, so that evidently the distribution of the mass becomes a matter of vital importance. In order to adhere strictly to the law of similarity, we should require to construct the model exactly proportional to the full scale aerodrome in every detail, using like material for like parts throughout. It is evident, however, that the conditions do not necessitate any such extreme measures. We need to simulate such a model *in all essential respects*, and we must appeal to our former investigations to ascertain what points must be considered *essential.*

Referring to § 129 in which the law of corresponding speed was reviewed in its relation to the stability equation, we see first and foremost that the total mass must vary as the cube of the linear dimension; this is analogous to the density condition in the case of a boat; it carries with it also the same condition as

to the position of the centre of gravity being similarly situated. In the second place we have the condition that the moment of inertia shall be *equal to that of the exact scale model*; that is to say, proportional to the fifth power of the linear dimension. This evidently applies not only to the moment of inertia about the transverse axis, as involved in the equation of longitudinal stability, but also about the other co-ordinate axes; in all cases the moment of inertia is reckoned about an axis passing through the centre of gravity.

§ 133. Scale Model Experiments (continued).—It is evident from the foregoing discussion that before a model can be constructed of any given machine or design, not only must the weight and position of the centre of gravity be known, but also the moment of inertia must be calculated or measured about each of the three co-ordinate axes. In practice, this means that in the originating of a flying machine, the moments of inertia of the component parts should, in the first instance, be separately computed from the initial design, and the moments of inertia of position of these separate components should then be calculated from the location of their centres of gravity relatively to the centre of gravity of the whole machine; the total moments of inertia will then be the sum of the moments of inertia of the component parts as separately computed, and of their moments of inertia as due to their position.[1]

Having thus ascertained the moment of inertia of the intended design, and having settled the relative scale of the model it is proposed to employ, the moment of inertia of the latter, about each of its three co-ordinate axes, should be calculated on the basis of $\mathbf{I} \propto L^5$, and when the model is made it should be carefully adjusted to the calculated values,[2] at the same time the total weight being made rigorously proportional to L^3.

[1] By a well-known theorem in dynamics. The procedure given is advantageous as allowing for a re-arrangement or adjustment of the design with the least possible re-calculation.

[2] For the method of measuring moment of inertia, see § 172, also Appendix VI.

The functional parts of the model must, of course, be arranged strictly in accordance with the design, that is to say, there must be complete similarity of position in the arrangement of the aerofoil, tail-plane, fins, or directive organs, etc., between the model and the full-sized machine that it represents.

It is probable, in fact it is almost certain, that the conditions as to skin friction that apply in the case of model experiments with ships apply also with regard to aerial experiment, that is to say, the coefficient of skin friction diminishes to some extent as the size increases; in consequence of this it may be found necessary to make a correction on the score of skin friction, deducting on the basis of the coefficient proper to the model and adding on the basis of the coefficient proper to the full-sized machine, after the manner proposed by Froude and employed in experiments with model vessels. The two cases are not, however, altogether parallel; in the aerodynamical problem we have many considerations foreign to the conditions that obtain in the model ship experiments.

§ 134. Scale Model Experiments. Allowances.—The influence of the skin-frictional error is more far-reaching in the case of an aerodrome than in the case of a ship, and consequently the corrections required will be more extensive.

The direct correction for the relative reduction of resistance *on account of size* is the first, one may say, the *primary*, correction. This depends upon the difference in the coefficient of skin friction in the two cases, *i.e.*, the full-sized aerodrome and the small scale model. It, however, also depends upon other factors of which our knowledge, in the absence of an actual design, is limited; thus in the case of a model we know that we can make an aerofoil that is structurally sufficient without stays, struts, or guys of any kind, but in a full scale machine it is credibly established that this is not possible. Now all these guys, struts, etc., offer resistance which to all intents and purposes may be regarded as an augmentation of skin friction since this added resistance varies

as the square of the velocity, and further it increases in some ratio with the size of the aerofoil. Consequently, although the larger model or machine is less resisted owing to its lower coefficient, it experiences a greater resistance owing to these structural necessities; it will be only possible to ascertain by careful experiments and calculations whether and to what extent any allowance will be required.

As a consequence of any primary correction that may have to be made, others of a secondary character are required; thus if the resistance of the model be proportionately greater than that of the aerodrome, its γ value will be greater, and consequently it will possess a greater degree of stability, and allowance will require to be made for this, otherwise the small scale model may have a sufficient margin of stability, but the full scale machine may yet be deficient.

Given the extent of the primary correction, *i.e.*, the relative values of the gliding angle γ, the secondary correction could be provided by deliberately making the moment of inertia an appropriate amount greater (or less as the case may be) than calculated on the L^5 basis, in accordance with the equation of stability, so that the value of Φ shall remain constant. This correction may either be made in the model as compared with the design, or the correction may be made in the design to comply with the performance of the model.

§ 135. Scale Model Experiments. Summary.—The fundamental reason that corrections are required in accordance with the foregoing section, is that the influence of *viscosity* was ignored in the dimensional equation. This is definitely a weak point in the theory of corresponding speed, at least in the form enunciated by Reech and Froude. The author has shown (Vol. I., § 39) that in order that Froude's law should hold good *without correction* the kinematic viscosity of the fluid must be made to vary as the $\frac{3}{2}$ power of the linear dimension. The inconvenience of conducting flight experiments in specially prepared fluids is,

however, too great to allow this result to possess much practical interest; in spite of this it is highly probable that, when once the physical relations that exist between one fluid and another are properly recognised, many of the aerodynamic data required for the theory of flight will be more accurately determined from experiments in water than those conducted in air. It is worthy of remark in this connection that unless some great discovery as to the physical properties of fluids remains to be made, determinations of resistances and pressure reactions made in water, at velocities small in comparison to the velocity of sound, should apply to air in the ratio of the densities of the two fluids, *provided the product of velocity and linear dimension is, in the aquarium experiment, one-fourteenth of that represented in the actual conditions ;* fourteen being the ratio of the kinematic viscosities of the two fluids.[1] On the question of the resistance of the structural parts of the aerofoil or wing member, the struts, ties, etc., the magnitude of this can be determined approximately from independent data, the various elements of construction being separately assessed from tabulated experimental determinations.

It would seem desirable, at any rate during the earlier stages of practical aviation, to precede the construction of a full scale flying machine by two sets of small scale models, the first being on a quite small scale, perhaps $\frac{1}{16}$ full size, followed by a second trial in which the model could be conveniently made about $\frac{1}{6}$ full size, and furnished with all the constructional detail of importance in miniature. On this basis for a machine of 1,200 lbs. total weight, the first model would weigh about $4\frac{3}{4}$ oz., and could be constructed principally of mica, and could be flown indoors, the larger model would weigh some $5\frac{1}{2}$ lbs., it could be fitted with twisted rubber propulsion, and could be tested out of doors in any reasonably calm weather.

§ 136. Departures from Elementary Type.—The type of aerodone or aerodrome chosen for the purpose of investigation and

[1] Compare "Aerial Flight," Vol. I., *Aerodynamics*, §§ 25, 36 *et seq.*, and 137.

exclusively dealt with in Chapters II., III., and IV., etc., is one of the most elementary simplicity. There are certain forms of departure from type that are more apparent than real, and which evidently in no way affect the results of our investigation; for example, the aerofoil may consist of a number of superposed elements like the double deck machine of Lilienthal or the "Venetian blind" construction of Phillips, without invalidating the theory in any important respect. There are other modifications that probably require to be taken into account, as involving some departure from the conditions, thus if the aerodone is *accntric*, that is to say, if the centres of mass and resistance do not approximately coincide we have to deal with new conditions; for example, if the weight be hung some distance below the aerofoil, it is evident that since the velocities of the *mass* and the *aerofoil* on a curvilinear flight path are not equal, the Phugoid Theory will cease to apply with exactitude. Again, in the case of a machine that employs more than one aerofoil *in tandem* for the purpose of support, instead of a single aerofoil and a directive organ, it is doubtful to what extent the theory is applicable without some modification.

In the tandem arrangement it is evident that if the aerodone be constrained to follow a phugoid flight path, other than that of uniform gliding, it will require a variable applied torque, about a transverse axis, always acting in the direction of its rotation, thus when passing from left to right and *troughing* the applied torque must be counter-clock. If the distance from the leading aerofoil to the trailing aerofoil is small in comparison with the radius of curvature of the flight path this effect becomes negligible, just as the similar effect which exists also in the elementary type also becomes negligible when the tail length is small compared to the radius of curvature, one of the conditions of the initial hypothesis. It is evident therefore that this effect may be legitimately dismissed.

In a type of flying machine at present in some vogue,[1] the

[1] The Wright brothers are reported as employing a machine of this kind.

longitudinal stability is maintained by a horizontal rudder in advance of the main aerofoil. As a hand-operated device for regulating the flight of the machine the theory of such an arrangement is simple enough, but it would appear that it may also be used to secure automatic stability.[1] The theory under these circumstances is obscure. If the load per foot super on such a "leading plane" be greater than that on the aerofoil, the present theory probably covers the arrangement in question. We may regard the machine as one with a hypertrophied tail, on which the greater part of the weight is carried. If this explanation is found inadmissible, as must be the case if the pressure on the "leading plane" is less than that on the aerofoil, then the behaviour of this type of machine is a matter that awaits investigation.

The author believes that in general the tandem form is amenable to the same laws as the elementary type, and it would probably not be a difficult step to generalise the theory to include both types. It would appear possible, on the principle dealt with under the heading "Fin Resolution," § 99, that any such tandem combination might perhaps be resolved into an equivalent aerofoil and tail plane, and it would plausibly appear that we may have here the solution to the problem of obtaining an effective tail of great length without being penalised by excessive moment of inertia. This is one of the points left for future investigation.

§ 137. The Theory of Lateral and Rotative Stability.—The theory of lateral and rotative stability has not up to the present been subjected to so rigid an experimental examination and verification as the Phugoid Theory and equation of longitudinal stability. The general qualitative results unquestionably hold good; for example, the lateral oscillation as depending to some extent on the moment of inertia about the axis of flight, the damping effect and lateral stability obtainable by the employment of

[1] The flying models made by Clarke, of Kingston-on-Thames, are constructed with a "leading plane."

vertical fins, the importance of obtaining approximately neutral equilibrium (in respect of lateral motion) about a vertical axis, etc., these are all facts that may be demonstrated by simple experiment with mica models such as employed by the author and described in the present work.

The theory of *rotative stability* was actually developed as the result of an investigation to account for the anomalous behaviour of a model, which exhibited a form of instability that at the time was new to experience, at least so far as the author was concerned. It is of some interest to state that the aerofoil of this model had been designed with a view to testing the value of a tapered form whose ordinates were proportional to the curve of grading—approximately parabolic (comp. Vol. I., § 120). Thus the centre of pressure of the aerofoil must have been nearly neutral in respect of *aspect* and the aerodone had to rely almost entirely on its fins for its restoring couple. Now at the time of this experiment it was the author's practice to arrange the centre of pressure of the fin area well abaft the centre of gravity with a view to making sure that the aerodone should be directionally stable. The result of these two facts in combination was that the whole design of the aerodone seriously infringed the conditions of rotative stability, both as exhibited by experiment, and—as was subsequently ascertained—*by computation from the equation.*

Similar confirmation was obtained from another model in which the theoretical conditions were inadvertently infringed, but here the aerofoil was not of "neutral" form and the necessary data were not obtainable, the relation of the righting moment due to the aerofoil itself being an unknown quantity.[1]

§ 138. Application of the Theory of Lateral and Rotative Stability.—It has already been stated that the vertical fin is not without disadvantages; it lends itself to exact calculation better

[1] The plan-form of the aerofoil in this case was an ellipse with approximately segmental grading. Stability was restored by increasing the area of the leading fin.

than other arrangements by which lateral stability can be secured, but it offers an unnecessary extent of surface to skin friction. The importance of minimising resistance losses of every kind in the construction of a flying machine is so great that the very idea of resistance being incurred for the sake of calculation is inadmissible, except at most for initial experiments. It is consequently evident that the method of calculation or computation must be rendered more comprehensive to deal with such forms as from economic considerations may be found best.

Now in the case of the gliding bird there exists nothing in the way of a fin of any kind, although the streamline form of the bird's body must be regarded as possessing some considerable directive power, and thereby contributing to the rotative stability. The lateral stability proper is in all probability due, so far as it is independent of nervous control, to the considerations discussed in §§ 85—88 and to the upturned extremities of the primary feathers, a feature common to nearly all the larger birds when gliding or soaring.[1] This turning upward of the extremities is due to the natural flexure of the tips of the primary feathers owing to the pressure reaction, but it is none the less effective on this account.

In a flying machine it is evident that the provision for lateral stability will require to be proportionally more extensive than in the case of a bird, owing to the absence of any delicate mechanism of adaptation. It is, moreover, obviously better to give direct stability by the provision of adequate areas to the functional parts than to attempt to minimise these by any system of complex automatic adjustment.

The problem then resolves itself into determining the equivalent value in vertical fin area of wing or aerofoil terminations of known area and angle. The problem as thus presented is

[1] This feature may be observed in most gliding or soaring birds, but not always. In the swift and other members of the swallow family the wings have a very marked downward trend; even here the extreme tips of the wings are sometimes visibly flexed in the manner stated.

one for which it would be impossible, from theoretical considerations alone, to give more than an approximate solution.

As a practical solution, the first step is to investigate the critical point of rotative instability by means of a model designed to comply strictly with the requirements of theory, *i.e.*, with an aerofoil as nearly as possible neutral in aspect, and with fins of a form for which the aerodynamic data are accurately known. Then having verified the equation (or determined its defect), the proportions should be ascertained experimentally of the upturned extremities[1] that are equivalent to some definite fin-plan.

It is not anticipated that the fin system can be entirely dispensed with by the employment of inclined terminals, but rather that it shall be reduced to the minimum necessary to ensure that the aerodone shall be directionally neutral (comp. §§ 96 *et seq.*, and § 101).

§ 139. Conclusions.—Before passing on to the subject of the following chapter, which constitutes to some extent an independent section of the work, the conclusions that we have been able to reach may be enumerated, and a few words added as to the history of the present theory and of its experimental confirmation.

In the foregoing chapters it has been shown :—

(a) That the longitudinal stability of an aerodone is determined by purely dynamical considerations, without any adjustable organs or mechanism whatever.

(b) That the normal flight path may be regarded as a straight line trajectory in which the velocity of flight is just sufficient to produce the necessary lifting reaction of the aerofoil, and the gradient is at such an angle as may be required to supply by gravity the power required for propulsion, or in the case of an aerodrome propelled by motive power to supply (or absorb) the difference between the power supplied by propulsion and that expended in overcoming resistance.

(c) That the flight path otherwise consists of a series of

[1] Or such other feature as may be provided to give lateral stability.

undulations, whose amplitude is either constant or increasing or diminishing in accordance with certain equations which may be regarded as established beyond question as a first approximation.

(d) That the stability of the flight path depends upon the amplitude being either constant or diminishing, and the condition of such stability may be determined from the present theory in combination with known aerodynamic data.

(e) That in face of an isolated gust of wind the stability depends upon the relation of the velocity of flight or *natural velocity* to the velocity of the gust of wind; the latter in terms of the former must not exceed a certain critical value which may be deduced approximately from theoretical considerations alone.

(f) That in face of a synchronous periodic aerial disturbance the continued stability of an aerodone or aerodrome depends, under given conditions, upon the damping factor, a quantity that is calculable from the known data and which follows a logarithmic law.

(g) That lateral stability is intimately associated with the maintenance of direction and with a kind of stability termed by the author "rotative stability," and that the necessary conditions to ensure lateral, directional, and rotative stability can be satisfactorily met by suitably proportioning the various functional parts, without adjustable organs or equilibrium mechanism of any kind.

(h) That the law of corresponding speed enunciated independently by Reech and Froude in connection with model ships holds good *mutatis mutandis* for aerodones and flying machines.

Beyond the above the detail results reached are almost too numerous for recapitulation.

§ 140. Historical Note.—The series of investigations by which the foregoing results have been reached, published for the first time in the present volume, has been carried out by the author at odd times during the last fifteen years. In the first instance, about the years 1892–3, the general qualitative theory of automatic

equilibrium, both longitudinal and lateral, was developed. At that date the author had no knowledge of the work of Mouillard or Penaud; the account of these experiments has been added subsequently. The experiments of 1894 were devised with the unique object of putting to the test the theory in its then rudimentary form, and constituted in every respect a complete demonstration.

The next step of importance was made in 1896–7, when the quantitative solution to the flight path was found in the investigation of Chapter II. and the plotting of the phugoid equation, which was finally effected in October, 1897, in the form presented in Fig. 42, and which appears as a frontispiece to the present volume. The special cases had been solved about a year previously, and the result is to some extent embodied in a portion of the descriptive matter of Patent 3,608 of 1897, given in Appendix II.

In 1905[1] the author succeeded, by the employment of mica, in constructing very light aerodones of low velocity, and thereby effected the verification of the flight path curves of the Phugoid Theory, both as to the general form of the curves and as to the phase length and time relationship. It was in connection with these experiments that the influences of resistance and of moment of inertia were discovered. The damping effect of the former was first recognised, and a preliminary investigation as to the rate of damping was made; this and other observations resulted in the further discovery of the moment of inertia effect.[2] Both

[1] During the period intervening between 1898 and 1905 the author was too closely occupied in other ways to give much time to the present subject.

[2] The manner in which the importance of moment of inertia was eventually brought home to the author is perhaps worth recording. In endeavouring to construct a small low velocity indoor model with indiarubber propulsion it became evident that a further investigation was necessary. The author was already aware that a serious discrepancy existed in the damping rate calculation, but for the time being was unable to locate the fault. The miniature aerodromes, instead of damping, deliberately increased the amplitude of their flight path, and in watching their behaviour in flight it became evident that the cause was a kind of fly-wheel action; in other words, it was *visibly a moment of inertia effect.*

these influences were then subjected to rigorous theoretical investigation, with the results recorded. The investigations relating to lateral and directional stability followed shortly after, and that on rotative stability was undertaken owing to this peculiar form of instability having been accidentally encountered when making experiments with a quite different end in view. With the exception of a few embellishments and confirmatory experiments that have been added subsequently, the whole of the investigations were completed before the end of 1905.

CHAPTER IX

SOARING

§ 141. **Introductory.**—The phenomenon of "soaring" is not witnessed with sufficient regularity or frequency in the British Isles for it to impress itself on the ordinary observer as a distinct *mode* of flight. It is not at all unusual in fact to find the term entirely misapplied, and in view of the dictionary definition it is, perhaps, necessary to admit two distinct usages of the word, the one meaning being simply *to fly to a height*, quite regardless of the *mode* of *flight* employed, and the other being that commonly accepted and used in scientific works, the equivalent of the French term *vol à voile;* it is in the latter signification that the word is now employed.

In the active flight of birds, that is in flight such as that of the pigeon, where the wings are energetically employed (the *vol ramé* of the French), we are rightly accustomed to regard the work performed in the flapping of the wings as the source of the power expended in flight, and it is a matter of ordinary observation that under normal conditions, when the wing flapping ceases, the flight path takes on the whole a downward trend; under these passive conditions we know that the energy that is still being expended in the continued flight is derived from the *descent of the body under the force of gravity*, the angle of free descent being indicated by the symbol γ throughout the present work. The conditions under these circumstances are analogous to those of a cycle or motor vehicle when "free-wheeling" or "coasting"; the flight path must on the whole be at a sufficient downward inclination to supply the energy necessary for propulsion; the analogy also comprehends the

fact that in either case undulations may exist in the flight path, the latter even at times rising so that *work is done against gravity*, but this work is supplied from the kinetic energy of the body in motion which constitutes an energy reservoir, and any depletion has to be made good by a compensating descent at a greater angle than the natural gliding angle (γ), if the flight is to continue.[1]

The above facts are, as has been stated, a matter of common observation, and in still air there is no exception possible; it has, however, long been noticed that in windy weather the phenomenon of flight appears in an entirely different aspect, birds may be seen to glide for an indefinite period without any descent whatever, and without so much as a flap of the wings, or motion of any kind that could be of any material value for the purpose of propulsion. This is the type or *mode* of flight known as *soaring*, and it manifests itself under a considerable diversity of circumstances; the one important fact is that in all its varieties the presence of wind is essential.

The idea of sustained flight without any visible source of power is so foreign to the instincts engendered by modern training, that it has been the common experience of nearly all writers on the subject to have had the *facts* of observation received with doubt and incredulity; this scepticism, as constituting a barrier to the serious study of the subject, has frequently been made the subject of comment, and it is thus, perhaps, that at one time the problem was allowed to remain in the hands of persons ignorant even of the principles of relative motion, and those who, having no mechanical instinct to enable them to grasp the difficulties of the problem, could glibly explain the whole subject by a supposed analogy to the ordinary string-kite[2] (*cerf volent*)!

[1] The essential is that the velocity at the beginning and end of the observed flight should be the same.

[2] The term "*string*-kite" is employed in the present work in the same sense that "*paper* kite" is frequently used, in order to avoid possible confusion with the bird commonly so called.

In order that there should be no misapprehension as to the *facts*, the author has prefaced the present account of the theory of soaring flight by a brief description of the phenomenon as observed by him in these latitudes, including certain elementary deductions, and by quotations from the writings of authorities of unquestionable standing whose observations cover a wider field.

§ 142. Author's Observations on Soaring Flight.—Soaring flight, as defined by the preceding section, is distinguished from other *modes* of flight by two facts that are easy to recognise by attentive observation; firstly, that the bird when soaring does not sustain or propel itself by the performance of muscular work, as it does in active flight; secondly, that it does not continuously, or on the whole, lose altitude, as is the case in the gliding mode. Thus in appearance the soaring bird is a gliding bird that in some mysterious way glides continuously but does not come to earth.

The common swift (*Hirundo apus*) may, in a comparatively light wind, be seen to soar for considerable periods. The spectacle is not nearly so impressive as is the case with the larger birds, but as the swift is by far the most common example of a soaring bird to be witnessed *inland* in Great Britain, it is an example that we cannot afford to ignore. The author has on several occasions observed swifts to soar definitely without a wing flap for periods of 15 to 30 seconds, and ignoring a kind of spasmodic flutter that seems to take place at intervals of some 20 or 30 seconds, they may be seen to soar for periods of many minutes' duration. The "flutter" referred to is a very peculiar feature in the flight of the swift, it seems to be a kind of spasm that does not occupy more than a small fraction of a second; it leaves, however, a loophole for the sceptic, and prevents us regarding the evidence in this case as absolutely conclusive; there is a possibility that these spasmodic efforts contribute more considerably to the propulsion than in the author's opinion is the case.

More frequently than not the site chosen by the swift for

soaring is the immediate vicinity of some terrestrial obstacle by which the motion of the wind is disturbed; in this case it probable derives its support from the upcurrents developed, or otherwise from the variations in the wind velocity, in a manner subsequently to be discussed. On many occasions, on the other hand, the author has observed swifts soaring at greater altitudes, and *apparently* in no way associated with any obstacle or surface irregularity, although this is a fact of which it is difficult to make certain. Observations on the swift alone are not entirely conclusive as evidence of the fact of soaring flight, but so long as the fact is not questioned the demonstration is all that could be wished.

In January, 1893, the author witnessed the spectacle of some twelve or fifteen herring gulls following in the wake of the ss. *Germanic* the whole voyage from Liverpool to New York. The weather was stormy, and the passage occupied some ten days; in spite of this the gulls followed without difficulty, soaring continuously day and night. The author watched these gulls sometimes for upwards of an hour, and scarcely a single flap of the wing amongst the whole flight could be observed.[1]

[1] In an article on "Flight" in Newton's "Dictionary of Birds" the following passage occurs: "Gulls may be seen to soar for a prolonged period of time in front of or above cliffs on the shore when the wind comes from the sea. This, however, cannot be taken as an example of soaring proper, seeing that gulls can only remain suspended in the air without loss of relative horizontal motion under special conditions which do not apply in the case of the typical soaring birds. Gulls are not seen to soar except under conditions which point towards upward currents being a very obvious explanation of the phenomenon. In the case, on the other hand, of the typical soaring birds, such as those named above, soaring is observable under conditions and at heights where there is no sufficient reason to assume that upward currents exist. For example, eagles and adjutants are seen to rise continuously by soaring for miles above the ground or sea."

The above passage serves to illustrate how difficult it is to reconcile the statements of different observers, also the danger of *asserting a negative* unless the experience on which the assertion is made is universal.

Parenthetically it may be remarked that the latter portion of this passage is misleading. If upward currents exist in the atmosphere, other than as due to a terrestrial obstacle to the flow of the wind, then they will evidently

It is said that, generally speaking, the gulls change from an out-bound to a homeward-bound vessel when one or two days out, but on this occasion nothing of the kind took place.[1] The flight path consisted in what appeared to be gentle undulations, the birds describing orbits (relatively to the vessel) of oval form, the major axis being inclined backwards from the vertical, and probably of some 6 feet or 8 feet length.[2]

The herring gull may also be observed to remain in the air soaring for considerable periods of time at almost any point along the south coast where there are cliffs on the sea shore. The soaring here is evidently on a different footing and depends in most cases on the upcurrent set up the cliff itself. We shall find later that it is necessary to discriminate between certain different kinds of soaring, the simple case where the bird utilises an upcurrent does not differ, from the aerodynamic standpoint, from the gliding mode of flight, there is a superposed upward motion of the aerial system, nothing more.

In the autumn of 1906, on the west coast of Ireland, in particular Achill Island, the author observed the following gulls in soaring flight:—

Great Black-back (*Larus marinus*).	Numerous.
Glaucous[3] (*Larus glaucus*) . .	Two only.
Herring Gull (*Larus argentatus*) .	Very numerous.
Kittiwake (*Larus rissa*) . .	,, ,,

be greater at considerable altitudes, for the surface of the earth is a boundary surface to a fluid region to which the motion in the immediate vicinity is of necessity parallel, but it is not so constrained elsewhere (comp. § 145).

The reason that gulls do not soar to a height like eagles and adjutants is possibly connected with the manner in which they get their living. To *assume* that the mode of soaring is different seems to the author unwarrantable.

[1] The weather was thick and stormy, and no homeward-bound vessel was sighted.

[2] It is impossible to vouch for the accuracy of this estimate; the orbit may have been considerably larger than stated; distances at sea are deceptive.

[3] Some doubt attaches to identification.

In general, it was observed that in weather in which the gulls are able to soar in the open, the velocity of the wind is considerable, not very much less than the velocity of flight of the birds themselves. Observations were made for the most part on the nearly flat foreshore of Keel Bay, in the neighbourhood of Keel Lough.

With the wind in the south-west a pair of ravens were seen on several successive days to make use of the upcurrent of a cliff when returning from scavenging the foreshore at low water. Having reached the point where the cliff commences, these birds made no further wing effort, but merely sailed lazily up and up, till where the cliff merges into the steep of the mountain side, high above the "Cathedral Rocks," they became lost to view. On three separate occasions the author watched this simple, but effective, labour-saving device in operation; it was evidently part of a daily routine carried out with a familiarity born of long experience. What might happen when the wind is in some other quarter is an open question; the occasion did not arise, in all probability it is some other part of the foreshore that receives attention and the wind is still made humbly to do its duty, evidently whatever part of the island presents for the time being a "lee shore"[1] is that where the food supply will be most abundant, and it is there also that the wind provides the upcurrent for the journey back into the mountains.

On the slopes of Mweelin mountain, in September, a golden eagle[2] was seen soaring high on the mountain side; unfortunately, at too great a distance for minute observation. In contrast to the slow deliberate sailing of the eagle, one moment in a direct line, then in a mighty circle, could be seen the livelier but equally astonishing evolutions of a kestrel hawk; it was the hawk's hunting ground, the eagle was the usurper. First above, then

[1] A sailor's term, meaning that the *shore is to leeward of the water.*

[2] It would appear that this bird is now extremely rare in the British Isles; in 1906 only one pair were known to exist in the neighbourhood of Achill, reported locally as nesting in Clare Island.

far below, then completely lost to sight, and again high above, the hawk eventually drove the larger bird from the scene; the eagle took its departure still soaring, but in a straight course without "circling" or orbital motion of any kind.

§ 143. Soaring Flight Described by other Observers. — The albatros furnishes one of the most remarkable examples of soaring flight; its evolutions have been vividly described by many writers. To J. A. Froude[1] we are indebted for the following graphic passage:—"The albatros wheels in circles round and round, and for ever round the ship — now far behind, now sweeping past in a long rapid curve, like a perfect skater on an untouched field of ice. There is no effort; watch as closely as you will you rarely or never see a stroke of the mighty pinion. The flight is generally near the water, often close to it. You lose sight of the bird as he disappears in the hollow between the waves, and catch him again as he rises over the crest; but how he rises and whence comes the propelling force is to the eye inexplicable; he alters merely the angle at which his wings are inclined; usually they are parallel to the water and horizontal, but when he turns to ascend or makes a change in his direction the wings then point at an angle, one to the sky, the other to the water."

No less vivid is the description of the flight of the *condor*, given by Darwin;[2] the following extract will suffice. "When the condors in a flock are wheeling round and round any spot, their flight is beautiful. Except when rising from the ground I do not recollect ever having seen one of these birds flap its wings. Near Lima, I watched several for nearly half an hour without once taking off my eyes. They moved in large curves, sweeping in circles, descending and ascending without once flapping. As they glided close over my head, I intently watched from an

[1] "Oceana" (1886), p. 76.
[2] "Voyage of H.M.S. *Beagle*," Chapter IX.

oblique position the outlines of the separate and terminal feathers of the wings; and if there had been the least vibratory movement these would have blended together, but they were seen distinct against the blue sky. The head and neck were moved frequently and with force, and it appeared as if the extended wings formed the fulcrum on which the movements of the neck, body, and tail acted. If the bird wished to descend, the wings for a moment collapsed; and then when again expanded with an altered inclination, the momentum gained by the rapid descent seemed to urge the bird upwards, with the even and steady movement of a paper kite. In the case of any bird *soaring* its motion must be sufficiently rapid so that the action of the inclined surface of its body on the atmosphere may counterbalance its gravity. The force to keep up the momentum of a body moving in a horizontal plane in that fluid (in which there is so little friction) cannot be great, and this force is all that is wanted."

Some information of importance may be gleaned from the description of the evolutions of the soaring bird given by Mouillard in his "L'Empire de l'Air"; the following paragraphs may be quoted as to the point:—

"Sans vent le voilier tombe, son vol n'est plus possible, il est obligé de devenir rameur; c'est ce qui fait qu'il est rarement matinal, parce que la matinée est ordinairement calme, surtout dans les pays chauds.

"Admettons maintenant l'existence d'un courent d'air, ce qui arrive presque toujours a une certaine hauter dans l'atmosphère.

"La scène change, le voilier décrit des cercles, s'élève en l'air a une grande hauteur, puis de là se laisse glisser dans la direction ou il veut aller, même contre le vent."

And later he says:—

"Un voilier qui s'élève par un temps calme rame ordinairement jusqu'à une centaine des mètres, et arrivé à cette hauteur commence à décrire ses ronds, moitié ramant moité planant, diminue les battements à mesure que l'élèvation augmente et

finit par les cesser tout à fait; ce qui démontre que l'air n'est immobile qu'à la surface du sol."

Langley[1] gives the following account of the soaring of the turkey buzzard:—"On the only occasion when the motion of one near at hand could be studied in a very high wind, the author (Langley) was crossing the long 'Aqueduct Bridge'[2] over the Potomac, in an unusually violent November gale, the velocity of the wind being probably over 35 miles an hour. About one-third of the distance from the right bank of the river, and immediately over the right parapet of the bridge, at a height not over 20 yards, was one of these buzzards, which for some object which was not evident, chose to keep over this spot, where the gale, undisturbed by any surface irregularities, swept directly up the river with unchecked violence. In this aerial torrent, and apparently indifferent to it, the bird hung, gliding, in the usual manner of its species, round and round in a small oval curve, whose major axis (which seemed toward the wind) was not longer than twice its height from the water. The bird was therefore at all times in close view. It swung round repeatedly, rising and falling slightly in its course, while keeping on the whole, on one level, and over the same place, moving with a slight swaying, both in front and lateral direction, but in such an effortless way as to suggest a lazy yielding of itself to the rocking of some invisible wave."

The foregoing observations must be considered sufficient not only to place any question of *fact* beyond doubt, but also to define the characteristics of soaring flight as known to observation.

§ 144. The Different Modes of Soaring Flight.—In a letter to *Nature*,[3] in 1883, Rayleigh laid it down that in order that a bird

[1] "The Internal Work of the Wind" (S. P. Langley); "Smithsonian Contributions to Knowledge."

[2] Washington City.

[3] *Nature*, XXVII., p. 534.

should consistently maintain its flight, without working its wings, either—

(1) *The course is not horizontal;* or

(2) *The wind is not horizontal;* or

(3) *The wind is not uniform.*

These alternative conditions, first clearly enunciated as above, must be looked upon as the essential basis on which all explanations of soaring must be founded.

It is evident that if a wind were *uniform* and *horizontal* the condition of the bird in continued flight, so far as the dynamics of flight is concerned, would be precisely the same as for still air; the difference is merely one of relative motion; therefore soaring is impossible under these conditions. If the horizontal velocity of the wind be greater than the maximum velocity at which the bird is able to travel, that is if it exceed the value of V_n by more than a certain amount, it is evident that the bird will be unable to stem the current; this, however, does not affect its power of flight *as such* one way or the other, it merely affects the utility or otherwise of the exercise of that power.[1]

The first of Rayleigh's conditions is framed to cover the case of a downward flight path. It is evident that if the flight path is not on the whole horizontal and that the maintenance of flight is due to this cause, the bird (or aerodone) will eventually come to earth. The case is one of ordinary gliding, hence we have already excluded this by our definition of soaring, and only conditions (2) and (3) remain, thus Rayleigh's dictum may be expressed in the following modified form:—*In order that a bird should remain continuously in flight without performing work and without loss of altitude*, that is to say, *in order that a bird should soar*, either—

(A) *The wind being uniform possesses an upward velocity component;* or,

(B) *The wind is not uniform.*

[1] Thus a horizontal wind exceeding the maximum velocity of flight, renders flight impossible to a bird that requires to return to its point of departure, but it may be of inestimable value to a bird in migration.

So far we have not demonstrated in what way these conditions render it possible for soaring to take place, the problem has been discussed as a negative proposition. We shall in the following sections show that these conditions form the fundamental basis of two distinct *modes of soaring*, both of which are practised extensively by birds in flight. In some cases it is probable that the soaring bird takes advantage of favourable meteorological conditions of both kinds, at one and the same time; such mixed conditions without doubt frequently exist. This, however, in nowise prevents the subject being dealt with in theory as if the two conditions are entirely separate and distinct.

There are two kinds of want of uniformity that will be discussed as coming under condition (B); firstly, one in which there is a fluctuation due to a general turbulent condition of the atmosphere,[1] as evidenced by the "gusts" that commonly form part of the phenomenon of *wind;* and, secondly, that due to the proximity of an obstacle to the flow of the aerial stream, such as a building or other obstruction by which a "dead-water" region is caused and maintained. The first is thus a want of uniformity of *velocity* expressed as a function of *time* for any particular point in space; the second is a want of uniformity of *velocity* as a function of *position in space* for any particular instant of time. These two kinds of want of uniformity give rise to two recognisably different modes of soaring; a subdivision of the mode due to condition (B).

§ 145. The Vertical Component of the Wind. Meteorological Considerations.—The explanation of *soaring* on the assumption that the bird is sustained by an upcurrent, or by an upward trend of the wind, is one that has frequently been advanced, and also on which doubts have been repeatedly cast. There would appear to be considerable prejudice existing against the idea of vertical motions of the air, and before discussing the probable

[1] "Aerial Flight," Vol. I., *Aerodynamics*, §§ 37, 131.

influence of upcurrents on the phenomenon of soaring, it is desirable to examine the question of such currents from a meteorological standpoint.

At the outset it may be pointed out that since the whole of our everyday experience of wind motion relates to movements of the air in the immediate vicinity of a *boundary surface* of the atmosphere, the whole of our ordinary wind data relate to motions parallel to that surface, that is on the whole to horizontal motion; this fact alone is sufficient to account for an innate dislike to any other kind of wind. Probably if we had spent our lives at the top of the tower a few thousand feet in height we should have become accustomed to specifying the direction of the wind both in *altitude* and *azimuth*, and the motion of a vertical wind would not have seemed strange or unusual.

In an article by C. S. Roy in A. Newton's "Dictionary of Birds," the following passage occurs:—"That the direction of the wind even at great heights, and above a comparatively smooth sea or plain, is by no means always parallel to the surface of the globe is more than probable; but we know of no reason for assuming that the upward currents are sufficiently predominant over the downward currents to justify us in looking on the former as capable of explaining observed facts as to soaring." This passage is delightfully ingenuous; firstly, it is evident that the upward and downward currents, measuring them by their *flux*, must *on the whole be exactly equal*, otherwise there will be an accumulation or an attenuation of the lower strata of the atmosphere which we know is not the case, so that we certainly have no reason to suppose that either predominates. Secondly, it is evident that it is entirely unnecessary to the proposition that the upward currents should predominate, unless it be first shown that more than half the area of the earth is simultaneously available to the soaring bird. Thirdly, it is unnecessary that the upcurrent theory should explain every case of soaring as implied, it is sufficient that it should explain those that are not otherwise explicable. The

above is quoted as showing, in connection with wind motion, the *bias* under which the human mind labours.

Now as to positive evidence; of this there is no lack. The phenomenon of *land* and *sea breezes* is one of which the *rationale* has been understood for a very long time.[1] When the influence of the solar rays begins to be felt in the early part of the day, the surface of the land quickly becomes warmed and gives up some of its heat to the air in its vicinity, which thus becoming specifically lighter than the surrounding air tends to rise.[2] The regions covered by water (rivers, seas, and oceans) do not so readily become heated, partly because the sun's rays penetrate to

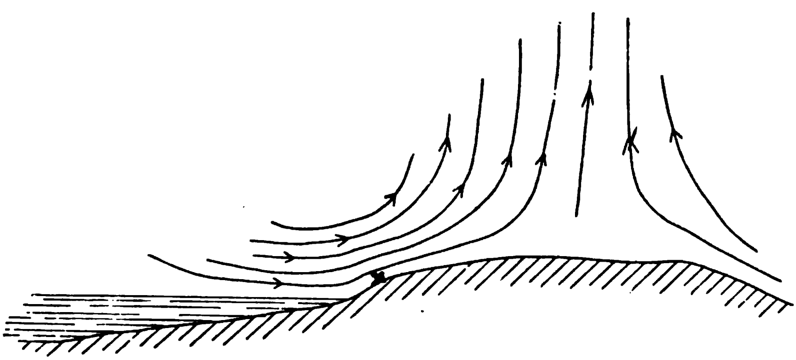


FIG. 110.

a greater depth, and therefore have to heat a considerably greater quantity of matter, and partly because the specific heat of water is greater than that of dry soil, and also probably because more of the heat vanishes in supplying the latent heat of evaporation of the water that is taken up in the state of vapour. The well-known consequence of this is that in the middle part and early middle part of the day, when the power of the sun has made itself felt, there is a strong upcurrent over the regions occupied by land

[1] Reference may be made to any encyclopædia or text-book of Meteorology.

[2] There is, perhaps, an academic objection to the use of the phrase "tends to rise"; this objection, however, is without effect on the discussion.

the displacement of which has to be made good by air flowing in laterally across the coast line. This is illustrated in the case of an island in Fig. 110. Later in the day, when the solar heat has declined, the land loses heat rapidly by radiation, and the conditions are, to some extent, reversed.

The above is the universally accepted explanation of the *sea breeze* that springs up in the course of the forenoon at the "sea-side" in the summer in our own latitude and in a more pronounced manner still in lower latitudes where the mid-day heat is more intense. It is scarcely necessary to remark that in rough weather the sea breeze is on too small a scale to

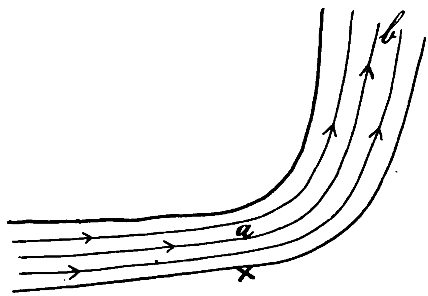


Fig. 111.

overcome the greater aerial disturbances of which the atmosphere is the prey; it is, however, worthy of note that if there be a slight breeze in the early morning in a direction opposed to the "sea breeze," that is to say, a breeze off shore, it is said by yachtsmen that the sun *eats up* the wind, for such a breeze in fine weather nearly always drops later in the forenoon, when we may regard it as having been neutralised by the sea breeze that would otherwise have arisen.

Thus the *sea-breeze* and the *up-current* are two parts of one and the same stream. The position of an observer on the coast line, Fig. 110, is comparable to that of an observer situated at the bend of a river, Fig. 111. If he measures the velocity of the stream in his immediate environment (*a*), he will have some idea

of its magnitude at (*b*). Now a sea-breeze of ten or fifteen miles per hour is not uncommon, so the up-current by which such breeze is generated may be expected to have a velocity of this order of magnitude. Again, from the purely theoretical point of view, let us suppose that the column of heated air is but one degree C. hotter than the surrounding air, then its density will be approximately $\frac{1}{300}$th part less than normal, and if the height of the heated column be 300 feet the difference of pressure by which it is propelled will be equivalent to a "head" of one foot; this, by the principle of Torricelli, corresponds to a velocity of 8 ft./sec., which is more than sufficient to sustain a gliding bird without loss of altitude.[1] It is of interest to note that in the passage already quoted from Mouillard's work he speaks of the *voilier* (soaring bird) *commencing* to soar at about 100 metres altitude, up to which point active flight requires to be employed if the weather is calm. The passage also, "C'est ce qui fait qu'il est rarement matinal, parce que la matinée est ordinairement calme, surtout dans les pays chauds," is of considerable interest. The word *matinée* must evidently, from the context, be translated as "*early* morning."

§ 146. The Vertical Component of the Wind (continued).—The question of the vertical component of the wind is actually far broader than might be supposed from the foregoing discussion. The considerations that apply to the generation of a sea-breeze in reality apply universally to the origin of almost every kind of aerial disturbance. Thus the changes of density due directly and indirectly to the solar heat constitute the mainspring from which nearly all wind energy is derived. There are exceptions of some slight importance, as, for instance, electrical action, the tidal action of the moon and sun on the atmosphere, also the internal heat of the earth; but the heat energy, whether solar or terrestrial,

[1] It is only necessary that the up-current should exceed the velocity of least descent in order that a gliding bird should be indefinitely sustained. Compare "Aerial Flight," Vol. I., *Aerodynamics*, § 176.

acts on the density of the atmosphere from point to point either by change of temperature or by change of water content, and so gives rise to *vertical motions*, to which the horizontal currents are counterpart. Thus in considering the horizontal component of the wind first and the vertical as a secondary and doubtful quantity we are reversing the order of cause and effect.

In spite of the above, it is probable that the vastly greater proportion of the energy *stored* in the atmosphere is in the form of horizontal motion, so that the customary view is to some extent justified. Using an analogy, we might express the position by comparing the vertical motions of the air to the piston movement of an engine, and the horizontal motion that results, and persists, to that of the fly-wheel.

In the summer of 1905 the author observed a particularly clear example of the type of upward flow that takes place under the conditions discussed in the preceding section when becalmed off Yarmouth, in the Solent. About mid-day, approximately at the turn of the tide (low water), the whole eastern portion of the Isle of Wight and mainland in its vicinity could be seen mapped in the sky. Over the land the sky was closely studded with a number of bolster-like cumulus clouds, apparently situated at a height of some few thousand feet. Each cloud forms the head of an ascending column, with a network of interspaces through which return currents descend from higher altitudes. The broad avenues and masses of blue sky, faithfully representing the waterways beneath, also are occupied by the descending currents.

Perhaps one of the most striking and convincing cases of an upward current on record is that most graphically described by Santos Dumont[1] in connection with a ballooning adventure. He says: "Shortly after this I let the balloon go down again, hoping to find a safe air current, but when within 300 yards of the ground, near the Var, I noticed that I had ceased descending. As I had determined to land soon in any case, I pulled the valve rope and let out more gas.

[1] "My Airships" (A. Santos Dumont), p. 54.

"I could not go down. I glanced at the barometer, and found indeed that I was going up. Yet I ought to be descending, and I felt, by the wind and everything, that I must be descending. Had I not let out the gas?

"To my great uneasiness, I discovered only too soon what was wrong. In spite of my continuous apparent descent, I was, nevertheless, being lifted by an enormous column of air, rushing upward. While I fell in it I rose rapidly higher with it.

"I opened the valve again; it was useless. The barometer showed me that I had reached a still greater altitude, and I

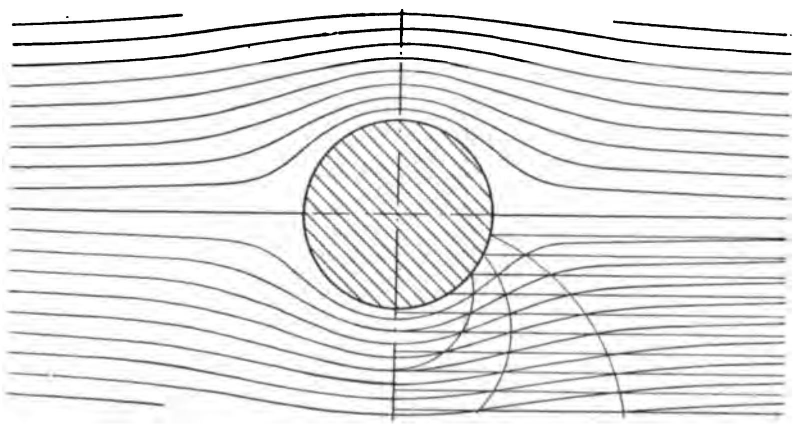


FIG. 112.

could now take account of the fact by the way the land was disappearing under me. I now closed the valve to save my gas. There was nothing but to wait and see what would happen.

"The upward rushing column of air continued to take me to a height of 3,000 metres (almost two miles). I could do nothing but watch the barometer. Then, after what seemed a long time, it showed that I had begun descending."

In general it may be stated that cyclones are regions of ascending currents; waterspouts, sand "pipes," and tornadoes, are also evidence of upward motion. It is in fact the potential

energy released by the ascent of the lighter air (and descent of the heavier) that passes into the vortical system, originally of sluggish motion, and renders it dangerous by reason of the enormous velocity that it acquires.[1]

§ 147. The Vertical Component of the Wind as dependent upon Geographical Conditions.—The vertical component of the wind as due to the physical features of the country, *i.e.*, mountains, cliffs, etc., is of a kind that is too self-evident to require any argumentative support. When a wind passes over a ridge, such, for instance, as a range of hills, the various layers flowing like strata one above the other, are diverted at first to the same extent as

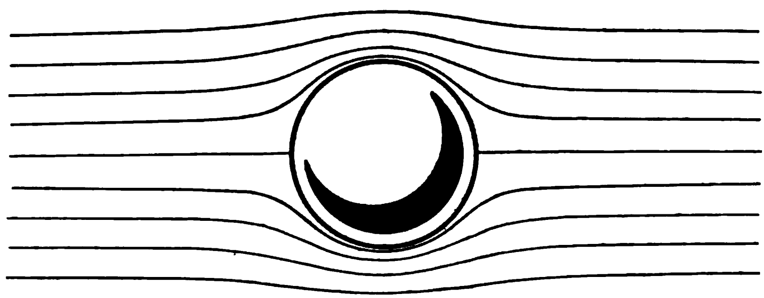


FIG. 113.

the ground, and in the upper layers to a lesser degree. If, instead of a ridge, the hill or mountain is isolated, or, as it is sometimes termed, of "sugarloaf" form, the disturbance close to the ground is parallel to its surface, as before; but the layers above are less deeply affected owing to the fact that the air can pass round the hill sideways as well as over the top.

The behaviour of a wind in passing over a ridge of an isolated

[1] When a mass of matter in rotation or circulation is drawn towards the centre of motion, its velocity is increased to an extent equivalent to the work done in overcoming centrifugal force. It is perhaps of some interest to note that the regions of the earth where the soaring bird performs its evolutions on a grand scale are also the regions where the phenomena mentioned may be most frequently observed.

hill may be diagrammatically illustrated by the hydrodynamic plottings given in Figs. 112 and 113, representing the flow of a "perfect fluid" round a cylinder and a sphere respectively; the horizontal axis of symmetry may be regarded as representing the ground surface, and the upper part of the diagram a ridge of semicircular section or a hemispherical hill, as the case may be.

In practice, in cases such as those illustrated, the motion is not of the form shown. Owing to the fact that the air is not a perfect fluid, the flow will be of the *discontinuous type*, with a dead-water region in the lee of the obstacle; this is unquestionably the case with forms far less abrupt than those figured,

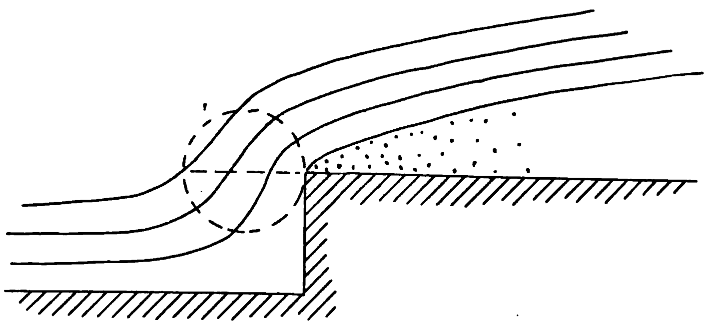


FIG. 114.

hence the shelter afforded by a range of hills or mountains; if the flow were of the Eulerian form such shelter would not exist. The general character of the stream-lines, on the windward slopes of the ridge or hill, is not, however, greatly different in the two types of flow, so that we may look upon Figs. 112 and 113 as roughly representing the conditions, so far as the ascending currents are concerned.

For gentle undulations, where the gradient is not too great, we may evidently take the upward velocity of the air current as approximately proportional to the gradient; the assumption here is that the air stream follows the surface of the ground, and that its velocity is not seriously affected.

In the case of a vertical cliff the form of flow will closely resemble that calculated on the Rayleigh-Kirchhoff basis,[1] this being represesented in Fig. 114. The surface of discontinuity of course becomes the seat of turbulent motion, as indicated diagrammatically in Fig. 115, in which an attempt has been made to indicate the stream flow as it must actually take place. The region of greatest up-current is evidently immediately in advance of the upper edge of the cliff, and the "useful" region may be roughly defined as a circle (shown dotted in Figs. 114 and 115), whose centre is slightly above the level of the plateau, and whose diameter is about equal to the height of the cliff.

It is evident that similar up-currents will be generated by other

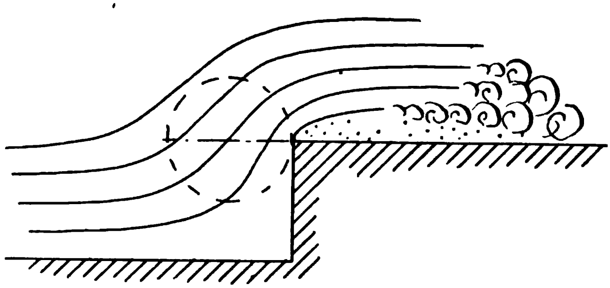


FIG. 115.

kinds of obstruction, such as houses, walls, etc., and, further, in some cases the up-current may arise from the motion of a body through the air in lieu of the air moving past the body. In this way ships, especially sailing vessels, may conceivably become the cause of an aerial disturbance that the soaring bird can turn to its advantage.

§ 148. The Up-Current in its Relation to Soaring Flight.—It is not by any means every case of soaring flight that is to be attributed to an upward motion of the air. There are, however, many cases in which this explanation is clearly adequate, and is indicated by the conditions. There are also many cases in which

[1] "Aerial Flight," Vol. I., *Aerodynamics*, §§ 94 *et seq.*

the upward motion of the air, though perhaps not in itself sufficient, unquestionably plays a prominent part.

It is in the author's opinion comparatively rarely that in these latitudes a sufficiently powerful up-current arises from the simple heating of the air to support a bird without loss of altitude. We may take it that the velocity of the soaring bird is usually about 35 to 50 feet per second, and that its natural gliding angle is approximately 1 in 5 or 1 in 6; it may be somewhat less, but that it is greatly less is scarcely probable. Consequently the downward velocity of a bird in gliding flight *relatively to the air* is commonly about 7 to 8 feet a second, and it will therefore require an up-current of a velocity equal or superior to this in order that soaring should become possible.

In connection with this subject it must be remembered that it is not only necessary that the conditions should be *occasionally* favourable, but that they should be favourable with sufficient frequency or regularity for the bird to take advantage of them in connection with its means of livelihood. If the necessary up-current exists over some particular region half a dozen times in the course of a year, and no more, it is hardly likely that we shall find that on these occasions the air will be infested with eagles and adjutants, or other of the larger exponents of this mode of flight. Neither will an essentially marine species leave its fishing ground in order to give a demonstration of soaring flight for no useful purpose. The most we can expect to witness on these occasions is that many of the birds that usually fly in whole or in part by wing exercise have taken to soaring, and that active flight is only resorted to occasionally and at infrequent intervals.

The connection between the frequency and regularity of conditions that admit of soaring and the presence—one may almost say the very existence—of the soaring bird that makes use of these conditions, is probably more intimate than has been hitherto imagined. A most suggestive passage occurs in Darwin's "Voyage of the *Beagle*," as follows:—"This day I shot a condor.

It measured from tip to tip of the wings eight and a half feet, and from beak to tail four feet. This bird is known to have a wide geographical range, being found on the west coast of South America, from the Strait of Magellan along the Cordillera, as far as eight degrees north of the Equator. The steep cliff near the mouth of the Rio Negro is its northern limit on the Patagonian coast; and they have there wandered about 400 miles from the great central line of their habitation in the Andes. Further south, among the bold precipices at the head of Port Desire, the condor is not uncommon; yet only a few stragglers occasionally visit the sea coast. A line of cliff near the mouth of the Santa Cruz is frequented by these birds; and about eighty miles up the river, where the sides of the valley are formed by steep basaltic precipices, the condor reappears. From these facts it seems that the condors require perpendicular cliffs." It is curious that Darwin should not have associated the fact to which he here testifies with the explanation of the soaring flight, which obviously suggests itself. The idea can hardly have escaped him, but presumably as a matter of observation the direct evidence was insufficient to justify the suggestion being presented.[1] In spite of this, the author feels that the circumstances are such as to render it almost certain that the flight of the condor depends very much upon the existence of these cliffs and the up-current to which they give rise.[2]

Up-current soaring may be seen at almost any point along the coast in this country where cliffs abound, especially in the south, where the prevalent wind (south-west) lends itself appropriately. In suitable weather the gulls can be seen gliding, and continually gliding, rising or falling at will, in front of the almost perpendicular chalk cliffs, where, with a breeze of some 15 or 20 miles

[1] It is not usually supposed that the up-current region in front of a cliff is so extensive as evidently it is (Figs. 114 and 115). Modern hydrodynamic theory did not exist at the date of Darwin's observations.

[2] It might be added, the *existence* of the condor may depend upon these geographical features, both in the sense of its past evolution and as to its future continuance.

an hour over a considerable region, the up-current would appear to be at least twice that actually necessary for mere sustentation. In a letter to *Nature*[1] Mr. S. E. Peal gave a sketch diagram and description of the soaring of some of the larger birds, those especially mentioned being pelican, adjutant, vulture, and cyrus,[2] the point of observation being situated near Sibsagar, Assam, that is, in the valley of the Brahmaputra, at the foot and on the slopes of the Naga Hills. The sketch is reproduced in Fig. 116, and the particulars given and to be inferred from the description are as follows:—The birds rise by flapping the wings vigorously, and begin to soar when they reach an altitude of from 100 to 200 feet from the ground. The flight path then consists of a series of large sweeping circular movements of *about* 150 feet

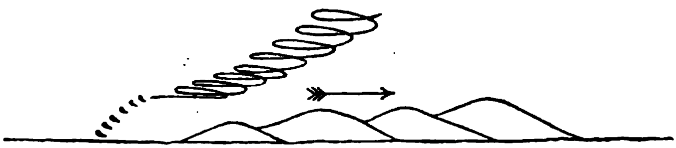


Fig. 116.

diameter, rising from 10 to 20 feet at each "lap." When soaring in this way the wings are rigidly extended and quite motionless; the birds frequently soar thus to a height estimated at 8,000 feet. The velocity of soaring flight is computed as from 15 to 35 miles per hour. The general trend of the spiral that constitutes the path of flight is to leeward, the loss of position, or drift, being about 20 to 50 feet each lap.

If we take the velocity of flight to be the mean of the limits estimated, *i.e.* = 25 miles an hour, the time taken to complete one lap of the spiral is approximately 13 seconds, which, on the rate of ascension stated, is about one foot a second. The probable rate of descent *through the air* of one of these birds when soaring

[1] *Nature*, XXIII., p. 10.

[2] The name here given does not appear to be in general usage. The author has been unable to identify this bird.

is 7 ft./sec., so that the upward velocity required to account for the phenomenon is 8 ft./sec. total.

If we take the difference between the maximum and minimum estimates of the velocity of flight as due to the superposed motion of the air, that is as due to the horizontal component of the wind, we obtain for the latter $\frac{35 - 15}{2} = 10$ miles per hour, or say 15 ft./sec. Thus the motion of the wind as estimated from the data supplied has an upward trend of about 1 in 2, and a velocity of between 11 and 12 miles per hour. It is of some interest to note that the mean upward course of the spiral flight, taken from Mr. Peal's sketch, corresponds with the upward angle of the wind as given by the above estimate. This coincidence suggests that if the foregoing reasoning is correct, the stratum or column of ascending air may be one of quite limited dimensions, a kind of stream in fact which the birds in their flight are careful not to quit.

Mr. Peal offers an explanation different from that here given, but as it appears to involve the usual fallacy of supposing that the bird can draw upon the translation energy of a horizontal air current, after the manner of a string kite,[1] it has not been thought suitable for discussion in the present work.

Lord Rayleigh in discussing Mr. Peal's observations points out the impossibility of the explanation offered, and advances the alternative one that the layers of the air at different altitudes are moving at different velocities. On this basis he shows on dynamic principles that a bird could extract energy from the wind. He says, however: "*A priori*, I should not have supposed that the variation of velocity with height to be adequate for the purpose; but if the facts are correct some explanation is badly wanted." The author is quite in agreement with this passage; in the light of present day knowledge it is evidently impossible to account for soaring in the manner suggested.

About the same date (1883) Mr. Hubert Airy offered an alterna-

[1] See footnote, § 141.

tive explanation, founded on the supposition of horizontal vortices in the atmosphere, due to, and constituting part of the phenomenon of *wind.* The author does not believe that in the case in point this explanation is indicated by the facts as presented, but the subject may be of great importance in its relation to soaring of another kind, and further reference will be made to this suggestion.

§ 149. The Up-current in its Relation to Soaring Flight (continued).—The late Mr. W. Froude was one of the most staunch adherents

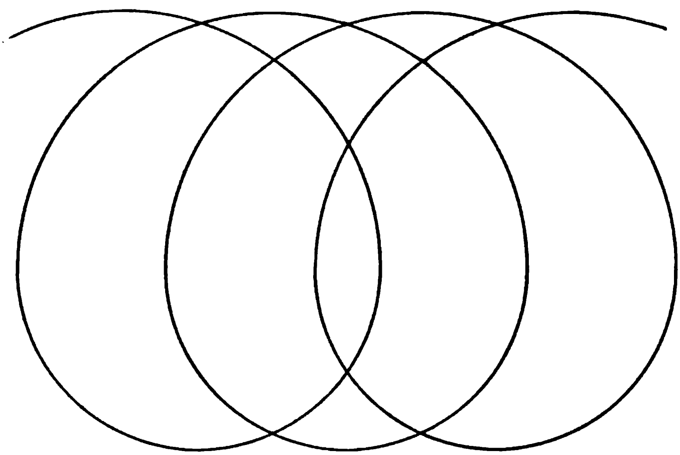


FIG. 117.

to the up-current theory of soaring, and a passage[1] in one of his letters to the late Lord Kelvin, shows that he fully appreciated the conditions essential to this view of the problem. Froude, however, endeavoured to apply the up-current explanation universally to all cases of soaring, and though many of his arguments are of very great interest, they are at times scarcely convincing, and eventually Froude himself voluntarily admitted[2] that the explanation in some cases was quite inadequate.

[1] Proc. Roy. Soc. Edin., XV., pp. 256–8 (1888).
[2] Proc. Roy. Soc. Edin., XVIII., pp. 65–72 (1891).

The form of the flight path of a soaring bird has been given by many observers, and in the kind of soaring we have now under consideration this consists of a number of circles, often with a superposed motion of translation, Fig. 117. When the gain in altitude is taken into account this form becomes an oblique spiral. Many theories have been built on this peculiarity, it being sometimes explained that the bird in going with the wind acquires an enormous velocity, and when it turns to go against the wind it can utilise this velocity in order to gain altitude. All such explanations, founded on the assumption of a wind of uniform velocity, involve a fundamental fallacy, and are inadmissible; on the principles of relative motion the position of a bird in still air or in air moving horizontally *and with uniform motion* is identical.[1] In spite of the absurdity of an argument of this kind, unless supplemented by some additional hypothesis touching the *irregularity of the wind*, it has appeared again and again; Mouillard[2] has presented it; Esterno[3] has advocated it; Marey[4] has taken it seriously and devoted nearly half a chapter to it; S. E. Peal[5] has offered it, and many others have fallen into the same pit. Rayleigh drew attention to this fallacy in 1883, when he enunciated the necessary alternatives already cited.

It has frequently been stated by observers that the circular orbit described by the soaring bird is not horizontal, but at a slight inclination; this supposed inclination is usually made to fit in with the requirements of the fallacious theories offered, and the orbit is not always described as leaning in the same direction.[6]

[1] In the sense of the article. Compare § 114. Assuming such an explanation as valid, a bird could soar in still air, for, by gliding *in the direction of the earth's rotation*, it could acquire an *enormous velocity*, and on turning to face the current the altitude to be gained would be *measurable in miles!*

[2] "L'Empire de l'Air," p. 43.

[3] D'Esterno, "Du Vol des Oiseaux," p. 40.

[4] Marey, "Le Vol des Oiseaux," Chapter XX.

[5] *Nature*, XXIII., p. 10.

[6] See Marey, "Vol des Oiseaux," § 17.

In view of the influence of perspective as affecting the apparent attitude of these flight path curves, and the disagreement between different observers, it would appear that this feature, in the absence of more conclusive evidence, may be safely ignored. The author believes that in general any attempt to found a theory of soaring on the horizontal orbital form of flight path is doomed to failure; the soaring bird moves in circles for the same reason that a man sits in a chair,—*because he wants to stay where he is.*[1] There are doubtless exceptions, but these are discussed in a later section.

When the up-current by which a bird soars is due to the wind passing up a slope or over a range of hills, the ability of the bird to utilise the up-current does not depend solely upon its upward velocity, as is the case when the air is rising vertically; it depends also upon the steepness of the gradient; there is a certain minimum inclination that must be exceeded. Thus if the gradient be exactly equal to the gliding angle, there is one critical wind velocity—the natural velocity V_n—at which the bird can soar; if the wind is too slow the bird will sooner or later come to earth; if the wind velocity is too high, the bird must either come to earth or be swept over the top of the ridge. Soaring is therefore only just possible on such a gradient, and in practice the angle needs to be considerably steeper than the gliding angle, so that there shall be a reasonable range of wind velocity available. It is possible that careful observations on the soaring of birds on slopes of known gradient may furnish data that will enable the gliding angles of different species to be determined with considerable exactitude.

In estimating the natural gliding angle γ and natural velocity V_n, the considerations dealt with in the author's "Aerodynamics" require to be taken into account. Thus the velocity of slowest fall is 1/1·315 of the speed of least resistance, and the gliding angle of slowest fall is 1·155 times the gliding angle of least resistance.[2]

[1] This was clearly the case in the example given in § 142.

[2] Vol. I., §§ 164, 174–5.

§ 150. Dynamic Soaring.—We now pass to the consideration of the second of the alternative conditions under which soaring may become possible (§ 144), *i.e., the wind is not uniform.* This is the third of the three conditions as laid down by Rayleigh. It will be shown that we now have to deal with an entirely different kind of soaring from that so far discussed, one of which the theory is of far greater difficulty, involving complex dynamical considerations. A distinction between the two kinds of soaring being desirable, the author has termed that kind we are about to discuss, *i.e.*, that which depends upon want of uniformity of the wind, *dynamic soaring.*

There is more than one kind of aerial disturbance that results in a want of uniformity (comp. § 144), and there are correspondingly different kinds of dynamic soaring; all, however, depend upon one fundamental principle, the utilisation of the *internal energy* or, as it has been termed by Langley,[1] the *internal work* of the wind. This term does not carry the same meaning as in thermodynamics, but rather an analogous meaning; it signifies in the present usage the kinetic energy *additional to that of mean velocity, but excluding that of the thermodynamic system.*

It is established that *wind* comprises in addition to its main motion of translation a superposed motion of turbulence;[2] and just as we may regard the energy of these two component motions as separately put into the wind, so we may recognise in the resultant motion two separate *energy accounts* which are virtually independent one of the other.

We are accustomed, in considering the aerodynamics of flight,[3] to regard the aerofoil or wing spread of a bird as dealing exclusively and uniformly with a stratum of air of some definite limited thickness, instead of, as is actually the case, dealing with an indefinite stratum in a variable degree, and in the *hypothesis*

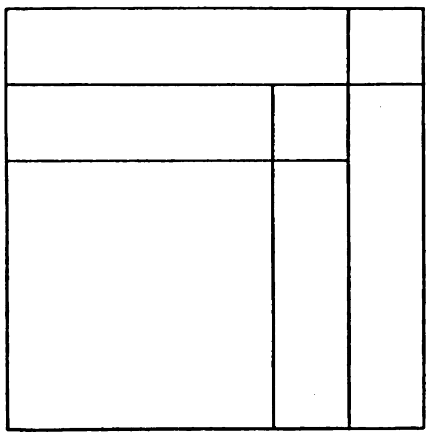
[1] "The Internal Work of the Wind," "Smithsonian Contributions to Knowledge,"

[2] Vol. I., §§ 37, 131.

[3] Vol. I., §§ 160, 172 *et seq.*

of constant "sweep" this method has been shown to give results in very close agreement with experience. Employing, again, the same idea, it is evident that the turbulence energy of the wind with which we are immediately concerned is that contained in the stratum of air with which the bird is supposed to deal, not of necessity confined to that coming within the sweep area, but *perhaps* that further included by the area of the periptery.[1]

It is evidently fundamental that the bird or aerodone cannot change the total momentum of horizontal motion of the air dealt



A C B D

FIG. 118.

with, without itself definitely undergoing a change of velocity, for any such action involves a horizontal force applied *from without*, consequently the mean horizontal velocity remains unchanged and the energy of translation of the wind cannot be touched. Thus if a bird act on some portion of the wind to impart to it change of velocity in the direction of flight, itself undergoing retardation, then it must sooner or later act on some other part of the wind, imparting to it a change of velocity opposite to that of flight, so that it may itself undergo acceleration and so preserve its *mean* velocity through the air unchanged.

[1] "Aerial Flight," Vol. I., *Aerodynamics*, Appendix V.

Now if the motion of the wind were originally *uniform*, any such procedure as above implied would mean imparting energy to the wind, otherwise *superposing turbulence on an existing motion of translation*. If, however, the wind is initially in a state of turbulence, and the bird by the exercise of intelligence or otherwise act in such a manner as to *equalise* the velocity of the air with which it is able to deal, it will have at its disposal the energy of turbulence so removed from the wind.

Taking an example, let us suppose that the bird, flying in air having a mean velocity represented by the line $A\ B$ (Fig. 118), deals with two equal masses of air, one having a velocity represented by $A\ C$, and the other a velocity $A\ D$, and if the bird by means of temporary variations of its own velocity and altitude (its initial and final states being the same) increase the velocity of the first mass of air to equal $A\ B$ and decrease that of the second to like degree, then it will have to supply energy equal to twice the area $A\ B$, $B\ C$ *minus* the square on $B\ C$, and it will have at its disposal energy equal to twice the area $A\ B$, $B\ D$ *plus* the square on $B\ D$. But $B\ C = B\ D$, hence there is an available balance of energy in the bird's favour equal to twice the square on $B\ C$.

It may be noted that the magnitude of the mean velocity $A\ B$ is a matter of no importance; it *cuts out*, thus for given conditions as to turbulence the motion of translation is unimportant, it has nothing to do with the problem except so far as in practice the turbulence cannot exist without the motion of translation. In the limit we might suppose that the turbulence alone exists, then the bird's duty will be to bring all the air with which it deals to rest, and the energy at its disposal is obviously that shown, *i.e.*, due to the integration of the velocity squared.

Before discussing the actual nature and theory of *dynamic soaring* it is very important to fully comprehend the underlying principles, of which the above may be taken as a preliminary exposition. The matter has been dealt with from a quantitative

standpoint in Vol. I., Appendix V., from which the following demonstration is taken substantially unaltered.

§ 151. Dynamic Soaring. Preliminary Investigation.—Without discussing the means whereby a soaring bird operates to play off one portion of the wind against another, we may, from the foregoing considerations, form an outside estimate of the *available energy*. Thus, if we prescribe some conventional form as representing the motion of turbulence, such as a uniform reciprocating motion in the line of flight, or a simple harmonic motion, or a compound harmonic or circular motion, of known velocity, we can calculate the turbulence energy per *unit volume*, and we may convert this into an equivalent thrust force per unit area of the stratum " handled "; if, then, we know the extent of this area in the case of any particular bird, we can determine the gliding angle γ, the minimum value of which is a quantity otherwise known. Conversely we may, starting from the gliding angle and other data, determine the minimum velocity of turbulence on the convention chosen that will render soaring flight possible.

A question that presents some difficulty is the estimation of the *area of the stratum handled.* At first sight this might be supposed to be the " sweep " of the aerofoil, *i.e.* $= \kappa A$ (Vol. I., §§ 109, 160, " Aerodynamics "), but the energy estimated on this basis from known fluctuation data appears to be insufficient.

The conception of the *peripteral area* (Vol. I., § 210) suggests that, as in the case of the propeller blade, the cyclic or peripteral system may distribute the momentum over a much greater mass of fluid than that coming within the sweep of the aerofoil, and so a larger mass of the air than that coming within the sweep area will be "handled" in the sense of the present discussion. On this basis the sectional area of the stratum from which the energy can be drawn is given by the expression, $\frac{1+\epsilon}{1-\epsilon}\kappa A$ (comp. Vol. I., § 210).

In the following example the turbulence velocity is computed necessary to provide the requisite energy to a hypothetical albatros, whose data are:—

Weight .		$= 14$ lbs.
Area . .		$= 5$ square feet.
n . . .		$= 12$, hence $\kappa = 1{\cdot}195$ and $\epsilon = {\cdot}75$.
γ .	taken	$= \frac{1}{7}$.

The computation will be made on the basis of the *peripteral area*, and figures will in the first place be obtained on the basis of uniform motion, either a uniform reciprocation, or a circular motion may be presumed, the assumption being in either case that the whole of the available energy is utilised. It will be shown subsequently that there are other conditions limiting the proportion of the total energy that can be usefully employed.

Now resistance to flight $= W\gamma = 14 \div 7$ pounds $= \frac{14 \times 32{\cdot}2}{7} = 64{\cdot}4$ poundals.

And peripteral area $= \frac{1+\epsilon}{1-\epsilon}\kappa A$, or, mass of air handled per foot traversed,

$$= \frac{1+\epsilon}{1-\epsilon}\rho\kappa A = \frac{1{\cdot}75}{{\cdot}25} \times {\cdot}078 \times 1{\cdot}195 \times 5 = 3{\cdot}3 \text{ lbs.}$$

Let v = velocity of turbulence under supposed conditions, we have

$$\frac{3{\cdot}3 \times v^2}{2} = 64{\cdot}4$$

$$\therefore\ v^2 = 39;\ \text{or, } v = 6{\cdot}25.$$

If, instead of supposing uniform motion, we take fluctuation to be of simple harmonic form, and let v_1 be the maximum velocity of the fluctuation; then we know that the energy for a given value of v_1 is half as great as for the same value of v on the previous supposition, hence in order that the requisite energy should be present, $\qquad v_1 = \sqrt{2} \times v,$

or, $\qquad v_1 = 8{\cdot}8.$

§ 152. Dynamic Soaring. Historical Development of Modern Theory.—The present-day theory of soaring of the kind under discussion may be regarded as the outcome of many suggestions and theories that have been offered from time to time during the last thirty years. Thus the earliest clear statement of the connection between the wind *fluctuation* and the soaring of birds with which the author is acquainted is due to Mouillard.[1] The following passage is of considerable interest, not only on account of the explicit manner in which the matter is stated, but also as containing the germ of an idea that has been developed independently by Mons. A. Bazin and by the author into a means of demonstrating the principles of dynamic soaring to an audience.

Mouillard says:—

"Le coup de vent est une puissance qui est l'âme de l'ascension: c'est la baguette qui frappe le cerceau de l'enfant, qui lui donne la force de rester debout,[2] et même de franchir des élévations. Supposons que nous abandonnions ce jouet à une descente rapide; l'attraction lui communiquera un mouvement qui le fera rouler jusqu'au bas. Si en bas il se trouve une montée, le cerceau poussé par sa vitesse acquise, par son inertie, remontera à un hauteur égale à sa descente, moins les frottements sur le sol et la resistance de l'air."

* * * * *

"Si maintenant nous supposons autre chose, qu'on puisse, lorsque le cerceau est on train de remonter, deplacer le sol, de maniere a ce qu'il aille en sens contraire du jouet, c'est a dire lui venir dessus, nous activerons encore l'ascension en lui communiquant une force supplémentaire, indépendente de son individu, dont la resultante sera encore une élévation."

Unfortunately, although here Mouillard gives the gist of the matter, the whole of the passage cited is given as a kind of supplement following on an explanation that is quite inadequate

[1] "L'Empire de l'Air," p. 47.

[2] **Mouillard's analogies are not always easy to understand!**

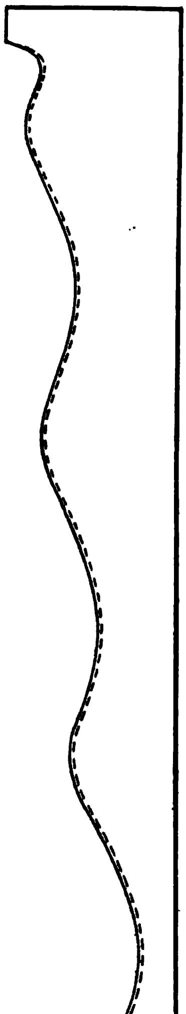


Fig. 119.

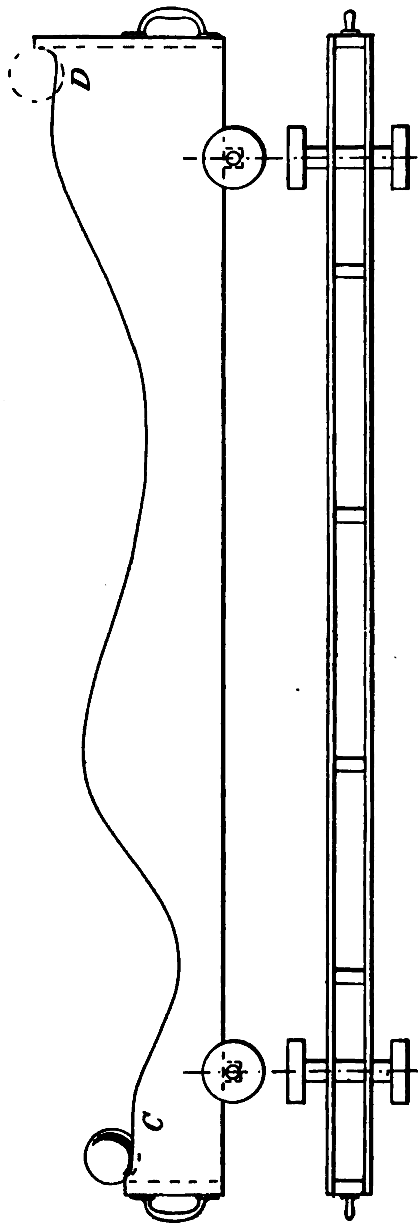


Fig. 120.

and which rests on the "string-kite fallacy," to which allusion has already been made.

In 1885, Rayleigh, in a letter to *Nature* (§ 144), suggested the variation of the wind at different altitudes as a possible solution, although the conditions enunciated in his letter will bear a much wider interpretation. Hubert Airy, about the same time,[1] propounded a similar theory, but suggested further that the irregular motion of the air due to horizontal vortices may constitute a factor of importance. Next we have Bazin,[2] who, in 1890, was the first to demonstrate the principles of dynamic soaring by means of a "switch-back" model (Fig. 119); this device was "re-invented" independently by the author a few years later, and exhibited to the Birmingham Natural History and Philosophical Society, in 1894; the author's model is represented in Fig. 120. The coincidence is most remarkable, the author's model has one less "wave" than that of M. Bazin, but the two are otherwise almost identical.[3] In 1893, Langley published his investigations on "The Internal Work of the Wind."[4] Although (apart from the meteorological portion of this work) Langley had no new fact or theory to present, the *rationale* of dynamic soaring is described by him in a particularly lucid and simple way; the publication is one well worth consulting, as presenting the subject in an elementary form divested of all mathematical complication.

§ 153. Dynamic Soaring. Theory of the "Switch-back" Model.— The switch-back model is intended to represent the type of dynamic soaring in which the flight path lies in a vertical plane, and in which the bird in flight encounters masses of air moving with different velocities. From the Phugoid Theory we may

[1] *Nature*, Vol. XXVII., p. 590.

[2] Marey, "Le Vol des Oiseaux," § 192.

[3] Bazin's "montagnes russes" model was not known to the author till many years after his paper to the Birmingham Natural History and Philosophical Society.

[4] "Smithsonian Contributions to Knowledge," *l.c. ante.*

evidently regard the soaring bird or aerodone as if it were travelling on a "track" in space whose form may be any one of the series of curves that comply with the equation, or in practice the metamorphosis of such curves as due to the added conditions of resistance and rotational inertia. Owing to the fact that a bird in flight may alter its "constants" at will it is further evident that we are not confined to curves of the phugoid series; in fact the bird can, within certain limits, carve out any curve it likes, without departing from the analogy. We may regard the resistance to flight as closely analogous to a frictional resistance in which the gliding angle γ represents the angle of friction, so that the conditions would be approximately represented if we suppose the ball, instead of rolling, to slide along the track.[1] If the analogy is to hold it is evident that the condition must be imposed that the velocity of motion must at no time fall below a reasonable limit.[2]

Now when a bird is gliding through the air in soaring flight, it has the power of adapting the form of its flight path to the changes in the wind velocity; but we have no means of directly representing this power of adjustment in the model. Instead of adjusting the flight path to the pulsations it is evident that for the purpose of demonstration we may do the converse and adjust the pulses to a prearranged flight path, so in the "switch-back" model the ball is arranged to run on a rigid track to which an oscillatory motion is imparted in a longitudinal direction. Thus in Fig. 120, if the ball be placed at the lower end of the "switch-back" C and set in motion, it may be made *by a properly applied series of pushes and pulls* to run up hill to the point D. The correct way to apply the pulses to the apparatus is to impart a motion counter to the "direction of flight" when the ball is

[1] In this case, of course, there need be no actual lifting of the ball (or sliding block); the mountains of the "switchback" may be made of uniform height.

[2] If the velocity of a bird or aerodrome becomes too low in relation to that of least resistance, a disproportionate increase of γ takes place (compare Vol. I., §§ 164, 176).

ascending, and in the same direction when it is descending. This means that the maximum applied *push* takes place when the ball is "troughing," and the maximum pull when it is "cresting."[1]

In reality, of course, the wind does not move *en masse* as is the case with the "switch-back" track in the model; but as the bird is not concerned with the motions of the wind at distant points this fact has no influence. In the model the motion of the ball is practically frictionless, the energy imparted to it being exhibited by its increase of altitude, or, if a sliding block be substituted for the ball, whose coefficient of friction represents

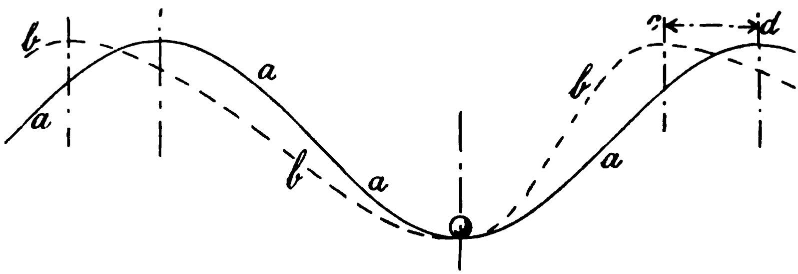


FIG. 121.

the gliding angle, the energy is expended in overcoming the frictional resistance.

Now the form of the "switch-back" track gives the form of the flight path *through the air*; that is to say from point to point relatively to the air in which it is supposed the bird is flying. Let this be represented in Fig. 121 by the curve *a a a a*. It is evident that the motion relatively to a point fixed in space will be somewhat of the form given by the line *b b b b*, for, during the passage of the ball (representing the bird in flight), from crest to crest the track makes the reciprocation *c* to *d* and back again. If, now, a piece of paper be cut to the shape of the track (the

[1] The push or pull represents acceleration, positive or negative, of the apparatus.

curve *a a a a*) and the motion of the ball be followed out by sliding the supposed track to and fro through the range *c d*, representative positions being given in Figs. 122 and 123, it will be seen that the normal to the track, which in the case of the

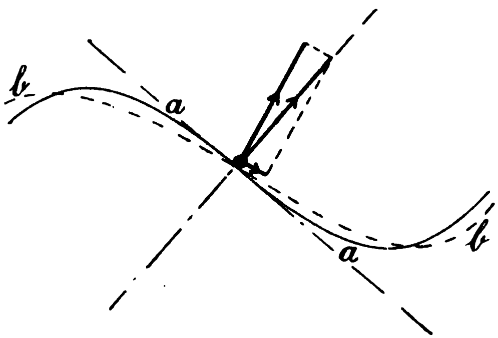


Fig. 122.

ball represents the track reaction, has a propulsive component on the real flight path *b b*, so that were the air to offer no resistance to flight the velocity of the bird would receive a substantial increase during each descending and each ascending

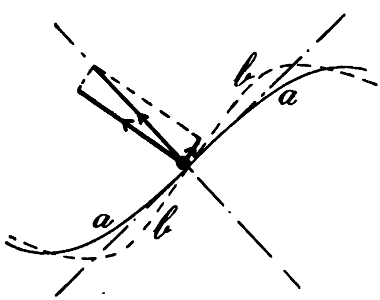


Fig. 123.

period. We consequently have a means of propulsion that may be in actuality employed to overcome the flight resistance, and, if sufficient, to enable the bird to increase its altitude in addition.

If we substitute for the normal reaction of Fig. 122-3 a

reaction inclined to the direction of the normal, Fig. 124, as representing the resultant of the lifting and retarding components of

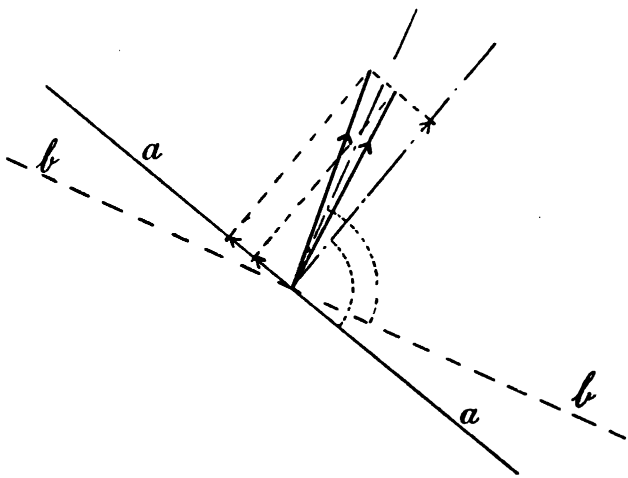


FIG. 124.

the flight reaction, the condition of no energy loss, that is, the condition of soaring at any point, is that this resultant shall be

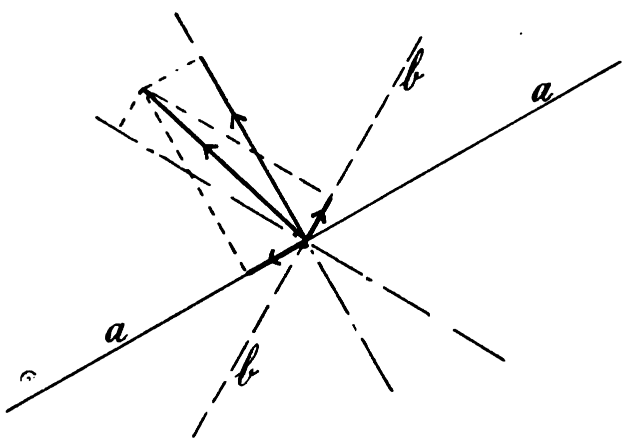


FIG 125.

normal to the curve *b b*. If it is in advance of the normal the bird is receiving energy faster than it is being expended; if

it lies behind the normal the supply of energy is insufficient. These two conditions are indicated in the figure.

It is evident that since the propulsive component falls to zero twice during every phase, while the resistance remains much as before, in order that soaring should be possible for the whole flight path, the propulsive component must be considerably in excess at the intermediate points, as illustrated in Fig. 125.

The foregoing theory is the basis of the following quantitative investigation.

§ 154. Theory of Dynamic Soaring. Quantitative Treatment.— Let A B, Fig. 126, represent a small element of the flight path

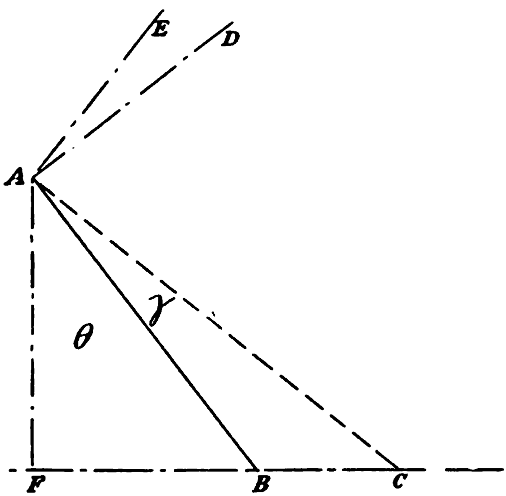


Fig. 126.

relatively to the air in which the bird is travelling, the direction of flight is down-hill (following the lettering). Let A C represent the same flight path element relatively to the "mean wind." Thus in the model the path A B represents a portion of the switch-back track, and A C the actual path of the ball relatively to a point fixed in space.

Let A D be normal to A B, and let A E be the direction of the

aerial reaction on the bird or aerodone; that is the resultant of the normal reaction and the resistance. Then the condition of "just soaring" for the element of the flight path under considéra-tion is that $A\ E$ is normal to $A\ C$, that is to say, the angle $B\ A\ C = \gamma$.

Now draw $A\ F$ perpendicular to the horizontal line $C\ F$, then $F\ C$ is the horizontal distance covered by the element of the flight path under consideration,[1] and $B\ C$ is the distance moved by the wind pulse in the same time. We require to determine the conditions under which the motion of wind fluctuation is least for a given horizontal component of flight path. This will

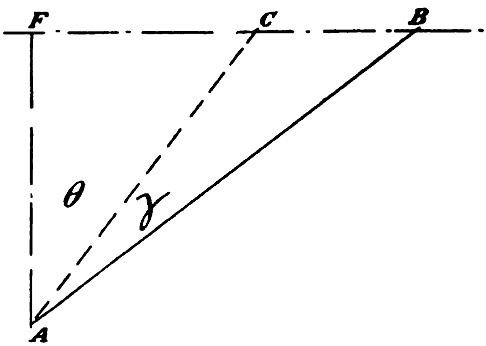


FIG. 127.

be defined by the relative magnitudes of $B\ C$ and $F\ C$; given γ we have to solve $\frac{B\ C}{F\ C} =$ minimum.

This may immediately be recognised as the problem of the efficiency of an element of the blade of a screw propeller, the solution of which we already know,[2] if as in Vol. I, § 204, we represent the angle $F\ A\ B$ by the symbol θ the solution is

$$\theta = \frac{90^\circ - \gamma^\circ}{2}$$

or the mean of the angles θ and $\theta + \gamma$ is 45°.

[1] Relatively to the mean wind.

[2] "Aerial Flight," Vol. I., *Aerodynamics*, §§ 202 *et seq.*

Again, let Fig. 127 represent an ascending element of the curve in which $A\ B$ as before is the flight path relatively to the air in which the bird is soaring at the time being, and $A\ C$ is the same path plotted relatively to the "mean wind" (relatively to a fixed point in the switch-back model); the condition of "just soaring" is as before when $B\ A\ C = \gamma$. Here $F\ C$ is the horizontal component of the true flight path, and the required condition is that $\frac{B\ C}{F\ C}$ is minimum. This condition may be otherwise expressed, $\frac{B\ C}{F\ C} + 1 =$ minimum, or $\frac{F\ C}{F\ B} =$ maximum, which again is

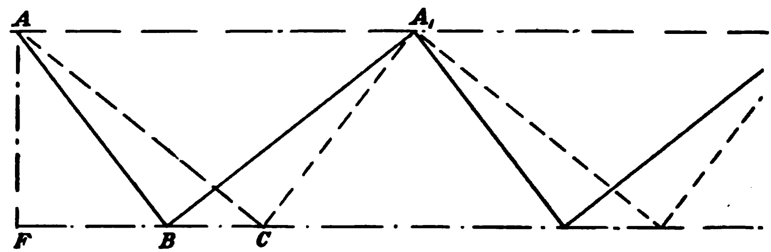


Fig. 128.

identical with the problem of maximum efficiency of the screw propeller.

Let us now imagine a flight path composed of alternate elements, as in Figs. 126 and 127, to form a zigzag, Fig. 128. There are difficulties in such a supposition, but these can for the moment be ignored. We must suppose that the change of direction at each angle of the zigzag path is effected without loss of energy by some external agency. Let us further assume that the individual elements are so small that the velocity of flight does not undergo changes of sensible magnitude. Then, whilst the bird (or aerodone) travels from A to A_1 (Fig. 128), the wind pulse motion is from C to B and back again; hence the velocity of the wind fluctuation in terms of the mean velocity of flight is $2\frac{B\ C}{A\ A_1}$. But the efficiency E in the sense used in the analogous

propeller theory is $\frac{F\,B}{F'\,C}$, so that $2\,\frac{B\,C}{A\,A_1}$ may be expressed in the form

$$2 \times \frac{1 - E}{1 + E}$$

the value of E being calculated from γ as in Vol. I., § 204. Alternatively the diagram may be drawn to scale for any stated value of γ and the ratio thus obtained graphically.

The results are given in Table I. for stated values of γ, both as to efficiencies and corresponding values of wind fluctuation in terms of flight velocity, as given by the above expression.

It is evident that the present case corresponds to the uniform motion conditions of § 151, conditions that are there dealt with

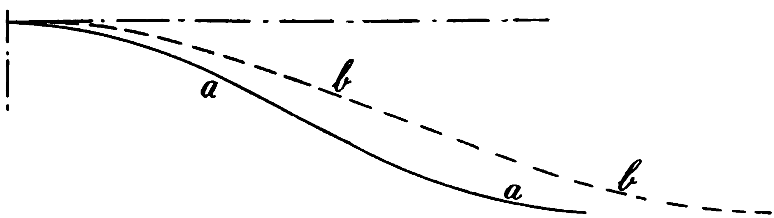


FIG. 129.

on a purely dynamical basis. We shall institute a comparison between the two aspects of the subject when we have considered one or two further examples of a less artificial kind.

§ 155. Theory of Dynamic Soaring. Quantitative Investigation (continued).—The zigzag form of flight path studied in the preceding section enables the theory to be initially presented in a very simple form, but the conception is essentially artificial. In practice such a form of flight path is not possible; the zigzag must be replaced by a path of undulating form, and the theory must be extended to deal with the modified conditions.

We will now suppose that the flight path is a curve of sines; that is to say, it is composed of a vertical harmonic motion superposed on a horizontal uniform translation. We will for the time

being adhere to the supposition that the wind pulses consist of simple alternations in the line of mean flight of uniform motion, and that the vertical amplitude of the flight path is of sufficiently small magnitude to permit us to regard the flight velocity as sensibly uniform.[1]

We will now, as in the preceding section, draw the flight path relatively to the air, and relatively to the mean wind. Thus in Fig. 129, the curve *a a* may be regarded as representing the contour of the switch-back railway, and the curve *b b* the path of flight relatively to a fixed point. Now these two curves (under the supposed conditions) differ only in respect of their

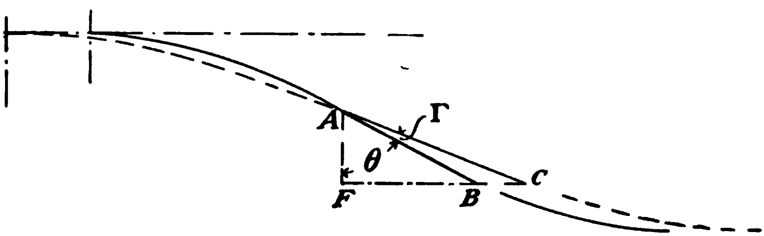


FIG. 130.

horizontal scale; that is to say, they are both sine curves. Let us suppose that the one curve is made to slide horizontally over the other, as in § 153, the intersection representing the passage of the bird in flight. Then if we examine the two relative paths at their intersection, owing to both curves being sine curves, their angles will be always related so that $\cot \theta / \cot (\theta + \Gamma)$ will be everywhere constant, the sign Γ being employed to denote the difference between the angles, now no longer equal to γ. That is to say, that if Fig. 130 represents a small element of the flight path, the relation $\frac{F\,B}{F\,C}$ is constant, or the efficiency is the same at all points.

But the propulsive force of which Γ is the measure now varies

[1] Relatively to the mean wind.

from point to point, being zero at the top and bottom points of the curves and maximum at the points of greatest inclination. And the condition essential to soaring is that the mean value of Γ over the whole curve is equal to γ, hence from the known properties of the sine curve, if Γ_1 is the maximum value of Γ,

$$\Gamma_1 = \frac{\pi}{2}\gamma.$$

The highest possible efficiency, therefore, that is to say the least velocity of fluctuation, is given by employing $\frac{\pi}{2}$ times the actual γ value, proper to the bird, as the γ of Table I.

TABLE I.[1]

γ°	E	$\frac{v}{V}$
2°	·932	·070
3°	·900	·105
4°	·869	·140
5°	·839	·175
6°	·811	·210
7°	·782	·245
8°	·756	·280
9°	·730	·315
10°	·704	·350
11°	·680	·385
12°	·658	·420
13°	·633	·455
14°	·611	·490
15°	·589	·520
16°	·567	·550

[1] The value of $\frac{v}{V}$ given in the table is calculated as stated in the text. It is, however, worthy of remark that the single expression, $\frac{v}{V} = \cdot 035\,\gamma^\circ$ gives the same result for all ordinary values of γ°, or if γ be expressed in circular measure (radians), $\frac{v}{V} = 2\,\gamma$.

§ 156. Theory of Dynamic Soaring. Harmonic Wind Pulsation.— When we introduce the further condition that the motion of the wind pulse shall be harmonic the problem becomes still more complex. This condition is justified as a theoretical basis for investigation on the general ground that every periodic motion may be regarded as a series of superposed harmonic components, and in practice the fundamental component is commonly the most important.

The supposition that the horizontal component of the flight path is uniform will be adhered to, and the form of the flight path will, as in the preceding section, be taken as a sine curve.

It will also be taken as part of the hypothesis of the present investigation that the amplitude of the flight path is small.

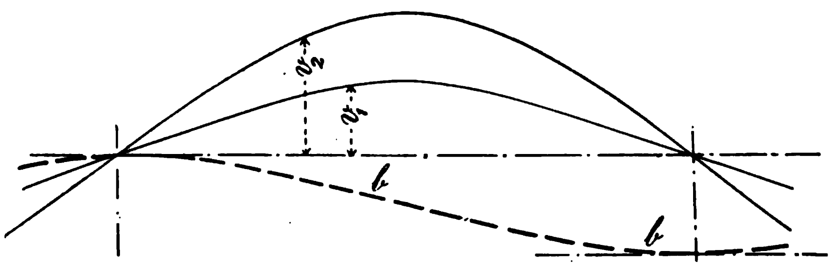


Fig. 131.

Let V be the constant velocity of flight.

„ v_1 = vertical velocity component of the flight path.

„ v_2 = velocity of the wind pulse.

From the conditions v_1 and v_2 are both variables whose change is harmonic, and whose phase is the same.

Let the curve b b, Fig. 131, represent the sine curve flight path; then the values of the variables v_1 and v_2 may be represented by two other sine curves one quarter phase out of step with the flight path curve, in the manner shown.

Let Fig. 132 represent a diagram of velocities at any instant. The aerodone situated at A is travelling relatively to the mean wind in the direction A C with velocity V; B C = v_2 represents the velocity and direction of the wind pulse; consequently A B

is the flight path and velocity relatively to the air in which the aerodone is travelling.

The angle Γ ($= C\,A\,B$) is that by which the propulsion is effected, and the condition of soaring is, as before, that the mean value of Γ is equal to or greater than γ.

We require to find an expression for Γ in terms of v_1 and v_2 whose manner of variation is known.

It would evidently be possible to lay off a series of values of Γ as a curve, from a number of measurements made from a geometrical construction, and to integrate such curve by means of a planimeter, the procedure here suggested being to construct a series of diagrams such as Fig. 132, assigning values to v_1 and

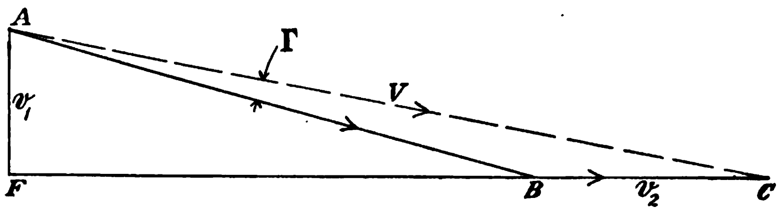


FIG. 132.

v_2 in accordance with the curves of Fig. 131, and then obtain the Γ values by measurement.

If in the present investigation we confine ourselves to the special case where the amplitude and gliding angle (γ) are both small, we have v_1 and v_2 both small compared to V, and the simple relation $\Gamma \propto v_1\,v_2$ obtains. But v_1 and v_2 vary in like ratio, hence,

$$\Gamma \propto v_1^2.$$

Now, since the curve of v_1 is a sine curve, the curve of v_1^2 is a sine curve of half the phase length,[1] Fig. 133, and the curve of Γ is likewise of this form; consequently the mean value of Γ is equal to half its maximum value. Thus, if we write Γ_1 for the maximum value of Γ we may express the conditions of just soaring,

$$\Gamma_1 = 2\,\gamma.$$

[1] A well-known result.

The solutions that we have effected in this and the preceding sections can only be considered as approximations, the assumptions that have been made are not, strictly speaking, justifiable. The assumption of uniform horizontal velocity holds good only where the amplitude is small, both in relation to the phase length of the flight path and absolutely compared to the value of H_u. If the relative amplitude is great so that the flight path curve is steep, the assumed condition breaks down, owing to the fact that a uniform horizontal velocity betokens an increased velocity on the steep portions of the curve; this is an error that can be condoned if we suppose that the form of the wind pulse motion is modified to suit, that is to say, it is not strictly uniform or accurately harmonic as the case may be. If the amplitude is considerable in relation to H_u, then the velocity will be perceptibly greater

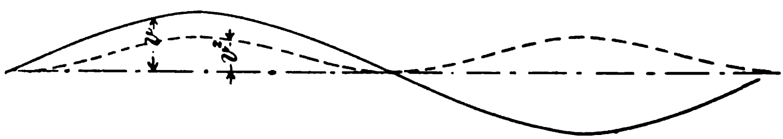


FIG. 133.

when "troughing" than when "cresting," just as it is in the phugoid curve.

In spite of defects of hypothesis, the author believes that the magnitude of the errors introduced is not serious, and that the results stated may be looked upon as satisfactory approximations to the truth.

§ 157. The Form of the Orbit in Dynamic Soaring.—It not infrequently happens that a bird may be seen soaring against the wind, making but little or no headway relatively to the earth. Under these circumstances the form of the flight path appears as an orbit, commonly of oval form, approximately elliptical.

The same flight path, if observed under different conditions, will appear as an undulating curve of a form closely resembling the phugoid curves of Figs. 37, 40 and 42; the difference between

the undulating path and the orbit is merely one of relative motion.

In considering the flight path as an orbit it is only necessary to suppose that a plotting be made on a co-ordinate chart travelling in the mean direction, and with the mean velocity of the aerodone in flight. Thus, in the rudimentary case considered in § 154, if we suppose such a plotting to be effected, it is evident that the "orbit" will be constituted by an inclined straight line drawn to bisect the angle made by the component elements of the saw-tooth flight path, that is, the angle $A\ C\ A_1$ of Fig. 134: for, since the velocity is taken as constant the time taken for the aerodone to travel from A to C, and from C to A will be equal

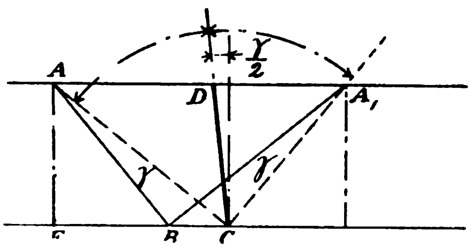


FIG. 134.

to the time taken for the co-ordinate chart to travel from A to D, and from D to A, part for part, this relationship being ensured by the construction given. It may be observed that by this construction the "set-back" angle of the straight line orbit path is equal to $\frac{\gamma}{2}$.

The condition $V =$ constant, assumed in §§ 154 *et seq.*, can only apply when the size of the individual path undulations is vanishingly small. In practice, in considering the form of the orbit we cannot afford to ignore the variations of horizontal velocity which are due to the interchange of kinetic and potential energy.

If the motions of the aerodone or bird in its flight path were the natural oscillations of the Phugoid Theory, we know that the

variations in the horizontal velocity would result in an orbit whose horizontal amplitude is $\frac{1}{\sqrt{2}}$ of its vertical amplitude. If we suppose that the oscillations of the soaring path *are* phugoid oscillations, then we must evidently consider the horizontal component of the phugoid oscillation superposed on the inclined line of Fig. 134. This is shown in Fig. 136, although owing to the artificial character of the initial assumption, *i.e.*, the saw tooth flight path, the figure can only be regarded as diagrammatic.

It is evident, at least in the case of a bird, that the flight path oscillations may be other than those of phugoid theory; in fact, by variations in the adjustment of the organs of flight, a bird

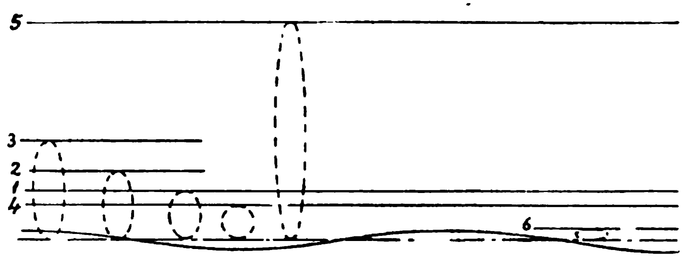


FIG. 135.

may for a given velocity (of flight) cause the undulations to be of greater or less length than that resulting from the phugoid equation.

Now the proportions of the orbit, on the approximate basis of §§ 29 and 30, are related to the values of H_n and the phase length. It may be readily demonstrated on the lines of § 30 that, on the assumption of an elliptical orbit, the ratio of the vertical to the horizontal axis is given by the simple expression,

$$\frac{\text{vertical axis}}{\text{horizontal axis}} = \frac{4\pi H_n}{L}$$

where L is the phase length. This is represented in Fig. 135 by a number of examples, giving the *proportions* of the elliptical orbits in several different cases. In setting out this

diagram a fixed length is taken as representing the value of L as indicated by the undulating line, and different values of H_n are defined by the numbered datum lines; the vertical axis of the ellipse has been drawn in each case as equal to H_n the horizontal axis being a constant quantity equal to $\frac{L}{4\pi}$ in accordance with

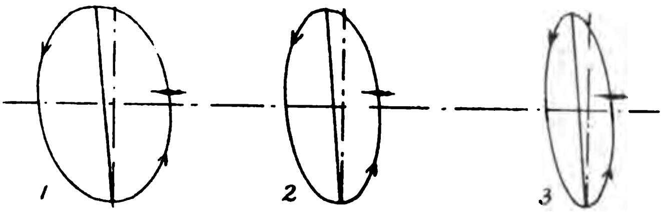


FIG. 136.

the equation. Obviously no attempt has been made to show the orbits of a proportionate or suitable size. The construction given enables one to readily visualise the type of orbit proper to any given L/H_n ratio, without actually plotting the ellipse.

Referring to Fig. 135, it will be seen that datum line No. 1 gives the orbit proper to the phugoid flight path, of ratio $\sqrt{2} : 1$.

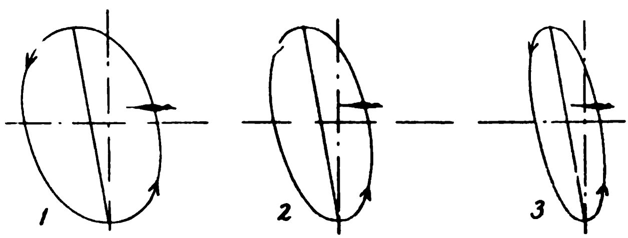


FIG. 137.

Datum No. 2 gives an orbit whose ratio is 2 : 1; in No. 3 the ratio is 3 : 1; No. 4 gives an ordinary trochoid; here the axes are equal and the orbit is consequently of circular form; similarly in the other examples.

It has already been remarked that a bird or machine with adjustable parts may have the power of varying the proportions

of its orbit at will, by carving out a curve of longer or shorter phase length for a given value of H_n, and the soaring orbit may not be by any means of constant form. Thus in Fig. 136, three examples are given in which (1) represents the orbit of the phugoid path, and Nos. (2) and (3) represent cases where the phase length is artificially shortened so that the ratio of the axes becomes 2 : 1 and 3 : 1 respectively.

The present method of delineating the orbit becomes more appropriate and exact when we deal with the sinuous form of

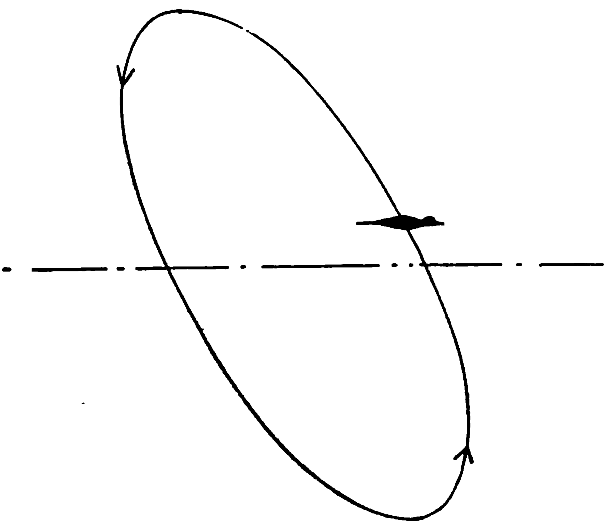


FIG. 138.

flight path discussed in §§ 155 and 156. If under the conditions of § 156 we examine the problem on the assumption of uniform velocity of flight, that is, on the supposition that the phase length is very small in relation to H_n, we find that the form of orbit will be a line having a general backward inclination, the maximum set-back angle being twice as great as in the case of the saw-tooth path, that is to say, it will be equal to the gliding angle γ, but the angle will diminish towards the upper and lower points, where it will terminate vertically. As a rough approximation

we may represent such a path by a straight line having the same mean set-back, and then add the superposed horizontal component as in the previous example; we may then draw the resulting orbit forms given in Fig. 137, which otherwise correspond to the similarly numbered diagrams in Fig. 136.

It is of interest to compare the orbit thus theoretically deduced with the form as known to observation. Fig. 138 represents the orbit as given by Basté;[1] this diagram is in close agreement with the theoretical result, the only difference appears to be that the set-back or slope of the major axis appears to be greater than we have deduced. This may be due to the gliding angle of the bird on which Basté made his observations being greater than

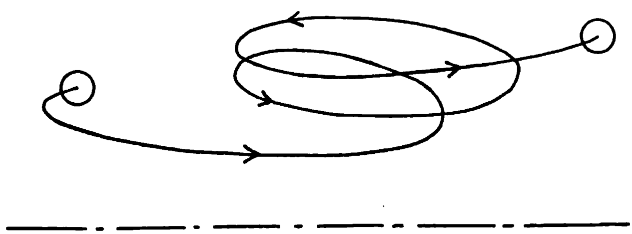


FIG. 139.

has been supposed in setting out Fig. 137, or the difference is one that might easily be due to the observer (Basté) having recorded an exaggerated impression of the feature in question. It is also possible that on the occasion or occasions when Basté made his observations the wind velocity was greater than the ordinary flight velocity of the bird in question, but that associated with this wind was an ample supply of fluctuation energy. Under these circumstances the bird would evidently be induced to employ a velocity of flight in excess of the ordinary which would account for the extravagant gliding angle betokened by the set-back angle recorded.

The major and minor axes of the ellipse or oval as observed by Basté, appear roughly to be ratio of 2 : 1, which corresponds

[1] Le Vol des Oiseaux; Marey, § 15.

to a phase length less than that of the phugoid path the proportion of $\sqrt{2} : 1$; this is given in diagram No. 2 of Figs. 136 and 137.

It is possible that the difference in the proportions of the observed ellipse and that of the phugoid equation is due to wing flexure, and not upon any conscious action of the bird in flight.[1]

A photographic record has been made by Marey[2] of the motion of the ball in Bazin's "switch-back" model; a diagram of this is given in Fig. 139. It would appear to denote that the velocity of the ball in relation to the length of the path undulations was too slow when this record was made to properly represent the actual conditions of soaring. The orbital path as recorded corresponds approximately to orbit No. 6 in Fig. 135.

§ 158. Dynamic Soaring in its Relation to the Phugoid Flight Path.—Let us examine the elementary case of §§ 154—5, in the light of the Phugoid Theory; that is to say, let us suppose that the wind pulse consists of an alternation of gusts of uniform velocity.

We will assume that the period of the wind alternation corresponds to the period of the phugoid curve, and that the two are arranged in respect of phase in accordance with the requirements of soaring theory.

The present point of view is of interest as bearing on the possibility of *automatic soaring*. The author has on several occasions, when experimenting with mica models in suitable weather, witnessed true soaring evolutions, models having stayed in the air and risen to a considerable altitude, the motions being undistinguishable from those of a living bird.

Let us suppose, in Fig. 140, that an aerodone when gliding uniformly, encounters at the point B an adverse gust, so that the value of H is increased, and let the increase be represented by a transference of the datum line from x_1 to x_2, then the flight

[1] Compare § 116.

[2] Le Vol des Oiseaux, § 192.

path passes from a straight line p_1 to a phugoid curve p_2. We now arrive at the point C one half phase further along, and the wind reverts to its former velocity, so that the datum line descends. Now if movements of the datum were proportional to changes in the wind velocity, the datum would return to its original position, and nothing would have been gained, but such is not the case. The movements of the datum are due to changes in the value of H consequent upon the changes of velocity, that

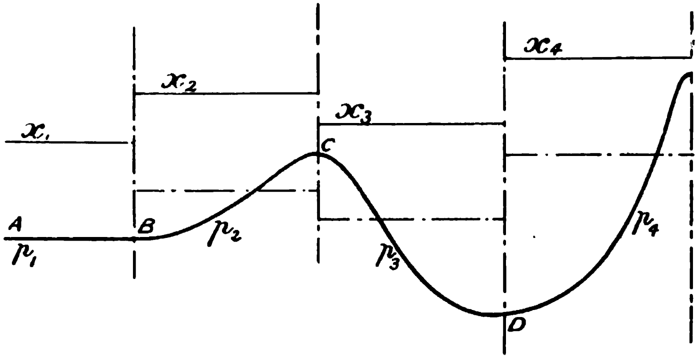


Fig. 140.

is to say, consequent upon the change of the velocity of the aerodone *relatively to the air.*

Now $2\,g\,H = V^2$. $\therefore \quad \frac{d\,H}{d\,V} = \frac{V}{g}$

and approximately for small finite increments,

$$\frac{\Delta\,H}{\Delta\,V} = \frac{V}{g},$$

we may take $\Delta\,V$ to represent the change in velocity that takes place periodically, and $\Delta\,H$ the corresponding variation in H, and we see that the value of the latter is proportional to the velocity for the time being, so that in the changes in the datum of Fig. 140 the rise will always be greater than the fall. The same fact may be rendered self-evident from the graphic construction given in Fig. 141, in which distances along the line

$O\ x$ represent velocities and the squares on these distances H values. Let the velocity V undergo an increase from x_1 to x_2 $= \Delta V$, then the change in H and rise in the datum is represented by the shaded area. Now when the aerodone is "cresting" let V undergo a decrease of the same value, from x_3 to x_4, then the decrease of H and therefore fall of the datum is represented by the corresponding cross hatched area, the net gain in altitude is the difference of the two.

Thus, referring again to Fig. 140, the fall from x_2 to x_3 is less than the rise from x_1 to x_2, and so at every oscillation there is a

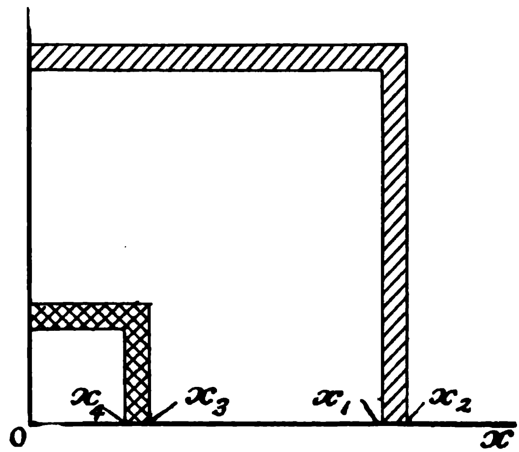


FIG. 141.

gain of altitude. If we take account of the fact that the aerodone experiences resistance to flight, we can exchange this gain of altitude for the necessary propulsive force; in other words, the energy gained from the wind instead of being stored as potential is expended in flight.

So far the result is precisely that which might be anticipated from the previous investigation, but we have now to encounter a difficulty of a serious kind. It will be noticed on reference to Fig. 140, that not only does the datum line (which is a measure of altitude) rise as the net result of the wind pulses acting on

the aerodone in flight, but also the amplitude of the curve increases at every step, consequently if nothing is done to damp down the phugoid oscillation, the aerodone will sooner or later find itself in the danger zone and instability will result.

Now we know that the resistance to flight is a potent factor in the damping of the phugoid oscillation, and since the increase of amplitude is caused by the same agency as that by which the resistance is overcome, there would seem at first sight to be some chance of the damping rate automatically varying in the required degree; it is questionable, however, if any such conclusion would be justified either from theoretical considerations or from observed fact. The author has endeavoured to formulate a *régime* under which the phugoid oscillation may be used as the basis of soaring flight *without an increase of amplitude*, but the difficulties have so far proved too great. In fact if the author had not himself witnessed the phenomenon of a simple aerodone soaring dynamically under conditions that appear to leave no room for doubt, he would certainly have been incredulous.[1]

Let us examine the experimental conditions. The model employed was one of about five grams weight (Figs. 56 and 57), with a comparatively low damping capacity (about *c* or *d*, Fig. 109). The natural velocity was 14 to 15 ft./sec. and the gliding angle approximately one in five, *i.e.*, $\gamma^\circ = 11^\circ$; the velocity of descent

[1] In the summer of 1905, when exhibiting model A 2 (Figs. 56, 57) to Mr. C. W. Dixon, at his residence ("Westbourne," Edgbaston), the author observed on several occasions that the flights were abnormally prolonged. On one occasion the model, launched from about 7 ft. above the level of the lawn, soared for some considerable period, rising at times to a height of about 20 ft., eventually alighting in a tree 13 ft. above the ground level. The model had already been in the air a long while before the time was taken; the actual period recorded was 11 seconds. Since the gliding path, in the ordinary way, occupied about three seconds, the probability is that the total time of flight exceeded one quarter of a minute.

There was a light wind with fitful gusts at the time of the observation in question, and the path of the aerodone was of a switch-back type, giving it every appearance of being a living bird.

There were present, besides the author, Miss Dixon and Mr. C. W. Dixon.

was therefore about 3 ft./sec. In order to soar this model seemed to require a light gusty breeze, but under the best of conditions anything in the way of a prolonged flight such as that recorded was a matter of so rare occurrence as to suggest that the fitting in of the wind pulses and the phugoid oscillations may have been a matter of pure chance. Even if such be admitted, and if subsequent experiment show that the occurrence of abnormal flights is a matter that can be explained by the doctrine of chances, the whole difficulty is not removed, for although the flight path consisted visibly of a series of undulations of considerable amplitude, much as in curve 5 of the phugoid chart, the aerodone never once lost its equilibrium or performed "tumbler" evolutions.

Only two possible explanations of this difficulty suggest themselves. Either the case was not one of dynamic soaring; or, the damping action increases very greatly with the amplitude of the path, so that although the oscillation damps very slowly for small amplitude, when the amplitude is great the damping is sufficiently rapid to prevent the flight path from entering the danger zone.

The first explanation, although possible, seems in every detail contrary to the impression of those present, the first few cycles of the motion were *visibly* dynamic soaring; the alternative explanation is one which requires proof, either experimental or theoretical, and is somewhat opposed to other of the author's observations. The difficulty is one that must be left for future investigation.

§ 159. The Relation between the Energy of Turbulence and the Velocity of Flight.—In § 151 it was shown that in order that there should be a sufficient store of energy in the wind for the flight of a given soaring bird, having a known gliding angle, the mean velocity of turbulence must reach a certain minimum value, this value depending upon the nature of the assumption as to the *form* of the disturbance to which the fluctuation is due. There

is no condition as to the flight velocity or the relation of the flight velocity to that of the aerial disturbance.

In the method of the preceding sections (§§ 154 *et seq.*), in which the manner of the employment of the turbulence energy is taken into account, we find as a result that the velocity of flight and that of turbulence are related; in other words, if soaring is to be possible the velocity of the wind fluctuation must exceed a certain minimum value in proportion to the velocity of flight.

Now these two results do not on the face of it appear to be in harmony, and it might readily be supposed that they represent two separate conditions that require to be fulfilled. Such is not the case: a careful examination of the hypothesis in both cases shows that in reality there is no want of agreement; the two conditions represent two aspects of the same problem and they are entirely in accord.

In the investigation of § 151 the basis is that whatever the velocity of the bird or aerodone may be, the value of γ is known. This is a convenient assumption, for γ is a quantity whose value is always known within fairly close limits, and which varies but little with the velocity of flight, and which is not very different in birds of different species. The investigation proceeds on the basis of the known data *of the aerofoil*, and if changes of velocity are contemplated the above assumptions involve, firstly, that the aerofoil undergoes no change either as to its sweep or its periptery; and secondly, that in the face of changes of velocity the conditions of least gliding angle are complied with; that is to say, the *form* of the aerofoil remains unchanged, but the weight supported increases as the square of the velocity.[1]

Now if we modify the above conditions to comply with the hypothesis of the switch-back method of investigation, we have firstly to suppose that the *area* be made to vary with variations of velocity, instead of the weight being variable, and in order that the pressure-velocity relationship of Vol. I., Table IX.,

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 159; also Chapter VIII.

shall be complied with the *area must vary inversely as the velocity squared*. But the sweep and peripteral area vary with the aerofoil area (the proportions remaining the same), so that we shall have the sweep and the peripteral area varying inversely as the square of the velocity of flight. And if, in accordance with the conditions, the gliding angle remains unchanged, in order that soaring should be possible, the turbulence energy of the wind per unit distance must also remain unchanged. Hence the square of the velocity of the wind fluctuation must increase in the proportion that the peripteral area and sweep are diminished. Therefore the velocity of the wind fluctuation must vary directly as the velocity of flight. *Thus the total turbulence energy of the air handled, and the energy available according to the switch-back theory, for an aerodone or bird of given weight both depend upon the existence of a definite relationship between the velocity of wind fluctuation and the velocity of flight.*

§ 160. The Efficiency of Dynamic Soaring. Energy Available in Terms of Total Energy of Turbulence.—We have now investigated the question of dynamic soaring from two different points of view; firstly in § 151 we have dealt with the quantity of energy existing in the wind in a possibly available form; and, secondly, we have in §§ 154 *et seq.*, on certain simplified hypotheses, found a means of expressing the quantity of energy that is usefully employed. We now require to find an expression for the latter in terms of the former.

Owing to the widely different lines of argument adopted in the two investigations, the results are not immediately comparable, and further treatment is necessary. On the zigzag basis, § 154:—

Let $\mathbf{E}$ = internal energy of wind per unit distance, *i.e.*, per foot of flight path.

„ A = area of aerofoil.

„ A_1 = area of stratum dealt with by aerofoil.

„ v = a uniform velocity of wind fluctuation.

„ V = velocity of flight along the zigzag flight path.

Let W_1 = normal component of weight sustained (poundals).
,, γ_1 = theoretical least gliding angle, as in Vol. I., Table VI.
,, γ = actual (experimental) gliding angle.
,, ρ = density of air.
,, κ and ϵ be aerodynamic constants as in Vol. I., § 181, Table III.

Now

$$\mathbf{E} = \frac{\rho A_1 v^2}{2}, \tag{1}$$

and on the basis that the whole energy of the peripteral area is available,

$$A_1 = \kappa \frac{1 + \epsilon}{1 - \epsilon} A$$

∴ by (1)

$$\mathbf{E} = \frac{\rho \kappa A (1 + \epsilon) v^2}{2 (1 - \epsilon)}. \tag{2}$$

Now by Vol. I., § 185

$$\frac{W_1}{A V^2} = \rho \kappa \beta (1 + \epsilon)$$

where by § 181 $\gamma_1 = (1 - \epsilon) \beta,$

or $\beta = \dfrac{\gamma_1}{1 - \epsilon}$

∴

$$\frac{W_1}{A V^2} = \rho \kappa \gamma_1 \frac{1 + \epsilon}{1 - \epsilon}$$

or

$$A = \frac{W_1 (1 - \epsilon)}{\rho \kappa \gamma_1 (1 + \epsilon) V^2}$$

∴ by (2)

$$\mathbf{E} = \frac{W_1 \rho \kappa (1 + \epsilon) (1 - \epsilon) v^2}{2 \rho \kappa \gamma_1 (1 + \epsilon) (1 - \epsilon) V^2}$$

$$= \frac{W_1 v^2}{2 \gamma_1 V^2}.$$

But the real γ is greater than the theoretical γ ($= \gamma_1$). Let us write $\gamma = n \gamma_1$ where n is a constant, or $\gamma_1 = \frac{\gamma}{n}$.

And we will arrange the expression as representing energy per foot per poundal sustained, thus,

$$\frac{\mathbf{E}}{W_1} = \frac{n\,v^2}{2\,\gamma\,V^2}$$

But of this the energy *usefully employed* is that which would propel one poundal through one foot in the line of flight, that is $= \gamma$. Hence the efficiency, *i.e.*, the energy usefully employed in terms of that in the air handled is,

$$\gamma \div \frac{n\,v^2}{2\,\gamma\,V^2}$$
$$= \frac{2}{n}\left(\frac{\gamma\,V}{v}\right)^2.$$

And for the conditions of best efficiency we know that for all useful values of γ, the quantity $\frac{\gamma\,V}{v}$ is approximately constant $= \cdot 5$ (§ 155, footnote to Table I.), or $\left(\frac{\gamma\,V}{v}\right)^2 = \cdot 25$.

Now the best value of γ obtainable in practice is commonly from $1{\cdot}5\,\gamma_1$ to $2\,\gamma_1$; that is to say, $n = 1{\cdot}5$ to 2, so that in general the soaring efficiency[1] will be from about ·33 to ·25.

[1] If the theoretical value of γ were actually realisable the soaring efficiency on the basis of the preceding section would become ·5, the bird thus employing usefully one half of the turbulence energy contained in the wind coming within its grasp. The reason of this limitation is not altogether clear.

It is of interest to note that, neglecting body resistance, the energy utilised in soaring flight in covering a given horizontal distance is, *under the conditions of least resistance*, equal to that expended in gliding uniformly over the same distance. This result is unexpected, but it arises from the fact that the reaction normal to the soaring flight path W_1 is less than the weight W.

Let E_1 and E = energy required to cover one horizontal unit distance soaring and gliding respectively. Let θ be the angle of the flight path to the horizontal, as in Chaps. II. and III.; and let the aerofoil area be supposed correctly proportioned for least resistance, so that the angle γ will have the same value in each case; then

$$E = \gamma\,W \qquad (1)$$

$$E_1 = \gamma\,\frac{W_1}{\cos\theta} \qquad (2)$$

But by resolution of forces $\frac{W_1}{W} = \cos\theta$, or $W_1 = W\cos\theta$, substituting in (2) $E_1 = \gamma\,\frac{W\cos\theta}{\cos\theta} = \gamma W = E$.

§ 161. Efficiency of Dynamic Soaring (continued). — The theoretical computation of least γ values in Vol. I., Chapter VIII., is founded on a hypothesis of an incomplete and imperfect kind.[1] It is for this reason that there is so large a discrepancy between the theoretical and actual values, apart from the influence of *added surface* (comp. Vol. I., § 181).

It cannot be pretended that the foregoing investigations on dynamic soaring are of an altogether exact kind. In view of the difficulty of attacking problems in aerodonetics by direct analytical assault, and the comparative impotence of the most advanced mathematical methods in the face of the simplest of problems in *real* fluid dynamics, some kind of stop-gap theory giving results of the right order of magnitude is the most that can at present be expected, and as such the theory here presented appears to be satisfactory.

As a numerical example we may take the hypothetical albatros of § 151. Let us take the velocity of flight $V = 50$ ft./sec. (Vol. I., § 187).

By § 155, since $\gamma = \frac{1}{7}$ $\quad \frac{v}{V} = \frac{2}{7}$

or $$v = \frac{2}{7} V = \frac{100}{7} = 14 \text{ ft./sec. (approximately).}$$

By § 151, on the peripteral area basis, the velocity of wind fluctuation required to give the necessary energy is 6·25 ft./sec.

And efficiency will consequently be

$$\left(\frac{6{\cdot}25}{14}\right)^2 = {\cdot}2 \text{ (approximately).}$$

But if we take the value of γ_1 given in Table VI., "Aerodynamics," on basis $\xi = {\cdot}02$, $\mathbf{n} = 12$ (to harmonise with $V = 50$ ft./sec. as in § 187), we have theoretical gliding angle = 1 in 16·8 (Table VI.), or $n = \frac{16{\cdot}8}{7} = 2{\cdot}4$; $\therefore$ efficiency $= \frac{{\cdot}5}{n} = {\cdot}208$, which is in approximate agreement with the previous determination based on the two separately computed values.

[1] "Aerial Flight," Vol. I., *Aerodynamics*, § 172.

The above is in no sense an independent check on the theory, it is merely a numerical illustration based on a hypothetical example.

The assumed value of the "actual" gliding angle of the albatros is possibly in excess of its true value, the difference between the values 1 : 7 and 1 : 16·8 suggests that this may be the case; it is, however, equally possible that the "plausible values" given in the tables of constants in "Aerodynamics" give too high a theoretical value for an aspect ratio **n** = 12; the question is only one that can be settled when more accurate experimental data are to hand. In any case it is most unlikely that the true gliding angle of the albatros is less than 1 : 10, it is in fact improbable that so small an angle is reached by any bird or other flying creature.[1]

§ 162. Dynamic Soaring as Determined by Different Kinds of Aerial Disturbance.—Dynamic soaring is a phenomenon that presents itself in much diversity of form, and the different types of soaring of this class depend upon certain quite distinct kinds of aerial disturbance.

In one variety of dynamic soaring, such as witnessed by the author from the ss. *Germanic* (§ 142), it would appear that there is a periodical wind fluctuation only remotely connected, if connected at all, with any terrestrial obstacle. In this variety it may be at least provisionally supposed that the fluctuation is that of turbulence pure and simple, and the wind consists of a "pack" of vortices consisting of cyclic motion containing rotation, and that the flight path of the bird in traversing such vortices meets with masses of air moving with different velocities of which it is able to take advantage in the manner already set forth. It is true that in the example cited there is considerable evidence that

[1] The only estimate extant of the flight angle of a bird that can be considered reliable is that of Bretonnière, who, from observations made on a number of storks, concluded that the gliding angle of this bird is 10°, or 1 in 5·7.

the wind pulsations possess some definite periodicity, and this may betoken that their existence is in some way associated with the ocean waves beneath; we have, however, at present no proof that such is the case.

The frequency and magnitude of the wind pulsation has been the subject of an investigation by the late Professor Langley,[1] who by means of anemometers of special construction showed that the fluctuations are of very considerable magnitude, more so than had

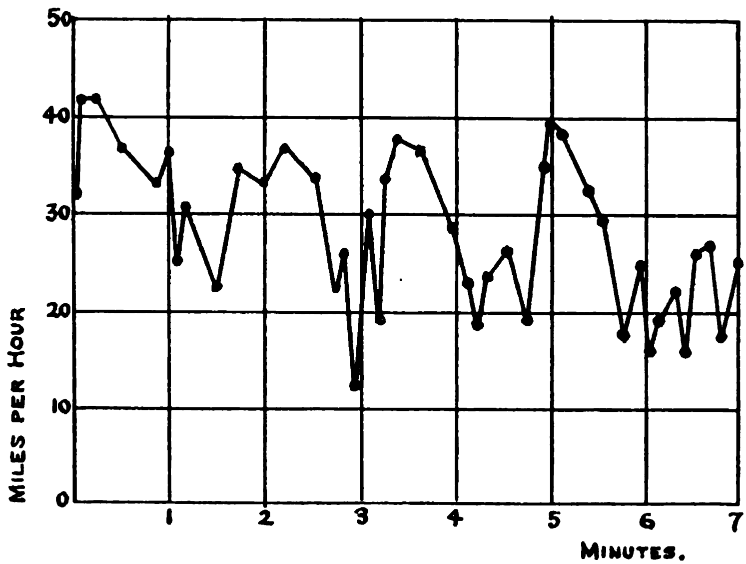


FIG. 142.

been currently supposed. In Fig. 142, which is a plotting given by Langley, it will be observed that in a wind having a maximum velocity of about 40 miles per hour there is a fairly constant fluctuation having a range averaging some 17 to 18 miles per hour, that is, about 25 ft./sec. Although the proximity of the observations is not sufficient for the periods recorded to represent those of most useful frequency from the point of view of the soaring bird, it is evident that the fluctuations that exist

[1] "The Internal Work of the Wind," Smithsonian Institute.

are of ample magnitude to provide the energy necessary to the soaring bird in flight; in the example cited the velocity fluctuation is approximately twice and the energy 4 times that necessary, on the hypothesis of uniform motion, to permit of the soaring flight of an albatros. If we take the harmonic basis the energy is still considerably greater than that required, after allowing for the loss of efficiency on the lines laid down. It is evident that a wind of far less internal energy than that given by the plotting will suffice for an albatros in flight.

It may be remarked that the records relate to the changes in the wind velocity in respect of *time* at one fixed point, and they do not, therefore, represent precisely the changes that a bird would have experienced in the course of its flight; evidently, however, although the effective fluctuation would be different, its general character and magnitude will be unaffected.

The relation of the fluctuation velocity to the mean velocity or "velocity of translation" is a matter of some interest. It would appear that the greater the mean velocity the greater *in proportion* is the velocity of fluctuation; this point is one on which Langley makes comment as follows:—"A prominent feature presented by these diagrams is that the higher the absolute velocity of the wind, the greater the relative fluctuations which occur in it. In a high wind the air moves as a tumultuous mass, the velocity being at one moment, perhaps, 40 miles an hour, then diminishing to an almost instantaneous calm. . . ."

It is very questionable whether the relation between the translational and fluctuation velocities of the wind depends alone upon the absolute magnitude of the former, even at altitudes that would preclude the possibility of local disturbance due to obstructions; nothing is known with certainty on this point. Mouillard, who was a very keen observer, noticed that in the soaring of birds, "Les vents n'ont probablement pas tous la même puissance de sustention. Il semble, en regardant attentivement les oiseaux, qu'il y a des jours ou l'air porte mieux que d'autres." It must certainly be supposed here that the proviso *for a given*

velocity is understood, so obvious a fact would not otherwise have been recorded; on the other hand it may have been a difference of vertical velocity component that gave rise to the observed variation.

§ 163. Dynamic Soaring as dependent on Dead-water Region.— In all probability many of the cases of dynamic soaring that have been observed depend upon the direct utilisation of the aerial disturbance set up by an obstacle, the flight path of the bird lying in part in the wake region or dead water and part in the live stream. In such cases the want of uniformity of the wind

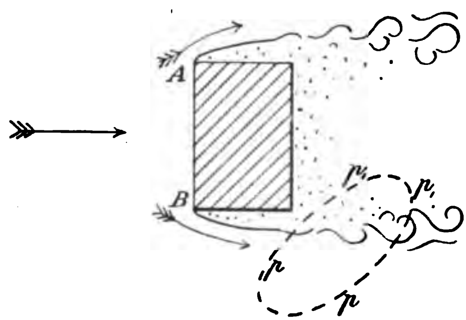


FIG. 143.

exists as a variation of velocity as a function of *place* instead of as a function of *time*.

In Fig. 143, let $A\ B$ represent in plan an obstacle of some kind causing a system of flow of the discontinuous type, and let $p\ p$, $p_1\ p_1$, be the flight path of a bird, part of which, $p\ p$, is situated in the live stream and part, $p_1\ p_1$, in the dead-water region. Then, the centrifugal component of the reaction of the bird in flight will tend, during the portion of the flight path $p\ p$, to diminish the velocity of the live stream, and during the portion $p\ p$ to impart velocity to the dead water. Thus the motion of the bird in its flight tends to *equalise* the velocity of the air coming within the peripteral system, and so the conditions of dynamic soaring

are fulfilled. The minimum wind velocity required under the present conditions can be readily computed from § 151.

§ 164. Dynamic Soaring. Mixed Conditions.—In cases such as discussed in the preceding section with reference to Fig. 143, it frequently happens that the obstacle by which the dead-water region is maintained gives rise to an up-current; in fact it may be said that this always occurs to a greater or lesser degree. A consequence is that soaring in the neighbourhood of an obstacle is usually of a nondescript kind; sometimes the bird can be seen to rise evidently borne aloft by an uprush of air, at other times it becomes equally evident that the soaring is of the dynamic kind, and that the dead-water and eddy currents are

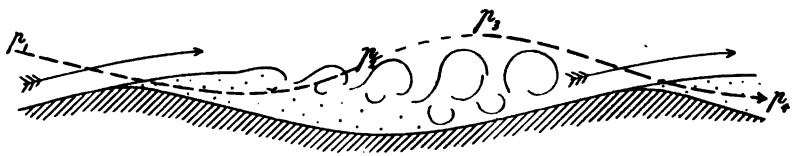


FIG. 144.

being utilised. It seems evident that many of the soaring birds are able to trap the available energy of the wind in whatever shape it presents itself, and it is almost certain that many of the larger soaring birds that habitually make use of natural up-currents are not above taking any opportunity of soaring dynamically that may present itself. Where a wind exists with an upward component, having at the same time horizontal fluctuations, it would seem from the remarks of many observers that the soaring is a mixture of the up-current and dynamic varieties. It is possible that when the conditions are not sufficiently favourable for either kind of soaring separately, by employing both the bird may be enabled to maintain itself in the air.

The form of flow of the air current or wind in the vicinity of

deep sea waves is, in all probability, of the discontinuous type, especially when the amplitude of the wave motion is considerable. If this were otherwise, *i.e.*, if the flow were of the Eulerian form, it would appear that a wind in the contrary direction to the waves would be of the same effect in maintaining the wave motion as a wind in the same direction, except for the immediate effect of surface friction. We know that this is by no means the case; a wind to be effective in creating or maintaining wave motion requires to blow in the same direction as the wave motion itself; hence the motion of the air involves discontinuity.

We may take Fig. 144 as representing the type of flow as we may credibly suppose it to exist, each wave crest giving rise to a surface of discontinuity and a dead-water region, the vicinity of the surface of discontinuity becoming a region of turbulence as indicated in the figure.

It is evident that if the system of flow is as thus depicted, a soaring bird will have an ample supply of energy at command, available by playing off the live stream against the "dead-water" as in examples already discussed, a possible flight path being indicated by the dotted line $p_1\ p_2\ p_3$, etc. Thus let us suppose that the bird descending in the region p_1 and gathering velocity from the wind, enters the "dead-water" region from which it emerges at p_2, it can evidently during this period impart much of its momentum to the "dead-water," and, as it were, leap once more into the live stream $p_2\ p_3$, where it again gathers momentum to repeat the operation at the next wave crest. The flight under these conditions will become a series of leaps, in which most of the sustentation will take place in the portion of the flight path, $p_1\ p_2$ and $p_3\ p_4$, etc., and comparatively little in the portions $p_2\ p_3$, etc.

It is probable that there are other ways in which a bird could make such a system as that depicted serve its purpose, but the one illustration will suffice.

The above exposition, although perhaps not the whole story, would appear to be at least a possible explanation of the bewilder-

ing mystery that attaches to the flight of the albatros and other marine species that skim and sail perpetually over the surface of the ocean, and whose *apparent* independence of anything in the nature of a source of motive power, has been for centuries the subject of comment and wonderment.

CHAPTER X

EXPERIMENTAL AERODONETICS

§ 165. **Introductory.**— The present subject is one of quite recent growth, in fact, it would appear that no comprehensive attempt has previously been made to deal with the question of stability in flight either by theory or experiment. Although other investigators have worked in the field, notably Penaud, Hargraves, Lilienthal, and Pilcher (apart from the builders of flying machines proper, such as Farman and the brothers Wright), the author has no knowledge of any systematic experimental work having been done or published, and the present chapter consists principally of an account of his own work.

The author's experiments comprise observations on the flight of aerodones or aerodromes of various sizes and proportions. The more important of these experiments, so far as data and results are concerned, have already been given (Chap. VI.), so that the present account is in the main a description of *method* and of the detail construction of flight models.

The construction of the flight models or aerodones, though originally a matter of comparative simplicity, is a subject into which, bit by bit, a considerable amount of technicality has crept, until the building up of a large low-velocity mica model, such as that illustrated in Fig. 55, is a matter that requires a certain amount of detailed instruction. Beyond this there are, in the handling of mica, and in the designing and building of models generally, many points in respect of which the author's experience will doubtless prove of value to others who may contemplate entering this fascinating field of research.

§ 166. Method of Experiment.—The author's method of experiment and apparatus are of the most primitive simplicity; a tape measure, a stop watch, and a launching staff (Fig. 2), together with the flight model itself, constitute the complete outfit.

A necessary adjunct to the above apparatus is a large room or other enclosed space. Failing this, it is necessary to experiment out of doors, a procedure that involves many tedious delays, waiting for suitably calm moments when the weather is favourable. At the best, outdoor experiments, unless on a rather large scale, are unsatisfactory.

Much of the author's work has, in spite of difficulties, been done in the open, but all the low velocity experiments (under 10 ft./sec.) have been conducted indoors, in most part in a room approximately 28 ft. by 20 ft., and in a few cases (the models of § 72), in a room about 60 ft. by 30 ft. The larger the room available the better, it is useful to have plenty of width as well as length, a room 50 ft. wide by 60 or 70 ft. long would, for most purposes, be found of ample size.

The aerodones may be launched either by hand or by means of the launching staff. In general, except for ballasted aeroplanes,[1] and for models of very rapid descent,[2] the author has found hand launching quite satisfactory, but a certain degree of skill, soon acquired by practice, is necessary, both in order to avoid giving the model initial rotation and to ensure imparting to it the correct velocity ($= V_n$), and gliding angle.

It is evident that any initial want of accuracy in the launching of the aerodone will affect its flight path, the amplitude of the resulting phugoid will be greater the greater the inaccuracy; it is thus necessary to ignore flights in which the amplitude is

[1] The ballasted aeroplane is difficult to launch with certainty by hand, owing to its having two alternative flight paths, $\cos\,\Theta = \pm\ 1$. There is but little difficulty when the launching staff is used.

[2] For models of rapid descent the launching staff is convenient, as, by the greater initial altitude, a longer flight is obtained. In the case of a model whose γ value is small the flight is limited usually by the length of the room, in which case the launching staff is of no advantage.

considerable, and in general the flights recorded by the author in the measurement of the quantities V_n and γ have been nice uniform glides in which the phugoid oscillation if existent at all is barely perceptible.

From the above considerations it is manifestly desirable to have a range of flight of at least two or three phase lengths, and where the amplitude is fairly constant it is evident that if the length of flight be made an exact multiple of the phase length (L_1), any error due to the phugoid oscillation will vanish, even though the flight path be of sensible amplitude. With due precautions the error in the determination of V_n due to the method of launching is far less than the possible error due to the method of timing by a stop watch, and the inaccuracy in the determination of *tan* γ certainly need not exceed one or two inches out of a total drop of 80 inches or thereabouts, or a probable error not greater than 2½ per cent.

§ 167. Construction. Materials Employed.—It being necessary to the designing of any appliance or machine to understand something of the materials utilised, even if only in order to be able to calculate approximately the weights of the various component parts, a few words are required as to the materials employed in the making of flight models.

Mica. Sp. gr. 2·8.

The mineral mica is obtained either in more or less regular lumps or slabs as when first taken from the earth, or as transparent or semi-transparent plates or laminæ, the form in which it as a rule finds its way to the market. There are several kinds of mica, but the only kind with which we are concerned is the mica of commerce, *i.e.*, *Muscovite mica*, frequently but erroneously called *talc*. The slabs in which the mineral is found in Russia consist in reality of rough crystals belonging to the monoclinic system, possessed of a very pronounced plane of cleavage; so markedly is this the case that it is possible to split mica up into laminæ in a manner and to an extent not possible with any other known mineral.

Mica is most conveniently obtained from a mica merchant ready cleft to some few thousandths of an inch in thickness and cut to sizes of rectangular form. The lamina employed for any particular purpose or job is preferably cut from a piece of about the required size. The cost of large sheets or laminæ increases (as might be expected) in a manner quite disproportionate to the area.

For the further splitting up of laminæ when thicker than required, a thin blade made of clock-spring, and sharpened to a point like a lancet, is found to be most serviceable. If the blade of a knife be used, it is, owing to its thickness, prone to damage the mica when the laminæ are of but a few thousands of an inch in thickness. A clock-spring of about 1/100 inch gauge is quite as stout as is necessary for the purpose.

Down to a thickness of about two thousandths of an inch the behaviour of mica in cleavage is very consistent, and is such as to suggest that the process of splitting and splitting again might be carried on *ad infinitum*. A limit is, however, soon approached: it is difficult to obtain with any degree of certainty laminæ of much less than 1/1000 inch thick. For a small model it is very often that a lamina of but half this thickness is desired, and advantage should be taken of the accidental production of very thin laminæ of this degree of tenuity, such as are sometimes split off accidentally, these being put carefully on one side until required.

For many purposes in the making of flight models the mica is required of varying thickness in its different parts. Occasionally laminæ will split off of markedly different thickness at one end to the other, and sometimes these may be employed with advantage. In general, however, it is easier to scrape the mica down from point to point to the varying thickness required; examples will be cited where this may be done to advantage. When mica is less than one-thousandth of an inch in thickness the scraping requires to be done with great care, but in such cases resort may be had to etching with

hydrofluoric acid; the process still requires the exercise of considerable judgment.

The proneness of mica to cleave at the slightest provocation, useful as it is for the preparation of thin laminæ, is a great source of weakness when the mica is considered as a structural material. Thus when used for large models it is necessary to *bind* the edges in some way to prevent the development of cleavages; for this purpose paper or silk may be employed, made to adhere by ordinary gum or mucilage; in other cases, especially for outdoor models, where damp is to be feared, gutta-percha tissue may be effectively used, or the edges may be treated with a solution of celluloid.[1]

A further difficulty that is experienced, owing to cleavage, is that of the attachment of mica laminæ effectively to the shaft

FIG. 145.

or backbone of the aerodone. However securely a mica plate be cemented in place, it is always able to escape by cleavage, a mere film of mica being left adhering to the cemented surface. This feature is convenient enough when preparing a model for a trial flight, as the tail plane, fins, etc., can be readily removed for adjustment, but it is equally inconvenient for the finished model. The method of overcoming this difficulty is by reinforcing the mica plate on the opposite face to that by which the attachment is to be effected, and then scraping a furrow at the point of attachment so as to expose the full thickness of the mica plate to the action of the cement, thus the latter obtains a grip on the whole on the constituent laminæ instead of a merely superficial hold on the outer surface. The above is illustrated diagrammatically in Fig. 145.

[1] Ordinary transparent celluloid dissolved in a mixture of 10 per cent. amyl acetate and 90 per cent. acetone, to the consistency of treacle.

§ 168. **Materials (continued).**—In addition to the mica used for the aerofoil, fins, and tail plane, the principal materials employed by the author are :—

Wood (for backbone), "White-wood" (or "Canary") planed to various thicknesses, from about 1 m.m. upwards; also veneer for use in very small models. The specific gravity may be taken as roughly = ·5.

Paper of various thicknesses, cigarette paper for binding edges (and covering mica surfaces in some cases), weight about ·0014 gram per sq. c.m. or ·009 gram per sq. in. Allow when gummed in place ·0018 gram per sq. c.m. or ·012 gram per sq. in.

Ballast.—The most convenient form of ballast is made by

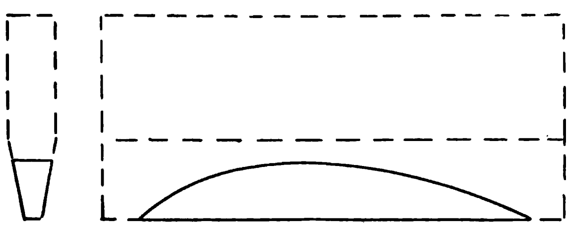


FIG. 146.

facing thick lead-foil with gutta-percha tissue, the two being warmed and pressed together. Stouter and heavier ballast of the same kind may be similarly prepared of several sheets arranged alternately. It is convenient to prepare composite ballast of this kind of some definite standard weight per square c.m., so that any desired addition to the weight of a model may be made approximately, without resort to a balance. Composite ballast of this kind has the merit of being very readily applied; it is rendered adhesive by warming and sets almost at once.

Split lead shot (fishing shot) may also be used, and are occasionally found convenient.

§ 169. **The Aerofoil.**—The aerofoil is constructed of a plate of mica, which may either be in one piece, or built up of two or

more laminæ joined together, if of any considerable size. For small models a single lamina of mica of appropriate thickness cut to the desired shape is all that is necessary; this is tied and cemented with fish glue or other adhesive to a "bolster" (Fig. 146), cut to the designed sectional form.[1] When required, as when working to some specified grading, the curvature can be maintained and regulated by means of cotton or silk ties arranged at intervals along the length of the aerofoil, as in Figs. 60, 62, and 63.

When the author's standard combination of parabolic grading and elliptical plan form is used, it is a property of the resulting aerofoil that the radius of curvature is constant at all points along its length; thus the aerofoil forms approximately part of

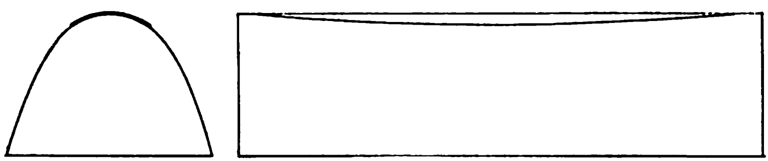


FIG. 147.

the surface of a cylinder. Strictly speaking, it is not a cylinder but rather part of the surface of a parabolic prism (Fig. 147).

The above fact renders the correct grading of a laminar aerofoil a particularly simple matter. A supporting pillow should first be made of somewhat the form of the parabolic prism of Fig. 147, but divided at intervals where it is intended to arrange the ties, Fig. 148; on this the aerofoil, cut to shape and otherwise prepared, is placed and secured by winding tape temporarily round the whole; the ties are then made fast and the bolster affixed in position, and the glue or cement allowed to set; the temporary winding is then removed and the aerofoil is complete.

The preparation of the aerofoil for the above mounting process depends upon circumstances; for very small models it is only

[1] Fig. 146 shows a convenient way of forming the bolster. The edges of the wood are first bevelled as shown and the bolster is cut from the bevelled edge on a fret saw machine.

necessary to reinforce the mica lamina at intervals with small squares of paper diagonally folded and gummed in position, to prevent the ties cutting into the edge of the mica, Fig. 149 (see also Fig. 63). In larger models it is necessary to reinforce also the whole front edge of the aerofoil, either with paper, gutta-percha, or celluloid, to render it capable of withstanding the impacts to which it is liable. In cases where it is required to reduce the

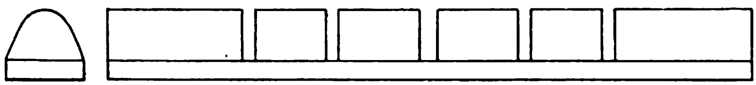


Fig. 148.

weight to a minimum, as for low velocity models, the preparation of the aerofoil is far more elaborate, the whole surface being scraped to graduate the thickness according to a prepared scheme; in this case the scraped surface may require to be in whole or in

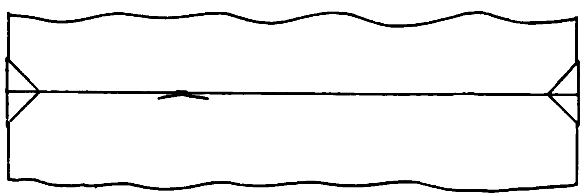


Fig. 149.

part covered with cigarette paper in order to prevent it from flaking (Figs. 55 and 60).

Where the aerofoil is built up of a number of smaller laminæ, these are given a coat of cigarette paper in the first instance where they are to be joined, and are then united with fish glue or gelatine. The surfaces before being served with the preparatory coat of paper are preferably scraped in an irregular way so as to expose several layers of the mica to the adhesive, and not to rely entirely on the surface lamina.

§ 170. The Aerofoil (continued).—In the preparation of the mica lamina for the aerofoil, consideration has to be devoted to the form required along that part where the curvature is given by the ties. It is evident, if the thickness of the mica is uniform, that since the only restraint is due to the cotton or silk cord in tension, the section will be an "elastic curve" (Fig. 150), instead of the desired arc form. In order to approximate more

Fig. 150.

Fig. 151.

nearly to the latter the thickness is preferably graded, being left greater in the middle and reduced towards the edges; the edges themselves, however, are left full thickness (Fig. 151) to

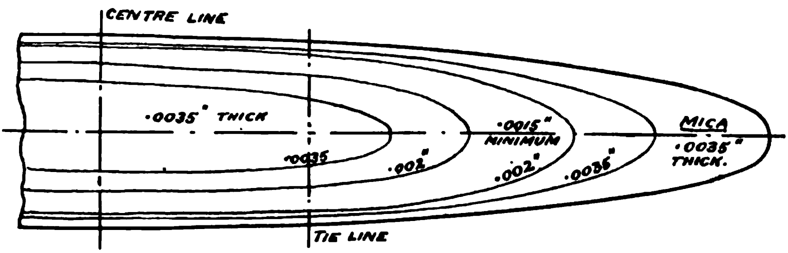


Fig. 152.

better withstand the rough treatment to which flight models are subject.

The grading of the thickness as above described is carried out by scraping, the disposition of the thickness in the finished aerofoil being indicated roughly in Fig. 152, by contour lines, appropriate thicknesses being given for an aerofoil 14 inches by 2 inches, for a model having a velocity of, say, 10 or 12 ft./sec.

An alternative plan has been tried for maintaining the curvature

of the aerofoil. Instead of tying, as described in the preceding section, ribs are arranged at appropriately spaced intervals, shaped to give the desired form, and tied and glued in place in a similar manner to the central bolster. This arrangement, although quite effective, suffers from certain disadvantages, not only is the moment of inertia about the axis of flight

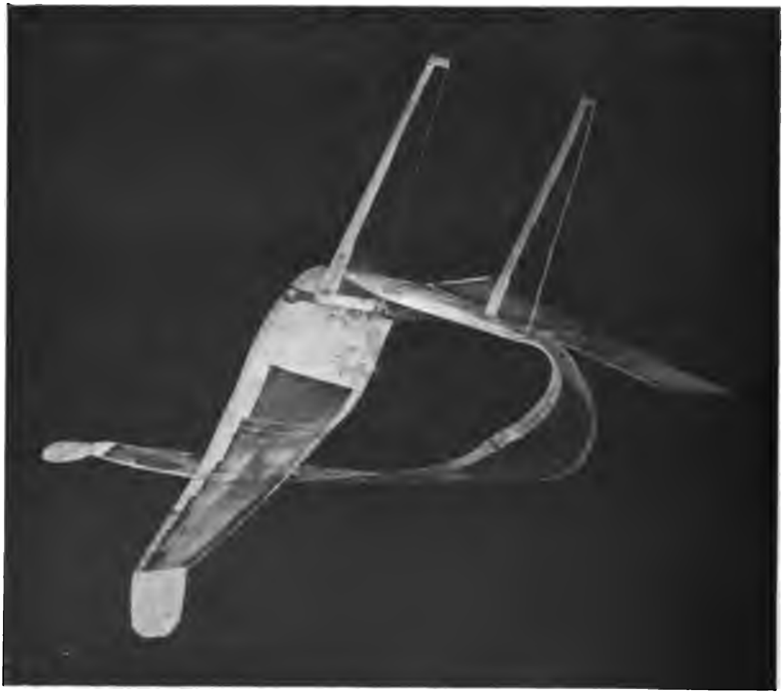


FIG. 153.

unnecessarily increased, but the wings are too rigid to escape injury for long. When the simpler method is used the wings are very supple; a model will sometimes fall, apparently hopelessly injured, Fig. 153, but the instant it is picked up the wings open out in a most surprising way and are quite undamaged.

§ 171. The Fin-plan and Tail-plane.—The fin-plan is constituted by the two fins, the backbone, and the ballast, thus including the whole aerodone with the exception of the aerofoil and tail-plane. These parts are taken as a collective unit on account of the measurement of moment of inertia.

In the method employed by the author for the determination of the moment of inertia, the fin-plan is suspended as a pendulum and its oscillation period is counted; the moments of inertia of the aerofoil and tail-plane being separately computed and added subsequently.

The reason that the aerodone is not swung bodily and the moment of inertia measured as a whole, is that owing to the exposed surface, normal to the direction of oscillation, and the consequent resistance, the damping is too rapid to permit of the oscillations being counted with sufficient precision. This difficulty would be removed if the determination were made *in vacuo*.

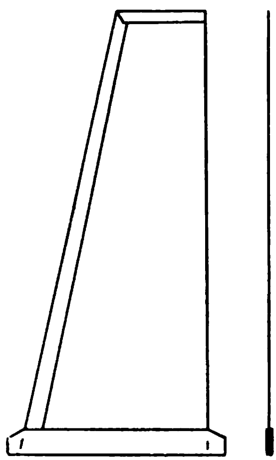


FIG. 154.

The *fins* are mica laminæ cut to the form required and glued to the backbone; for small models nothing more elaborate is required, but for large models, especially where it is intended that the job shall be permanent, the front edge is protected by a paper or other binding, and the attachment is strengthened by means of a stouter paper reinforcement (Fig. 154). If it is desired to attain the minimum velocity possible, the fins are scraped down as thin as consistent with proper strength, the leading edge is left full thickness, but the mica is thinned off towards the "trail" and towards the upper extremities.

The *tail-plane* is a lamina of mica cut to shape and similarly prepared, *i.e.*, except for very small models the edges should be bound or otherwise treated to avoid injury. The tail-plane is

attached to the backbone by glue or cement of some kind, the point of attachment being prepared as described in § 167.

The *backbone* is made either from a single slip of wood planed down to an appropriate scantling, Fig. 155 (*a*), or it may be built of two slips glued side by side with the fins inserted between, Fig. 155 (*b*). In the latter case the reinforcement is not necessary.

The *ballast* is attached to the front end of the backbone and is adjusted with the tail-plane temporarily affixed to bring the centre of gravity into its correct position, namely, slightly in front of the geometric centre of the aerofoil.

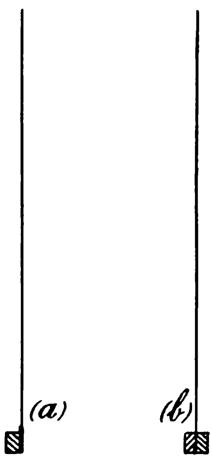


FIG. 155.

§ 172. The Measurement of Moment of Inertia.—The method of measuring moment of inertia employed by the author is one that has the advantage of requiring but little or no apparatus. The "fin-plan" of which the mass centre has been first determined, is suspended on a knife edge (Fig. 156) and set swinging, the time period being taken by counting fifty complete oscillations.

The calculation is then made as follows :—

Let l = length of pendulum, *i.e.*, the distance from the centre of suspension to the centre of gravity of the piece.

,, t = time of single oscillation.

,, λ = radius of gyration of the piece about its own centre of gravity.

Then λ is given by the expression :—

$$\lambda^2 = l\left(g\,\frac{t^2}{\pi^2} - l\right).$$

In the case of the "12 gram" model (§ 71), two trials were made :—

(1) $l = 2\frac{5}{8}$ inches $= \cdot 2185$ ft.

Fifty complete (double) oscillations occupied 31·6 seconds, or

$$t = \cdot 316.$$

$$\therefore \qquad \lambda^2 = \cdot 2185 \left(32{\cdot}2 \frac{\cdot 316^2}{\pi^2} - \cdot 2185 \right)$$

$$= \cdot 0235.$$

(2) $$l = 1{\cdot}2 \text{ inches} = \cdot 1 \text{ ft.}$$

$$t = \cdot 33 \text{ (50 double oscillations} = 33 \text{ seconds)}$$

$$\therefore \qquad \lambda^2 = \cdot 1 \left(32{\cdot}2 \frac{\cdot 33^2}{\pi^2} - \cdot 1 \right)$$

$$= \cdot 0255.$$

In the above example a repetition of the experiment showed the second of the two observations to be the most accurate, hence $\lambda^2 = \cdot 025$.

The square of the radius of gyration as above ascertained, multiplied by the mass of the piece, gives the moment of inertia about its own mass centre. Owing to the fact that the moment of inertia requires to be measured about the centre of gravity of the whole aerodone, there must be added to this the moment of inertia of the mass of the "fin-plan," supposed concentrated at its centre of gravity, about the mass centre of the aerodone. This may be effected in the usual manner by adding the square of the radius of gyration, as determined, to the square of the distance between the centres of gravity, and then multiplying the sum by the mass of the "fin-plan" to obtain the total value.

As an alternative a piece of ballast may be substituted for the tail, this brings the centre of gravity of the "fin-plan" almost into its final position, when the measurement, made in the manner already indicated, gives the final result *including the tail.* The author has employed both methods, the latter is perhaps a trifle the more convenient.

There is some tendency for the "fin-plan" to turn round into an undesirable position when suspended in the manner shown in Fig. 156. The author has tried as a substitute suspending by

fine silk fibres, but the results are not consistent. It is safest in any case to make two independent determinations from different points of suspension, the results of which should agree; a disagreement not exceeding 3 or 4 per cent. may be regarded as unimportant: it is difficult to get within this limit with silk suspension. The author has found the most satisfactory solution

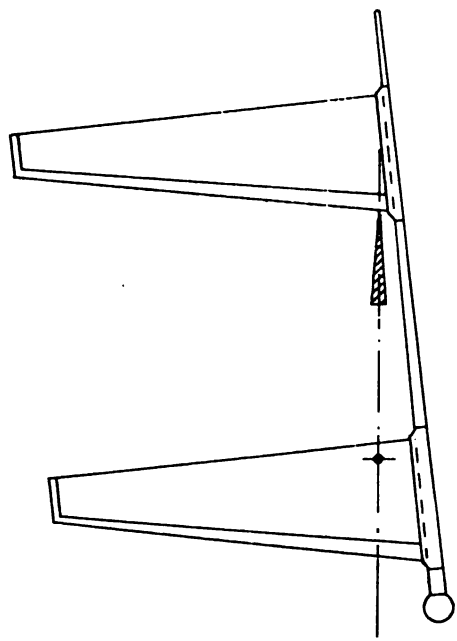


Fig. 156.

to the difficulty is to make the determination in some place where there is a draught, so that the current of air, by acting on the "fin-plan" like a weather vane, keeps it in the desired position.

When the tail-plane is not represented in the "fin-plan" determination by ballast, it is taken account of by calculation, its radius of gyration being assumed equal to the distance of its geometric centre from the mass centre of the aerodone; its

moment of inertia about its own mass centre is taken as negligible.

The moment of inertia of the aerofoil may be calculated from its mass and its radius of gyration in the usual manner; the latter may be assumed as approximately one quarter of the fore and aft dimension of the aerofoil; it may sometimes be a little more or less, but the error involved in making the above assumption is in no case important.

The following is an example of the calculation of the moment of inertia of the 12 gram model (Figs. 55, 60, and 61), of which the "fin-plan" computation has already been given.

	Grams.	λ^2	I gram. ft.^{2}
Fin-plan . .	2·32	·035 *	·081
Tail-plane . .	·87	·075	·065
Aerofoil . .	5·40	·00174	·0094
Total †	8·59		·1554

Or, in lb./ft.^{2} this becomes

$$\frac{\cdot 1554}{453} = \cdot 000344.$$

§ 173. Admissible Proportions of Models.—In designing an aerodone for any given purpose, some of the data are supplied by the conditions. If for example it be desired to verify the equation of stability in a room of given size, and it is decided that a minimum of four complete oscillations of the phugoid curve are to be observed, the values of H_n and hence also V_n can be at once calculated and become part of the initial data. Now within certain limits the size of the aerodone can be settled independently of its V_n value, but owing to the considerations discussed in § 82,

* This is the augmented value of λ^2, *i.e.*, the sum of the squares of the radius of gyration about the centre of gravity of the fin-plan and the distance of the latter from the centre of gravity of the aerodone.

† This is the total weight before addition of ballast: *Vide* § 174.

a superior limit of size exists. Thus for ordinarily careful workmanship it is difficult to give a stable flight path to an aerodone whose tail length in feet exceeds $\frac{V_n^2}{250}$.

Likewise, since the fore and aft width of the aerofoil is limited when the tail length is limited, and since the aspect ratio cannot be extended indefinitely, there will be a limit to the total weight; this is found to take place when W (poundals) is round about the value, $\frac{V^6}{1,000,000}$, where the aspect ratio is about 7 or 8, or in an extreme case the value of W may be made as high as $\frac{V^6}{500,000}$. It requires very careful design and exceptional workmanship to exceed this figure.

If conversely, as is the case where the problem is that of mechanical flight, the approximate value of W is given; the same equation may be employed to obtain the minimum permissible velocity, thus,

$$V_n^6 = 500,000\, W,$$

or,

$$V_n = 8\cdot 9 \sqrt[6]{W},$$

or if W_1 be the weight in lbs. the expression becomes

$$V_n = 16 \sqrt[6]{W_1} \text{ (approximately).}$$

Thus in the case of a man bearing aerodone, weighing in all some 220 lbs. (the weight of the Lilienthal machine with aeronaut), it will be difficult to obtain automatic stability at a less velocity than $16 \times 2\cdot 45 = 39$ ft./sec. or approximately 27 miles per hour; or, again, in the case of a flying machine of one half ton gross weight, $V = 16 \times 3\cdot 22 = 51\cdot 5$ ft./sec., or, say, 35 miles per hour is the minimum velocity.

There is admittedly much latitude in the value of the constant in the above equations; the exercise of skill on the part of the designer and constructor, in combination with the highest

class material, may doubtless (if required) enable the constants given to be materially improved; a limiting condition must soon be reached, however.

§ 174. The Design of an Aerodone.—In the design of an aerodone there are two sets of data to be determined, namely, *aerodynamic* and *aerodonetic*. The first, by the aid of the tables given in Vol. I., may be settled at once; the weight and velocity (W and V_n), being decided to come within the limiting condition of the preceding section; the area of the aerofoil for the conditions of least resistance is given at once by Table X. The appropriate angle of trail, β, is then ascertained from Table IV. or V., and the angle a is obtained from this by multiplying by the constant, ϵ (Table III.); the sum of β and a determines the form of the bolster from which the aerofoil derives its curvature.

The next step is to rough out the design and carefully calculate the weights of the different components, from which a computation is made of the moment of inertia, and the stability is worked out from the equation (§ 63).

Having ascertained that the degree of stability is that desired, or having revised the design to correct the value of the stability factor as may be necessary, the various parts of the aerodone are made and separately weighed in order to ascertain that the estimate is not being exceeded.

The parts are then put together and the results checked from stage to stage by weighing, and the moment of inertia by the method of suspension. The first putting together may be for a trial flight, the parts afterwards being made a permanent job.

The initial design and calculations should be complete in every detail, the results as recorded, and any alterations subsequently made, being entered on the drawing for reference.

As an example of the foregoing the "12 gram" model of § 72 may be taken. The calculations of the aerodynamic data may either be made for *known mass* or for *unknown mass*. The present model was dealt with by the latter method, and the mass

was subsequently made good by ballast[1] to a value afterwards calculated appropriate to the *least stable velocity* (presuming this to be the value required), as determined from the equation of stability.

The aerofoil was taken arbitrarily as 2 feet by 2 inches, thus $\mathrm{n} = 12$. The plan form was approximately elliptical,[2] and the grading parabolic. The tail and fin areas, tail length and other proportions, were settled from rough preliminary calculations.

Aerodynamic calculation :—

Equivalent aerofoil area (Vol. I., § 192),

$$= \frac{2}{6} \times \frac{2}{3} = \cdot 22 \text{ sq. ft.}$$

By Table IX., taking $\xi = \cdot 025$, $\frac{P}{V^2} = \cdot 0437$, or

$$K = \frac{W}{V^2} = \cdot 0437 \times \cdot 222 = \cdot 0097.$$

By Table V. (Vol. I., § 181), taking value of $\xi = \cdot 025$ as before, for least resistance, $\beta = \cdot 27$, or mean value for stated plan form and grading, say, $= \cdot 24$ (radian).

Aerodonetic calculation :—

Tan γ (estimated[3] probable value) $= \cdot 14$.

Tail data,

$l = \cdot 28$ ft.

$a = \cdot 0625$ sq. ft.

Constants,

$\rho = \cdot 078$

$$\left.\begin{array}{l} C = \cdot 7 \\ c = 2 \cdot 27 \\ \epsilon = \cdot 75 \end{array}\right\} \text{From the Tables, Vol. I., §§ 177, 180.}$$

Moment of inertia,

$\mathbf{I} = \cdot 00035$ (§ 172).

[1] The ballast is added as compactly as possible in the region of the mass centre, so as not to materially increase the moment of inertia.

[2] The wing extremities were left square instead of being rounded off; this is now the author's usual practice, see Figs. 60 and 62. The paper wing tips (Figs. 55 and 60) were added subsequently.

[3] The value ascertained subsequently by measurement was ·13.

Now, as in § 79,

$$H_n = \sqrt{\frac{\cdot 00035\ (103 + 716)}{4 \times \cdot 28 \times \cdot 14}} = 1\cdot 35.$$

Hence least stable velocity $= V_n = 8\sqrt{1\cdot 35} = 9\cdot 3$ ft./sec.

Aerodynamic calculation :—

Now $$K = \frac{W}{V_n^2} = \cdot 0097$$

$$\therefore \quad W = \cdot 0097 \times 9\cdot 3^2 = \cdot 84.$$

This is the weight in poundals that should give to the aerodone in question its least stable velocity as its V_n. In grams this becomes approximately[1] $\cdot 84 \times 14 = 11\cdot 75$.

Now the actual weight of the components as designed and made was 8·59 grams (§ 172), hence the model required 3·16 grams ballast to give it stability.

In the actual trial of this model, the ballast added to give stability was approximately that calculated, the gross weight as finally adjusted being 12·03 grams, but the natural velocity was true to the calculation, the measured velocity being 9·3 feet per second when the condition of stability was reached.[2]

§ 175. On the Angle of the Tail-plane and the Position of the Centre of Gravity.—Assuming that the aerofoil is of arc form of section, its mean angle to its direction of motion, that is, the angle formed by the chord of the arc to the line of flight, is given by the expression, $\frac{\beta - a}{2}$ (Vol. I., § 188), and since $\epsilon = \frac{a}{\beta}$ this becomes $\frac{1 - \epsilon}{2}\beta$. Consequently if the tail be a purely *directive organ*, that is to say, if, for the straight flight path, it carry no pressure reaction, either positive or negative, then we might suppose the tail-plane set at an angle $= \frac{1 - \epsilon}{2}\beta$ to the

[1] One poundal is approximately equal to the weight of 14 grams mass.

[2] Such exactitude of agreement between the calculated and experimental values is unusual; there is in fact a defect of about 7 cent., due to the error in the estimated value of *tan* γ.

chord of the aerofoil section (Fig. 157 (*b*)). But this presumes that the tail-plane is situated where it will be unaffected by the "wash" of the aerofoil.

If we take account of the latter we know (§ 63) that the residual downward velocity of the wake is represented in terms of the velocity of flight by an angle $(1-\epsilon)\,\beta$, so that if, as is usual, the tail-plane be situated in the wake of the aerofoil, the relative positions will be as represented in Fig. 157 (*a*), for the conditions of zero tail reaction.

Such a setting as that indicated in Fig. 157 (*a*) possesses many disadvantages in theory, and in practice is found to be unwork-

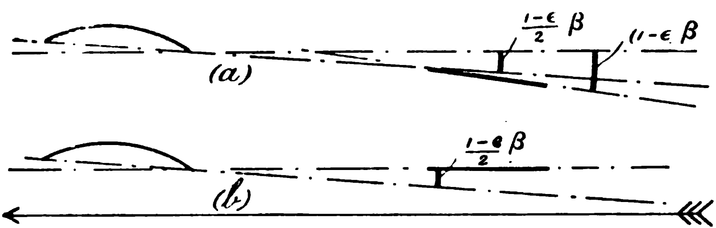


FIG. 157.

able. All actual settings of the tail-plane require to be made on the basis of a negative (*i.e.*, *downward*) pressure reaction.

It is of some interest to note that the positions of the tail-plane as determined on the assumption that it is situated in the "wash," or clear of the "wash," of the aerofoil, in both cases make an angle $= \frac{1-\epsilon}{2}\,\beta$ with the chord of the aerofoil section, but in the one case this angle is of positive and in the other of negative sign,[1] Fig. 157, for $(1-\epsilon)\,\beta - \frac{1-\epsilon}{2}\,\beta = \frac{1-\epsilon}{2}\,\beta$.

We know that in practice the leading portion of the aerofoil must be modified for the reasons stated in Vol. I., § 191, otherwise the peripteroy may become unstable. This is illustrated

[1] The acute angle made by the tail-plane to the chord of the aerofoil section, in Fig. 157, measured positive in a counter-clock sense.

in Fig. 158, where it will be seen that the effect is to increase the angle of the tail-plane as defined.

The possibility of an unstable periptery is a greater danger when the amplitude of the phugoid oscillation becomes considerable. In the extreme case, that of the semi-circle, when the aerofoil is launched by being dropped from rest, it is evident that the angle of the tail-plane must be positive, for otherwise when the aerodone is released the aerofoil will experience a pressure reaction on its leading edge in the wrong direction, and it will proceed on a flight path of opposite curvature than that intended. The author has frequently made models perform in this way, the same model being capable of

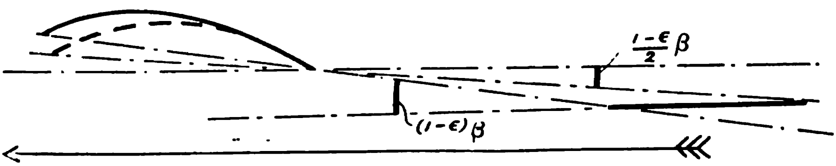


FIG. 158.

flight either right way up or upside down, in the manner of a ballasted aeroplane.[1]

In order to avoid irregularities of this kind, which apart from the objection stated are liable to detrimentally affect the stability of all phugoids whose amplitude is considerable, the tail-plane should be given a distinct upward trend,[2] its angle (as above

[1] It is surprising sometimes how well models will fly the reverse way up to that for which they are designed; when the aerofoil is of true pterygoid form the velocity of inverted flight is commonly higher than the normal velocity, but the gliding angle is sometimes quite good.

[2] It might at first sight be supposed that the sustaining of a pressure reaction by the tail would invalidate the investigation of Chap. V., and thus affect the truth of the equation of stability, since in this investigation the tail was taken as a purely directive organ. As a matter of fact, the modification of the conditions is without influence; the law of pressure on the tail is that of the *small angle* $P_\beta = c\ \beta\ P_{90}$, and thus is a straight line law. Hence the effect of an initial pressure reaction is merely to alter the *zero* without altering the influence of a given angular departure.

defined) being made nearly equal $(1 - \epsilon)\,\beta$, Fig. 158, this being a rough approximation to that which the author has found to give the best results in practice.

§ 176. On the Angle of the Tail-plane and the Position of the Centre of Gravity (continued).—Counterpart with the upward trend given to the tail-plane is the position that must be assigned to the centre of gravity. It is evident that if the function of the tail-plane were purely directive, the centre of gravity would need to coincide with the centre of pressure of the aerofoil. If, however, the tail-plane is destined to sustain a pressure reaction, the centre of gravity will need to be moved either forward or backward to suit according as the pressure reaction on the tail-plane is downward or upward. And since the pressure reaction on the tail-plane acts downwards, the centre of gravity must be displaced forward.

In practice the author has found that for his standard form of aerofoil the appropriate position of the centre of gravity is about ·4 of the width of the aerofoil from its leading edge, that is to say, the best results will almost invariably be obtained when distance of the centre of gravity from the leading edge in terms of the maximum width of the aerofoil is from 3/8ths to 7/16ths.

This displacement of the centre of gravity is approximately that which would be expected from the conditions. It would seem that the tail-plane must from its "setting" be moving at an angle of at least $\frac{1 - \epsilon}{2}\,\beta$ athwart the stream, and that consequently it must experience a *pressure* approximately $\frac{1 - \epsilon}{2}$ times that on the aerofoil,[1] or a *total pressure reaction*

[1] The value of the constant c is taken to be the same for both aerofoil and tail-plane, an assumption that is only justified by the absence of definite data. The actual value of this constant must be different, not only on account of the difference in the value of **n**, but also on account of the difference between the aeroplane and the pterygoid form. Comp. Vol. I., § 172, footnote.

$= \frac{(1 - \epsilon)\, a}{2\, A}$ times the load. In the case of the 12 gram model this will be—

$$\frac{(1 - \cdot 75) \times \cdot 0625}{2 \times \cdot 22} = \frac{\cdot 0156}{\cdot 44}$$

$$= \cdot 0354.$$

Or the displacement of the centre of gravity requires to be $= \cdot 035\, l$ where l is the tail length, or in the case in point, $\cdot 0354 \times \cdot 28$ ft. $= \cdot 01$ ft. or $\cdot 12$ in.

This is less than the actual distance between the centre of gravity and the geometric centre, but the centre of pressure is in all probability materially in advance of the latter.[1] Thus in the actual model the distance between the geometric centre and the centre of gravity is approximately ·2 in., so that the difference to be accounted for by the displacement of the centre of pressure is no more than ·08 in.[2]

[1] The centre of pressure of an *aeroplane* of the proportions of the aerofoil is very much in advance of the geometric centre (comp. Vol I., § 148), but this does not apply to the same extent to an aerofoil of pterygoid form. If the arc section be that adopted, not modified as in Vol. I., § 191, and of uniform *form* of section as in Vol. I., § 120, then in theory the centre of pressure should coincide with the geometric centre. Owing to the fact of the modification of the section by which it becomes of flatter form towards the extremities, and owing to the curtailment of the forward or dipping portion in accordance with Vol. I., § 191, already cited, the centre of pressure will in practice be situated in advance of the geometric centre, but only to a moderate degree.

[2] As a matter of fact, if any discrepancy exists, is in the opposite direction to that suggested by the above computation; the centre of gravity is in practice not so far forward as might be anticipated from the conditions. The displacement in the above computation has been based on a minimum estimate of the inclination of the tail-plane to the direction of the stream; if we take the probable value in view of the influence of the wash of the aerofoil (after due allowance for the curtailment of the dipping edge), it is at least double that taken, or, say, $= (1 - \epsilon)\, \beta$, and the position of the centre of gravity is more than accounted for without recourse to any allowance for a displacement of the centre of pressure. The matter calls for fuller investigation.

§ 177. **Methods of Steering.**—One of the points in aerodonetics that is most easily investigated by model experiment is the steering of an aerodrome or aerodone.

It is often found that a model when first flown has a tendency to steer either to the right or to the left, the flight path being the arc of a circle instead of a straight line. So much is this the case that some adjustment is usually required before it is possible to obtain flights of sufficiently constant direction for the determination of the true natural velocity and gliding angle.

If a model that has a bias in the one direction or the other be examined, it will be found that either the two fins are not truly

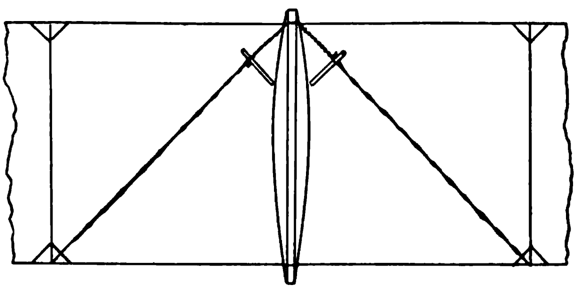


Fig. 159.

in line, or that the aerofoil has a twist or "wind." If the former is the case, the fins should be first set truly in the same plane or in parallel planes;[1] if after this the direction of flight is not constant, the error is due to a want of perfect symmetry of the aerofoil and can be corrected by twisting the aerofoil one way or the other.

When the model is a small one, it is sufficient to breathe on the glue by which it is affixed to the bolster and carefully apply the twist by taking the "two wings" one in each hand, between the forefinger and thumb. When the model is of any size, it is convenient to arrange a tourniquet as shown in Fig. 159, by

[1] It evidently is not important that the fins should be accurately in the same plane so long as they are truly parallel.

tightening which any desired degree of twist may readily be applied.[1]

It is easy to remember which way the steering twist is required from the fact that the rotary motion imparted to an aerodone in flight by a twisted aerofoil is in the same sense as would be the case if the aerofoil were mounted free to rotate about a vertical axis, and allowed to fall through the air, acquiring a rotary motion from its screw-like form; thus an aerofoil that has a corkscrew-like twist[2] causes an aerodone to steer to the right and *vice versâ.*

When for any reason a curvilinear flight path is desired, it may be given either by setting the leading fin slightly in the desired direction, or by giving an appropriate twist to the aerofoil, or both methods may be employed simultaneously.

§ 178. On the Ballasted Aeroplane.—The ballasted aeroplane is an article of such simplicity that but few words are necessary as to its correct proportioning or constructional detail.

In general, much of that already written as to the preparation and treatment of mica applies without alteration.

Formerly the author was in the habit of employing split shot for the purpose of ballasting, but more recently this has been given up in favour of the composite ballast described in § 168, as being more convenient to adjust and less liable to become detached.

There is a relation between the size of the mica used and its minimum thickness; if it is too thin the aeroplanes become unduly flexible; if, however, a very low velocity model is required, a great deal may be done to lessen the weight and retain stiffness

[1] It is usually found that a model when first tried has a slight bias in the one direction or the other, and the tourniquet may then be fitted on whichever side it is required. If, however, a model is approximately symmetrical, and it is desired to be able to steer it either to the right or left, a second tourniquet may be fitted as shown in the figure.

[2] The length being taken transverse to the line of flight. The same twist reckoned about the axis of flight is of the contrary sense.

by artistic scraping in the manner already described. For unscraped plates the minimum thickness it is desirable to employ is given roughly by the expression—

$$1000\ t = \sqrt{\frac{l^3}{60}}$$

where l is the length of the plane, and t the thickness, in like units of length.

§ 179. Some Vagaries of the Flight Path.—It must not be supposed from the apparent completeness of the present work that everything that relates to the flight path of an aerodone has been thoroughly worked out or explored ; in making experiments, or even in merely repeating experiments previously made, new and unexpected points frequently arise, and the deportment of some particular model under some particular conditions often seems inexplicable until by careful and repeated observation some clue is discovered to the behaviour observed. Usually anything in the way of an anomaly in flight is eventually traceable to the conditions assumed for the purposes of theoretical investigation not being complied with, either as touching the initial hypothesis or the limitations imposed by the nature of the investigations.

When a model is constructed to come well within the limits of stability, both lateral and longitudinal, its behaviour in flight is usually irreproachable; it yields results as to time period and phase length approximately in harmony with the equations of § 36, and its flight path is ordinarily quite consistent. It is when dealing with models whose flight path borders on the unstable, and whose lateral stability has but a very small margin, that irregularities are met with and anomalous cases of flight occur.

Thus, in experimenting with models Figs. 60 and 62, the "12 gram" and "4 gram" aerodones employed for the verification of the equation of stability, the time period of the phugoid

curve, when of considerable amplitude, was found to be greater than that calculated from the natural velocity; consequently the phase length also was greater. This anomaly is frequently met with in large low velocity models, and it appears that it is principally due to the size of the model in its relation to the flight path, or more correctly the length of the tail in relation to H_n. The author has found that, generally speaking, so long as the tail length is less than one-tenth of H_n, the agreement between the phase time and length and the velocity is fairly good; but if the tail length exceeds this value, an appreciable error, for paths of sensible amplitude, begins to be manifest, and when the tail length is made equal to $\frac{1}{4}$ of H_n, the error may amount to as much as 30 or 40 per cent. or thereabouts.

An alternative explanation of the abnormal phase length may be founded on the fact already discussed (§ 115), that the air in the periptery contains energy, and the influence of this is in effect to somewhat lower the effective value of g in the phugoid theory (§ 20), but investigation shows that the extent of the probable error from this cause is too small to account for the observed difference. Beyond this, in watching the models to which reference has been made, it is possible to see the hanging up or partial suspension of the motion at and about the crest of the flight path; this is the point at which the radius of curvature is least, and consequently the influence of tail length is most marked.[1]

§ 180. Vagaries of the Flight Path (continued).—When experimenting out of doors, the author has sometimes been astonished by obtaining long, uniform, almost horizontal, glides; in one case a drop of 2 ft. 6 in. only was recorded in a flight of over 50 feet length; this flight was obtained with the 12 gram model of § 72. Now the actual gliding angle of this model, when carefully

[1] The author is not thoroughly satisfied that every case of abnormal phase length is fully explained as above; it is possible that there are other contributory causes. The investigations continue.

ascertained indoors (§ 72), is given by the expression *tan* $\gamma = \cdot 13$; it is evident, therefore, that there is something abnormal about the above flight in which *tan* γ is less than ·05.

Owing to the low velocity of the model in question and to the fact that its flight path is on the verge of the unstable, we know that this anomalous flight was not due (as in the author's 1894 models) to the launching velocity being in excess of V_n, for an excess equivalent even to a few inches of extra "head" (H) would have been quite obvious from the amplitude of the resulting phugoid, if not sufficient to actually put an end to the stability. The explanation is consequently to be sought elsewhere.

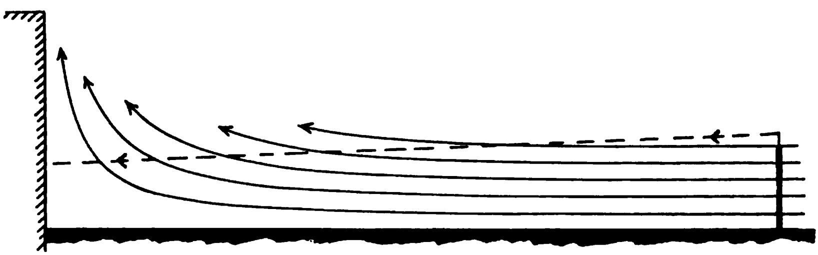


FIG. 160.

At the time of the experiment there was a very light air moving; the velocity, estimated from the rate of drift of the smoke from a cigarette,[1] amounted to approximately three feet a second in the direction of flight; this at once accounts for some portion of the diminution of gliding angle, for the length of the flight *through the air* was probably but $\frac{V_n}{V_n + 3}$ of its observed length relatively to the earth—that is approximately $\frac{3}{4} \times 50$ feet, or say 38 feet.

Now the correct fall where *tan* $\gamma = \cdot 13$ for a 38 feet flight is 5 feet approximately, so that there is still a discrepancy of 2·5

[1] The drifting of a smoke cloud is certainly the most convenient way of estimating the velocity of very light aerial currents. With two observers and a stop watch it is probably also the most accurate; perhaps better results could be obtained by timing the drift of a soap bubble.

feet between the actual value and that which would have been observed under still air conditions.

The next point noted as affecting the conditions was that the flight took place from an open space towards a building (Fig. 160), the aerodone striking the wall of the latter about 4 feet above the ground. Here we find two distinct factors tending to sustain the aerodone in flight, both contributing to the observed result.

It is in the first place evident that the air in the region traversed will have an upward component to its motion (comp. Figs. 114 and 115), and thus the apparent gliding angle will be diminished; secondly, since the aerodone automatically regulates its own velocity through the air, and its launching velocity is from the evidence of its flight path approximately correct, the initial velocity relatively to the earth will be 3 ft./sec. higher than the final velocity,[1] and the difference of kinetic energy due to this velocity difference will appear as a gain in altitude. The exact gain due to the first of these causes is difficult to assess, but the second accounts for a gain of about one foot, so that between the three different causes present to minimise the rate of fall we may regard the total discrepancy as roughly accounted for. The exact velocity of the air current might quite well have been as much as 4 ft./sec. at the moment of launching; fluctuations of this magnitude are constantly taking place in the velocity of the wind, even when this velocity is as low as a few feet per second.

The importance of the present example is as an illustration of how easy it would be to an inexperienced observer to record, in perfect good faith, fictitious and misleading results, and, further, the desirability of conducting all quantitative experiments in a suitable building. A motion of the air of one or two feet a

[1] The launching velocity *relatively to the wind* was approximately $= V_n = 9{\cdot}3$ ft./sec.; to this must be added the velocity of the wind, *i.e.*, 3 ft./sec., to obtain the velocity relatively to the earth at launching. At the end of the flight the air has no horizontal motion, hence the absolute velocity of the aerodone becomes V_n.

second velocity is scarcely noticeable, and yet it may materially vitiate the accuracy of experimental work.

§ 181. Vagaries of the Flight Path (continued).—In experimenting with the 4 gram model of § 72, Fig. 62, an opportunity occurred of studying a peculiarity in the flight path already observed in other cases, but for which no explanation had previously suggested itself.

In making the quantitative measurements of velocity and gliding angle it is very important to obtain a directionally straight flight path, and when adjusting a model in this respect, the regular difficulty is of course that the flight path, instead of being quite straight, will follow the arc of a circle of greater or less radius. By two or three trial flights it is usually possible (by twisting the aerofoil as already explained) to obtain a flight sufficiently straight for the purposes of measurement. In certain cases it may be observed that the flight path will be apparently straight for some distance and then suddenly change its direction without visible cause, and a semblance of regularity may often be noticed in the behaviour of a model under these conditions, the disturbance taking place at a certain definite point in the flight path, or one of two definite points.

The first impression one receives is that there must be a draught or other aerial disturbance at the point in question, but this theory is soon disproved by the fact that when the position of launching is changed the critical points in the flight path move also.

In the trial flights of the 4 gram model it was observed that the model would frequently make a perfectly regular flight measuring approximately 33 feet, Fig. 161, *b* and *c*, and then take a sudden turn to the right, but occasionally it would continue on its flight path undisturbed.

Various theories were tested to account for the peculiarity, but none would bear investigation. It was then noticed that occasionally a slight change of course would take place at a point about

17 feet from the point of launching in the same direction as before, *i.e.*, to the right, and that a further change would follow at the usual 33 feet distance, Fig. 161, *d*; this fact, coupled with direct observation, gave the clue to the anomaly; the alterations of course took place when the aerodone was *cresting*, and although the fact of the flight path oscillation was scarcely visible (so small was the amplitude), yet the aerodone itself could

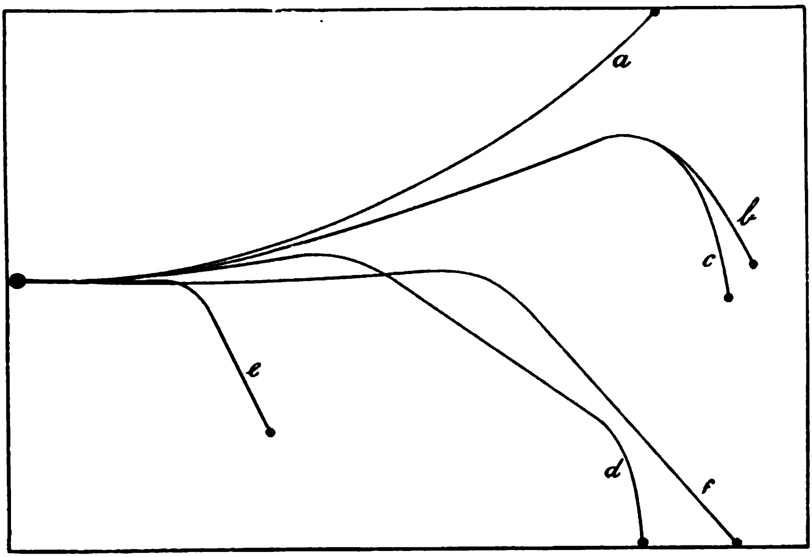


FIG. 161.

feel the change in its velocity, and, with a perversity usually associated only with animate matter, took advantage of the moment of its least velocity to behave in a crooked and disreputable manner.

If the foregoing account of the behaviour of the aerodone were correct, it was evident that the model was being in every case launched from the crest of its flight path, and that, if the velocity of launching were sufficiently increased, the points of deflection should be displaced to fall about halfway between the positions previously observed; the experiment was tried with the result anticipated, Fig. 161, *e* and *f*.

The explanation so far is evidently incomplete, for it has not been made clear in what way the change in the velocity due to the phugoid oscillation is able to effect a change in the direction of flight. It seems evident that if the aerodone were rigid in all its parts it would be impossible to account for a steering effort as due to a change of velocity; on the V^2 law the reactions of the component parts, *i.e.*, aerofoil, fins, etc., should vary precisely as the square of the velocity, and so their directive value would vary proportionally. The effect is evidently too pronounced to be accountable on the basis of the defect of the V^2 law; hence we must either regard it as due to the change of velocity in conjunction with some effect of elasticity, or else as due to something apart from velocity, say the curvature of the phugoid path.

Now the effect is one that it is just possible might be accounted for on the basis of the path curvature. Let us suppose that the fins and the aerofoil are so set as to tend to steer in different directions; then for a straight phugoid they may exactly balance each other, and so long as there is no oscillation the direction will be constant. When, however, the aerodone is cresting, the fins will be travelling through the air faster than the velocity of flight, for they are moving at a greater distance from the centre of path curvature than the mass centre of the aerodone; consequently their influence will be greater and their steering influence will preponderate.

If the feature in the flight path under discussion were observed only when the flight path curvature is considerable, then the foregoing might be regarded as an adequate explanation, but such is not the case; the deviation of the flight path will sometimes make its appearance before the existence of the phugoid oscillation is, by direct observation, at all obvious. It appears to the author that the true explanation is most probably to be found in the elasticity of the aerofoil, on the basis that the two wing extremities are not equally flexible, and under changes of apparent load, such as take place in the undulating flight path,

changes in the form of the aerofoil take place that determine the alterations in the course.

It is evident that a very small change per cent. in the apparent weight may be sufficient to determine the elastic yield of the wing terminals, and since the apparent weight is for small amplitude directly as the value of H, and since H_n is only of about 16 inches magnitude, a fraction of an inch in the path amplitude is a quantity that might make itself felt, and is one that would certainly not be noticed by direct observation. The yield may be an ordinary elastic yield that is greater in one wing than the other, or it may be a sudden change like that of the diaphragm of an oil can, owing to a kinking of the mica where the approximately flat extremity merges into the curved section.[1]

It is evident that if the aerodone possess a stability coefficient greater than unity, the deviation may be expected to take place at the first crest (or trough) of the flight path, or not at all, for the amplitude will diminish at each oscillation. The model on which the present observations were made possessed a value of Φ less than unity both by calculation and by observation; hence the uncertainty as to the point of deviation. The amplitude may be insufficient to bring about a deviation on the first oscillation, but it may have augmented sufficiently by the second or a later oscillation; hence there is some uncertainty as to when the departure from the rectilinear gliding path will manifest itself.

When repeated deviations occur at successive crests of the flight path, the latter becomes of polygonal form; the most the author has witnessed is three sides of such a polygon, but

[1] In the case of the "12 gram" model this sudden yielding of the extremities of the aerofoil by the kinking of the mica was actually observed. This model was originally designed and made with only two ties on each wing, the outer one being added subsequently to give stiffness to the extremities, and so remove the defect. In a low velocity model of this size one can observe every detail in the deportment of the model in a way not possible in an aerodone of small size or high velocity.

doubtless with a large enough arena, and a sufficient time spent in alteration and adjustment, a model could be made to repeat the performance with regularity.[1]

In the case investigated by the author the normal flight path, when not subject to deviation, curved somewhat to the left; this is shown in Fig. 161 (*a*); the flight paths showing deviation are shown diagrammatically, (*b*), (*c*), (*d*), (*e*) and (*f*), the latter being paths when the aerodone is launched in excess of its natural velocity.

[1] There would appear to be no reason why the change should not be made to take place during troughing; except that then the velocity is higher and the stability is greater, it consequently takes more to divert the aerodone from its course.

GLOSSARY

Words additional to those already given in the glossary to Vol. I. are marked.*

AERODONE (Author), from the Greek ἀερο-δόνητος, lit. *tossed in mid air; soaring.* To denote a gliding or soaring model or machine; in particular, any gliding or soaring appliance destitute of propelling apparatus or auxiliary parts.

AERODONETICS (Author, see *aerodone*). The science specifically involved in problems connected with the stability or equilibrium of an aerodone or aerodrome, or of birds in flight, and with the phenomenon of soaring. Equivalent to *Aerodromics*, as proposed by Langley.

AERODROME (Langley), from the Greek ἀερο-δρόμος, lit. *traversing the air; an air runner;* originally proposed to denote a gliding or soaring model or machine, or a flying machine of any kind. *Restricted* by the author to the latter signification; a fully-developed flying appliance; a power-propelled *aerodone,* or an aerodone furnished with directive apparatus.

AERODROMICS (Langley) originally proposed to denote the science concerned in the equilibrium, etc., of an aerodrome; equivalent to *aerodonetics* as used by the author.

AEROFOIL (Author), from the Greek ἀέρος and φυλλον, lit. an *air-leaf.* Denoting the organ of sustentation of an aerodone or aerodrome, or the spread wings of a bird. A supporting member (or members collectively) of undefined form: thus *pterygoid aerofoil,* an aerofoil of wing-like form; *plane aerofoil,* an aeroplane, etc.

APTEROID (Author), from the Greek *α*, *πτερόν* and *ειδος*, the converse of *pterygoid*. Thus *apteroid aspect*, with the greater dimension arranged in the direction of flight; the reverse to that which obtains in the wing plan-form of birds.

ASPECT (Dict.), proposed by Langley in its present usage to denote the arrangement of the plan-form of an aeroplane, or other aerofoil, in relation to the direction of flight.

*ATTITUDE (Dict.), used to denote the position of an aeroplane or aerofoil about a transverse axis relative to the direction of flight; analogous to the term *aspect*. The *attitude* of a given aeroplane or aerofoil is thus defined by its angle β: a change of attitude involves a change in β.

ICHTHYOID (Dict.), fish-shaped, here applied to denote a body of practical stream-line form.

PERIPTERAL. See *Peripter*y.

PERIPTEROID. See *Peripter*y.

PERIPTERY (Dict.), proposed by the author in its present usage as denoting the region round about the wing or in the vicinity of the aerofoil (Greek, *περι* and *πτερόν*), § 107. Hence *peripteral*, as in *peripteral theory*, *peripteral area*; *peripteral zone*; *peripteral motion*, comp. Vol. I., footnote 2, p. viii. (Preface). Hence also *peripteroid* motion (Greek, *περι*, *πτερόν* and *ειδος*), the form of flow proper to the inviscid fluid in a doubly connected region, resulting from the superposition of a cyclic motion on one of translation. Resembling the *motion in the periptery*, lit. *round-about-the-wing-like*.

*PHUGOID THEORY (Author), from the Greek *φυγη* and *ειδος*, lit. *flight-like*.[1] The theory dealing with longitudinal stability and the form of the flight path. Hence also *Phugoid chart*, *Phugoid curve*, *Phugoid oscillation*, etc. (Ch. II.)

[1] The appropriateness of the derivation is perhaps diminished by the fact that the word *φυγη* means flight in the sense of *escape* rather than the act of flying in the present signification.

PTERYGOID (Dict.), *wing-like.* Hence *pterygoid aspect,* with the lesser dimension in the direction of flight, as in the wing plan-form of a bird.

SWEEP (Dict.), proposed by the author in its present usage to denote the cross-sectional area of the stratum of fluid, supposed by hypothesis to be that to whose inertia the supporting reaction is due, § 160.

APPENDIX I

THEORY OF STABILITY—PENAUD, 1870

In § 65 reference has been made to the work of Mons. Penaud in connection with the stability and equilibrium of his flying models, and in particular to an argument advanced by him touching the theory of longitudinal stability.

In an article by him published in "L'Aéronaut," tome v., p. 2, Mons. Penaud not only describes two types of flight model, a "hélicoptère" and an aerodrome (the latter already given in Figs. 9 and 10), but gives also a full statement of his views of the theory of automatic stability. The author feels that, in spite of the rather inadequate nature of the explanation which Penaud was able to offer, the matter is of sufficient historical interest to demand its being quoted *in extenso*. Whatever faults may exist in Penaud's theory, this much is certain: the man himself had a clear understanding that an aerodrome could be made possessing complete stability within itself, without any adventitious aid in the way of equilibrating devices, a fact that since his time has been almost forgotten.

The portion of Mons. Penaud's article that has been thought worthy of reproduction is as follows:—

"Après avoir varié de toutes façons les proportions de mon héliocoptère; après l'avoir vu voler sur place, planer obliquement, s'élever comme un trait, la pensée me vint tout naturellement d'appliquer mon méchanisme à la propulsion d'un appareil du genre aéroplane, de façon a montrer la possibilité de ce système comme l'hélicoptère démontrait la puissance

de l'hélice. . . . Heureusement, après quelques recherches, j'imaginai un organe très simple, remplissant le but desiré.

"C'est un petit gouvernail horizontal incliné vers le dessous de plan sustenteur derrière lequel il se trouve. Ce dispositif peut, en définitive, se rattacher à la propriété que possèdent les surfaces convexes vers le bas, de tomber verticalement, sans chavirer, et à l'aide desquelles M. Joseph Pline réussit même à construire de jolis petits papillons qui planent obliquement vers le sol. Mais dans mon aéroplane il-y-a comme chez l'oiseau, séparation complète de la surface supportante et de la surface dirigeante qui, des lors, ne peuvent plus se nuire,

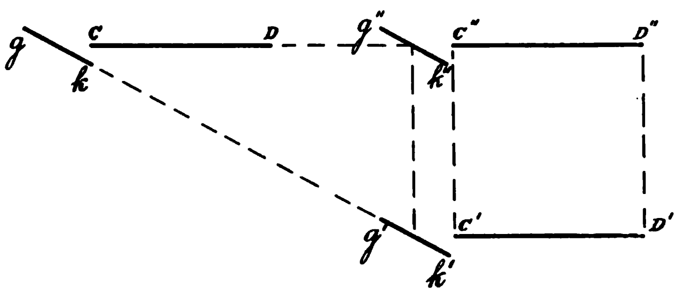


FIG. 162.

la translation est rendue sensiblement plus facile; enfin mon gouvernail représente mieux l'organe évidemment mobile dont serait pourvu un navire aérien conduit par un homme.

"Quoi qu'il en soit, la théorie d'un gouvernail fixe régularisant automatiquement le niveau de l'appareil n'ayant été exposée ni même etudiée par personne de ma connaissance je vais la traiter ici en quelques mots.

"Soit un plan horizontal *c d* chargé d'un poids placé au dessous de son centre de pression, et lié à un gouvernail *g k* incliné vers le dessous de sa surface.

"Supposons qu'il se meuve horizontalement avec une certaine vitesse qui l'amene en *c″ d″* au bout d'un temps donné. Pendant ce même temps il sera tombé, en vertu de la pesanteur d'une

certaine quantité $c'' c'$ dépendant de sa surface et du poids qui le charge en sorte qu'en définitive, $c\ d$ se trouvera au bout du temps considéré en $c'\ d'$ par suite $g\ k\ g'\ k'$; et l'on conçoit qu'en donnant une valeur convenable soit à la vitesse de translation, soit à la chute verticale, soit enfin à l'angle du gouvernail et du plan $g\ k$ puisse se trouver enfin de compte sur le prolongement de $g\ k$ comme sur la figure, en sorte que dans le mouvement réel, il n'aura fait que fendre l'air par sa tranche.

" Supposons maintenant que $c\ d$ s'incline légèrement vers le sol, sa vitesse augmentera, puisque à la force de propulsion se joindra une composante de sa pesanteur; mais alors pour une même chute $c'' c'$ le chemin parcouru horizontalement sera plus grand, la position finale de $c\ d$ et par suite de $g\ k$ sera a droite de $c'\ d'\ g'\ k'$, le gouvernail recevra donc le choc de l'air sur sa partie supérieure, il tendra a s'abaisser, et comme l'appareil tend a tourner autour de son centre de gravité, l'avant d se relèvera, et le plan $c\ d$ reprendra sa position horizontale. Si le plan tend au contraire a se relever; sa vitesse se ralentit aussitôt, le gouvernail reçoit l'air sur sa partie inférieure ce qui ramène encore $c\ d$ dans sa position naturelle en sorte que l'appareil se trouve forcé de descendre suivant la direction $g\ g'$.

" Si l'on suppose que $c\ d$ au lieu d'être absolument horizontal, soit légèrement incliné vers le haut ou vers le bas dans sa position normale, on peut voir avec un peu d'attention, que les mêmes effets du gouvernail se produiront encore de façon que l'on peut obtenir ainsi, soit un vol horizontal soit une montée ou une descente isochrones sous un angle determiné.[1] On peut même obtenir tous ces effets en faisant porter au gouvernail lui-même une partie du poids en placant le centre de gravité un peu vers l'arrière. La surface supportante est ainsi augmentée, ce qui est avantageux."

[1] In this passage Mons. Penaud appears to overlook the fact that an upward course requires a supply of energy or a relatively downward course an increased resistance.

The adjustment suggested would alter the value of V_n, but not the gliding angle, as Penaud supposed.

APPENDIX II

THEORY OF STABILITY; AUTHOR, 1897

The explanation of the automatic longitudinal stability of an aerodone or aerodrome given by the author in his Patent 3608 of 1897, to which reference has been made in the present work, is as follows :—

"When a machine constructed as hereinbefore described travels through the air with a sufficient velocity its weight is supported dynamically by the reaction of the air on the upper and under wing surfaces, the curved form of section developing a region of pressure beneath and rarefaction above in the same manner as a simple inclined plane, but with the advantage of greatly reduced resistance in the line of motion. Now if the velocity of travel is insufficient the air reaction will be deficient, and if the velocity be excessive the air reaction becomes greater than the weight of the machine, and consequently there is for any particular adjustment of a machine a certain speed at which its weight will just be buoyed up so that (so long as this critical speed be maintained) its centre of gravity will continue to move in a straight line; that is to say at this speed (which has already been referred to as the 'natural velocity') there is no tendency of the course of the machine to change. Now let us suppose that the machine without motor be launched horizontally at its 'natural velocity,' then at first no change in its direction of motion takes place, but as its velocity falls owing to the resistance of the air (frictional and otherwise), it gradually assumes a course inclined downwards, and after some oscillation finally settles down

to such an inclination as will by its descent enable it to be supplied with the energy necessary for its propulsion, the steepness of the descent under given conditions depends upon the rate of dissipation of energy by the resistance of the machine ; this angle may be termed the angle of 'recuperation.' If now we suppose the machine to be launched at a speed considerably above its natural velocity, then in the first instance it takes an upward course of gradually increasing inclination till its excess of kinetic energy has been absorbed as potential, the inclination of its course then gradually diminishes till it again becomes horizontal, when, its velocity having become considerably deficient, its course takes a downward trend, and thus it proceeds to describe curves in the air of approximately trochoidal form of gradually diminishing amplitude, slowly settling down to its angle of 'recuperation.' If the launching velocity be excessive (greater than one and a half times the natural velocity in most cases) the machine runs serious risk of being capsized, and the limit of longitudinal stability may be said to be reached somewhere about this point."

"The natural velocity of a machine may be modified at will by altering the relative angle between the wings and tail-plane, thus causing the former to meet the air at a greater or less angle."

"If in the course of its evolutions a machine (constructed as hereinbefore described) heel over sideways one way or the other, or if a rolling motion be set up, the first effect is for the machine to begin to slide down, so to speak, in the direction in which it is for the time being inclined, this motion is very quickly arrested, however, by the resistance of the 'fins' whose centre of pressure is arranged above the centre of gravity of the machine, and equilibrium is thereby restored. A similar result might be brought about by inclining the wings or the tips of the wings upwards to the right and left; but an arrangement of fins is specially valuable owing to its damping action on any side oscillations that may be set up."

"It is easy to understand when a motor is employed for the

purposes of propulsion that according to the magnitude of the thrust transmitted to the machine the 'angle of recuperation' of the machine is diminished, and if the thrust be sufficient the machine will retain its natural velocity with a horizontal course or even an upwardly inclined course, and it will moreover automatically adapt itself to whatever thrust may be applied, so that the effect of an increased thrust is only to increase the velocity of the machine in quite a transitory manner, the final result, after the trochoidal oscillations due to the disturbance have settled down, being a change in the course of the machine in an upward direction."

"Thus, in order to effect a change in the course of the machine in a vertical plane, I operate on the propelling mechanism by either increasing or diminishing the supply of working fluid or by such other means as may be appropriate to the motor employed. I may in certain cases introduce a counter thrust, instead of operating on the motor mechanism, by erecting a small plane or other obstruction perpendicular to the direction of motion of the machine so as to cause it to take a downward course."

"When only a temporary change of direction is required, as when evading an obstacle, the tail-plane may be used with effect for vertical steering, but only within the limits of the safe velocity of the machine."

Fig. 163.

APPENDIX III

THE AUTHOR'S 1894 AERODROME

In Chap. I. mention has been made of an aerodrome constructed by the author in 1894; a representation has been given in Fig. 24, and particulars of a successful flight in § 14. A photograph of this machine is given in Fig. 163, taken at the time of the experiments, and a scale drawing in Fig. 164.

The general construction is evident from the drawing, and follows closely on the lines of the aerodones fully described and figured in § 11. Owing to the greater size and weight of this machine, it is stayed by piano-wire guys in the manner shown, in order to provide longitudinal stiffness both vertically and horizontally.

The propulsion is, as figured, by twin propellers driven by twisted "elastic" (indiarubber), the data of the propelling mechanism being as follows:—

Propellers, 17½ inches diameter; two blades of approximately 6 square inches each; pitch recorded as 16 inches, but on remeasurement it appears to be nearer 20 inches. The construction of the propellers is shown in detail in Fig. 165; the blades are of sheet aluminium ·03 in. thick, (=·75 m.m.), mounted on arms formed by a single steel wire of ·1 in. diameter. The propeller blades are so fitted as to "feather" automatically if the aerodrome overruns the range of the propulsion; that is to say, when the twisted rubber is spent, the blades swivel approximately into the line of flight and do not act as a drag on the machine. This feathering of the blades is accomplished by mounting them

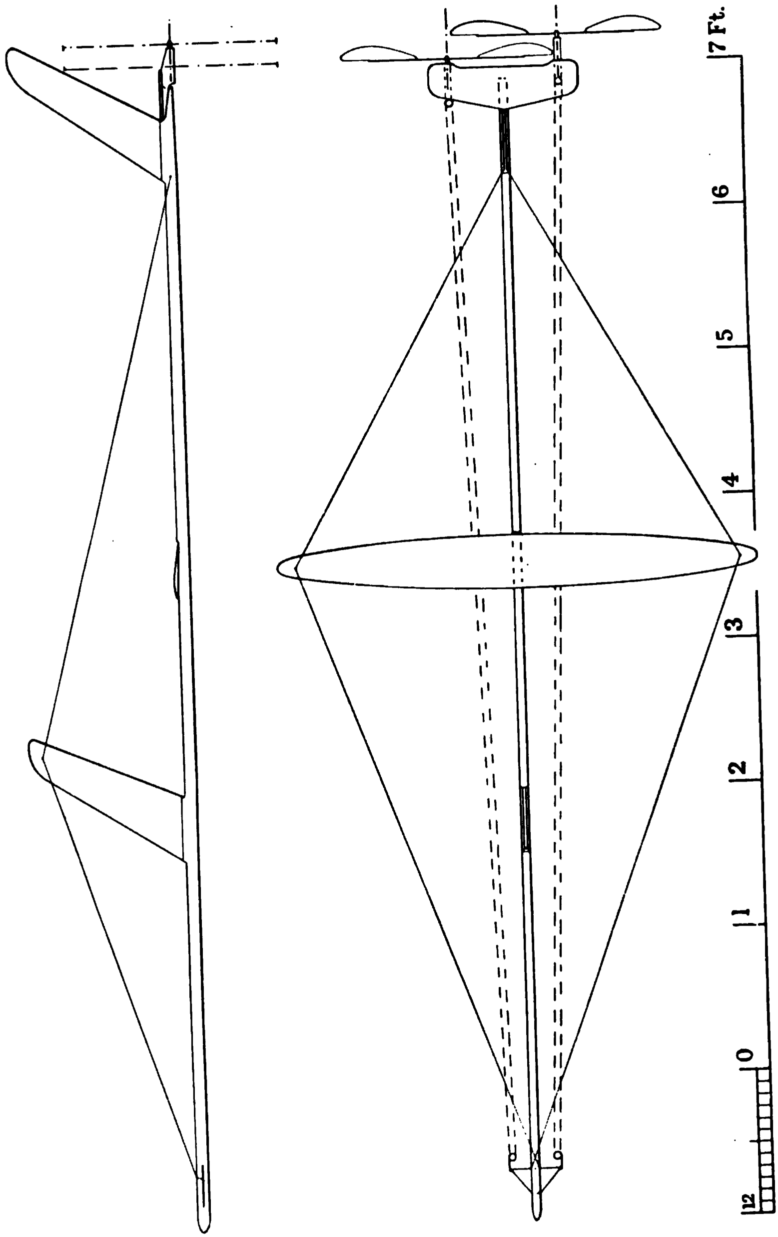


FIG. 164.

to pivot on the wire support (the aluminium being bent to form a hinge), and the provision of a stop consisting of a short strip of brass soldered in position to limit the angular motion and at the same time to locate the blade longitudinally.

Energy of Propulsion.—The energy of propulsion stored in the two indiarubber skeins amounted in all to about 1,000 ft. lbs. (loading energy); the total number of propeller revolutions being 500, representing an average of one foot pound of energy per revolution. This requires to be multiplied by a coefficient to give the energy available at the propeller shaft; in all probability, after allowance also for the propeller efficiency, not more than 50 per cent. of the loading energy is usefully employed in propulsion.

The total weight of rubber in the two skeins is recorded as ·7 lb., each skein being composed of six strands.

General Notes.—The aerodrome weighed with rubber complete 2½ lbs.; the angle made between the flat face of the aerofoil and the tail-plane as first adjusted was 3 degrees, but there is a record to the effect that the angle was subsequently increased to twice this amount, *i.e.*, 6 degrees. The author has no note as to which of these angles was employed when the flight recorded in § 14 was made.

The range of flight should theoretically amount to about 250 yards before the energy of the indiarubber is expended, and there would be probably another 50 or 60 yards covered while the aerodrome is coming to earth. Unfortunately, owing to obstacles, the full range of flight was never realised; the most satisfactory flight is that recorded, but here the machine finished its career prematurely in an elm tree, the propulsion energy being only about half expanded.

An attempt made to photograph the aerodrome in flight proved abortive.

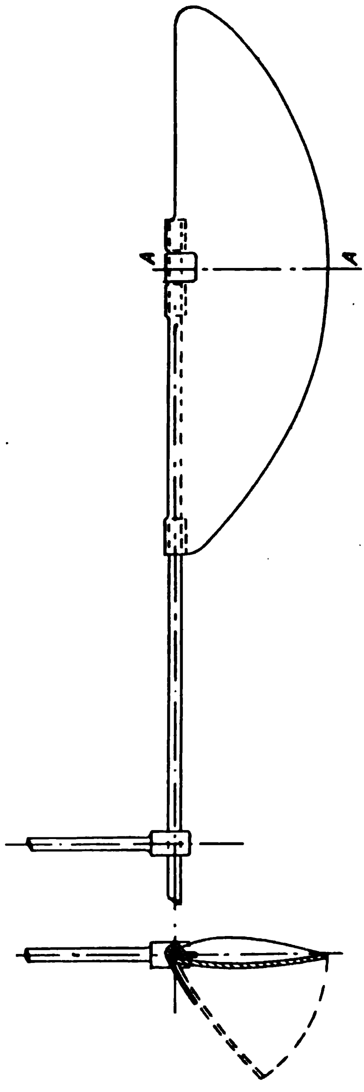


FIG. 165.

APPENDIX IV

SOLUTION OF THE CUBIC EQUATION

Equation (14), where *cos* $\Theta = 1$, *i.e.* for the highest and lowest points of a phugoid, is,

$$\mathbf{C} = \sqrt{H}\left(1 - \frac{H}{3\,H_n}\right)$$

in which **C** is known and it is required to find the value of H;

or
$$\frac{\mathbf{C}}{\sqrt{H}} = 1 - \frac{H}{3\,H_n}$$

$$\therefore \quad \frac{\mathbf{C}}{\sqrt{\left(3\,H_n\,\frac{H}{3\,H_n}\right)}} = 1 - \frac{H}{3\,H_n}.$$

Let $\frac{H}{3\,H_n} = x$; the equation becomes

$$\frac{\mathbf{C}}{\sqrt{3\,H_n\,x}} = 1 - x$$

which can be solved on a slide rule of the "Gravet" type as follows:—

Using the ordinary nomenclature for the four scales,

$$\left\{\begin{matrix} A \\ B \end{matrix}\right. \qquad \left\{\begin{matrix} C \\ D \end{matrix}\right.$$

(1) Set the cursor to **C** on scale D.

(2) Set $3\,H_n$ on scale B to $\mathbf{C}^2$, as indicated on scale A.

(3) Move cursor to 1 on scale B (this divides $\mathbf{C}^2$ by $3\,H_n$).

(4) Set the slide to read the quantity x on scale B at the cursor, so that scale D reads (opposite 1 on scale C) $1 - x$.

(5) Move the cursor back to its original position, and read H required on scale B.

The whole operation of thus solving the equation occupies hardly more than half a minute, and is accurate to within 1 in 500.

When C is negative—in the case of tumbler curves—the quantity $(x + 1)$ is negative, x being greater than unity. This fact does not complicate the slide rule manipulation, but it is convenient to dispose of the anomaly by modifying operation (4) to read:—(4) Set the slide to read the quantity x on scale B, so that scale D reads $x - 1$ (opposite "1" on scale C).

It is evident that since different values of H are given by values of C which differ only in their decimal point, such as C $= 15$, C $= 1{\cdot}5$, C $= {\cdot}15$, etc., it is necessary to watch the change involved in every operation, in the position of the "unit 1" on scale D.

It may be also noted that the inflected curves have two solutions for the value of H, the tumbler curves have each only one; this agrees with what is demanded by the equation.

In order to further demonstrate the method and manipulation of the slide rule in the solution of the cubic equation, the following examples are given:—

(a) *Inflected Curve.* C $= 2{\cdot}71$ $H_n = 64$.

(1) Set cursor to $2{\cdot}71$ on scale D. (Note: The left-hand "1" is "unit 1.")

(2) Divide $2{\cdot}71^2$, as indicated by cursor on scale A, by $3\,H_n = 192$ (on scale B). (Note: The left-hand "1" on scale D becomes ${\cdot}1$ since $1{\cdot}92$ is used for division instead of 192.)

(3) Move cursor to the left-hand "1" on scale B.

(4) Set slide to read simultaneously $x = {\cdot}0417$ on scale B at cursor, and $1 - x = {\cdot}9583$ on scale D opposite "1" on scale C (remember right-hand "1" is "unit").

(5) Replace cursor at 2·71 on scale D and read H on scale B. $H = 8$.

Again, process (1), (2), and (3), as before :—

(4) Set slide to read simultaneously $x = {\cdot}7783$ on scale B at cursor, and $1 - x = {\cdot}2217$ on scale D opposite 1 on scale C (remember still that right-hand "1" is unit).

(5) Replace cursor at 2·71 on scale D and read second value of H on scale B. $H = 149$.

In the curve in question the two values of H when $\cos \Theta = 1$ are :—$H = 8$ and $H = 149$, the highest and lowest points on the curve.

(b) *Tumbler Curve.* $\mathbf{C} = 15{\cdot}21$ $H_n = 64$.

(1) Set cursor to 15·21 on D. (Note : Left-hand "1" is unit.)

(2) Divide $15{\cdot}21^2$ on scale A by 192 on scale B. (Note: The right-hand "1" becomes unit.)

(3) Move the cursor to left-hand 1 on scale B.

(4) On moving slide to obtain a coincidence, we find but one: we read $x = 1{\cdot}815$ on scale B at cursor, and $x - 1 = {\cdot}815$ on scale D opposite "1" on scale C.

(5) Replace cursor at 15·21 on scale D, and read $H = 348$ on scale B.

The relationship between H and $\mathbf{C}$ when $\cos \Theta = 1$ may also be expressed graphically. In Fig. 166 ordinates $= H$, and abscissae $= \mathbf{C}$; the curve is plotted to half the scale of Fig. 42, $H_n = 64$.

This curve not only gives at once the relationship between H and $\mathbf{C}$ when $\cos \Theta = +1$ for phugoids of all amplitudes (within the limits of the plotting), but also exhibits clearly the existence of the two values of H for every positive value of $\mathbf{C}$ (that is, for values of H between zero and $3\,H_n$), and the single value only when $\mathbf{C}$ becomes of negative sign.

A curve is also plotted for *cos* $\Theta = -1$, (dotted line); here **C** is of negative sign throughout.

Let us draw, for any value of **C**, an ordinate cutting the curves at two points, then :—

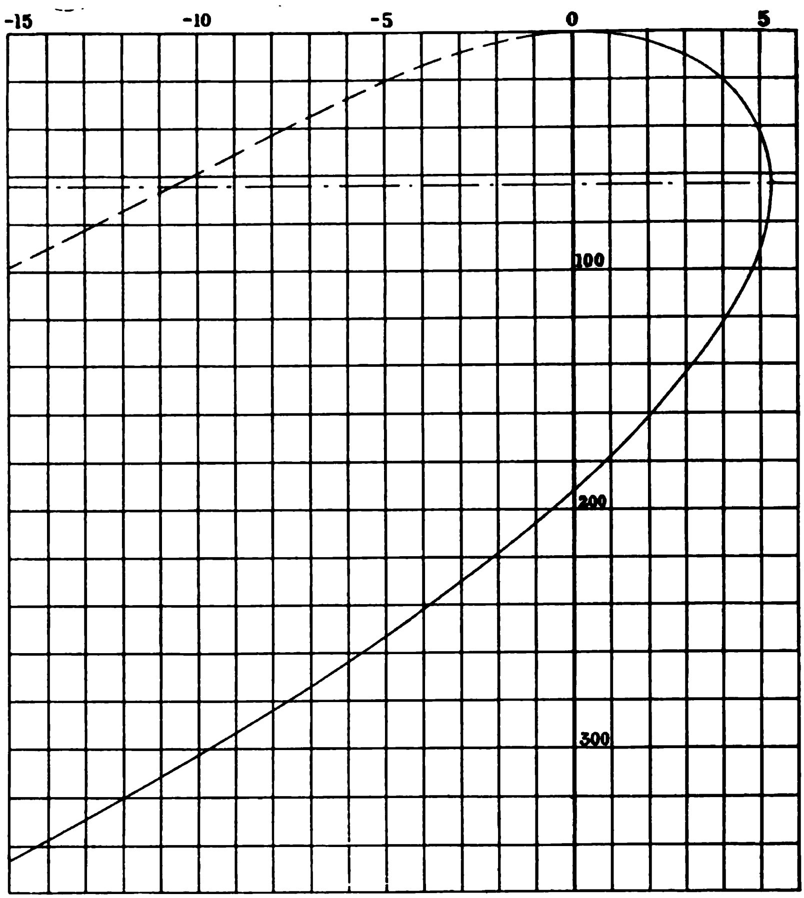


Fig. 166.

(a) If **C** is positive, we have, cut off on the ordinate by curve *cos* $\Theta = 1$, the maximum and minimum value of H for an inflected curve.

(b) If **C** is negative, we have, cut off by curve *cos* $\Theta = 1$, the

maximum, and by curve $\cos \Theta = -1$, the minimum value of H for a curve of "tumbler" type.

These curves may be initially plotted by assuming values for H, and calculating the corresponding value of **C**, thereby evading the necessity for solving the equation in the opposite sense, the curve may then be used in the manner indicated, and the solution of the cubic equation as such is avoided.

APPENDIX V

CALCULATIONS FOR THE PLOTTING OF THE PHUGOID CHART

(The reference numbers correspond to those given in Fig. 42 and the Frontispiece.)

Curve No. 1 is the special case of the straight flight path.

Curves Nos. 2 and 3 are plotted by the approximate method of § 31.

Curve No. 4. (Inflected type.)

$$H_n = 64. \qquad H_1 = 25. \qquad Cos\ \Theta_1 = 1.$$

$$\mathsf{C} = \sqrt{H_1}\left(cos\ \Theta_1 - \frac{H_1}{3\ H_n}\right)$$

$$= 5 \times \left(1 - \frac{25}{192}\right) = 4{\cdot}35.$$

For highest and lowest points on the curve, $cos\ \Theta_1 = 1$, and

$$\frac{H}{192} + \frac{4{\cdot}35}{\sqrt{H}} = 1$$

the solution of which gives alternative values of H (compare § 28).

$H = 25$, highest point of curve (already known).

$H = 113{\cdot}5$, lowest point on curve (value required).

Point of Inflection.—This is given by § 28, Equation (15).

$$H_i = \left(\frac{3\ H_n\ \mathsf{C}}{2}\right)^{\frac{2}{3}}$$

$$= \left(\frac{192 \times 4{\cdot}35}{2}\right)^{2} = 56$$

And by Equation 16, § 28,

$$cos\ \Theta_i = \frac{56}{192} + \frac{4{\cdot}35}{\sqrt{56}} = {\cdot}873$$

or $\Theta_i\quad 29^\circ\ 10'.$

Curve No. 5. (Inflected type.)

$$H_n = 64. \qquad H_1 = 16. \qquad Cos\ \Theta_1 = 1.$$

Vide § 28.

Curve No. 6.

$$H_n = 64. \qquad H_1 = 8. \qquad Cos\ \Theta_1 = 1.$$

$$\mathsf{C} = \sqrt{8} \times \left(1 - \frac{8}{192}\right) = 2{\cdot}71.$$

Lowest point on curve,

$$\frac{H}{192} + \frac{2{\cdot}71}{\sqrt{H}} = 1$$

whence $H\ (\text{max.}) = 148{\cdot}7.$

Point of Inflection.

$$H_i = \left(\frac{192 \times 2{\cdot}71}{2}\right)^{\frac{2}{3}} = 40{\cdot}75$$

and $$cos\ \Theta_i = \frac{40{\cdot}75}{192} + \frac{2{\cdot}71}{\sqrt{40{\cdot}75}} = {\cdot}636$$

or $\Theta_i = 50^\circ\ 30'.$

Curve No. 7. Special case. Semicircle.

Curve No. 8. (Tumbler.)

$$H_n = 64. \qquad H_1 = 8. \qquad Cos\ \Theta_1 = -\ 1.$$

$$\mathsf{C} = \sqrt{8} \times \left(-1 - \frac{8}{192}\right) = -\ 2{\cdot}946.$$

Lowest point on curve,[1]

$$\frac{H}{192} + \frac{-\ 2{\cdot}946}{\sqrt{H}} = 1$$

whence $H = 229.$

[1] The lowest point on the tumbler curve, *i.e.*, when $cos\ \Theta = +\ 1$, is calculated as a check on the plotting.

Curve No. 9. (Tumbler.)

$$H_n = 64. \qquad H_1 = 16. \qquad Cos\ \Theta_1 = -1.$$

$$\mathsf{C} = 4 \times \left(-1 - \frac{16}{192} \right) = -4{\cdot}333.$$

Lowest point on curve,

$$\frac{H}{192} + \frac{-4{\cdot}333}{\sqrt{H}} = 1$$

whence $$H = 245.$$

Curve No. 10. (Tumbler.)

$$H_n = 64. \qquad H_1 = 36. \qquad Cos\ \Theta_1 = -1.$$

$$\mathsf{C} = 6 \times \left(-1 - \frac{36}{192} \right) = -7{\cdot}125.$$

Lowest point on curve,

$$\frac{H}{192} + \frac{-7{\cdot}125}{\sqrt{H}} = 1$$

whence $$H = 275.$$

Curve No. 11. (Tumbler.)

$$H_n = 64. \qquad H_1 = 50. \qquad Cos\ \Theta_1 = -1.$$

$$\mathsf{C} = \sqrt{50} \times \left(-1 - \frac{50}{192} \right) = -8{\cdot}91.$$

Lowest point on curve,

$$\frac{H}{192} + \frac{-8{\cdot}91}{\sqrt{H}} = 1$$

whence $$H = 292.$$

Curve No. 12.

$$H_n = 64. \qquad H_1 = 100. \qquad Cos\ \Theta_1 = -1.$$

$$\mathsf{C} = 10 \times \left(-1 - \frac{100}{192} \right) = -15{\cdot}21.$$

Lowest point on curve,

$$\frac{H}{192} + \frac{-15{\cdot}21}{\sqrt{H}} = 1$$

whence $$H = 348.$$

The calculated values of r for the preparation of trammels for the plotting of the curves as above are appended.

4.		5.		6.		8.	
H	*r*	*H*	*r*	*H*	*r*	*H*	*r*
25	82·1	16	42·5	8	18·2	8	14·2
26	89·3	18	51·1	9	22·2	9	16·7
27	97·1	20	65·2	10	26·6	10	19·3
28	105·5	22·5	83·0	11	31·3	11	22·0
29	115·0	25	105·2	12	36·5	12	24·6
30	124·3	27·5	132·6	13	42·1	13	27·3
31	135·1	30	167·6	14	48·4	14	30·0
32	146·7	32·5	222·0	15	55·1	15	32·8
33	158·9	35	274	16	62·6	16	35·4
34	173·2	37·5	358	17	70·7	17	38·1
35	188·9	40	490	18	79·8	18	40·8
36	206	45	1155	19	89·7	19	43·5
37	224	49·8	∞	20	100	20	46·1
38	245	55	1408	25	177	22	51·3
39	269	60	794	30	329	24	56·3
40	295	65	585	40·75	∞	26	61·3
45	500	70	482	55	529	28	66·0
56	∞	75	419	60	435	30	70·5
70	668	80	378	65	382	32	74·8
75	538	85	348	70	345	34	79·2
80	461	90	328	75	320	36	83·2
85	411	95	310	80	302	38	87·0
90	376	100	296	85	286	40	90·7
95	350	110	276	90	276	45	99·1
100	330	120	262	95	267	50	106·6
110	301	129·9	252	100	260	55	113·4
113·5	293			110	248	60	119·3
				120	239	65	124·6
				130	233	70	129·5
				140	227	75	133·7
				148·7	224	80	137·5
						85	141·0
						90	144·1
						95	147·0
						100	149·7
						110	154·1
						120	158·0
						130	161·2
						140	164·0
						150	166·4
						160	168·5
						170	170·1
						180	171·8
						190	173·1
						200	174·4
						210	175·8
						220	176·8
						229	177·4

9.		10.		11.		12.	
H	*r*	*H*	*r*	*H*	*r*	*H*	*r*
16	25·6	36	46·1	50	56·2	100	78·1
17	27·6	38	48·9	52	58·4	104	80·7
18	29·8	40	51·9	54	60·8	108	83·4
19	31·9	42	54·6	56	63·0	110	84·8
20	34·0	44	57·4	58	65·3	115	88·0
22	37·7	46	60·1	60	67·7	120	91·0
24	42·3	48	62·7	65	73·0	125	93·9
26	46·4	50	65·4	70	78·0	130	96·8
28	50·4	55	71·8	75	83·0	135	99·4
30	54·3	60	77·7	80	87·6	140	102·0
35	63·8	65	83·3	85	91·7	145	104·5
40	72·6	70	88·5	90	96·0	150	107·0
45	80·8	75	93·5	95	99·8	155	109·25
50	88·2	80	98·0	100	103·5	160	111·5
55	95·0	85	102·5	110	110·3	165	113·65
60	101·3	90	106·5	120	116·3	170	115·65
65	107·0	95	110·3	130	121·8	175	117·65
70	112·2	100	114·0	140	126·6	180	119·6
75	117·0	110	120·4	150	131·0	185	121·5
80	121·3	120	126·2	160	135·0	190	123·3
85	125·3	130	131·4	170	138·5	195	125·0
90	129·0	140	135·9	180	141·7	200	126·6
95	132·5	150	140·0	190	144·8	210	129·8
100	135·5	160	143·5	200	147·5	220	132·6
110	141·0	170	146·6	210	149·8	230	135·3
120	145·7	180	149·5	220	152·0	240	137·9
130	149·8	190	152·1	230	154·0	250	140·1
140	153·2	200	154·5	240	156·1	260	142·3
150	156·4	210	156·7	250	158·0	270	144·4
160	159·3	220	158·7	260	159·2	280	146·4
170	161·5	230	160·5	270	161·0	290	148·2
180	163·6	240	162·2	280	162·4	300	149·8
190	165·5	250	163·6	290	163·6	310	151·4
200	167·2	275	166·9			320	152·9
210	168·8					330	154·3
220	170·2					340	155·6
230	171·5					348	156·7
240	172·6						
245	—						

APPENDIX VI

MOMENT OF INERTIA

THE METHOD OF DOUBLE SUSPENSION

In certain cases, as for example when determining the moment of inertia of the body of a bird, it is not convenient to employ the method of § 172 owing to the difficulty of accurately locating the centre of gravity. In such cases the *method of double suspension* may be employed, the value of λ^2 being deduced from two determinations of the time period, made with two different points of suspension a known distance apart.

When a single determination of the time period is made, the length of the pendulum (*i.e.*, the l of § 172) being unknown, a curve may be plotted for a series of assumed values of l, the ordinates representing the values of λ^2 corresponding to values of l as abscissae. The resulting curve is a parabola passing through the origin and through a point distant from the origin by an amount equal to the length of a *simple pendulum* corresponding to the observed period.

By plotting two curves from origins O_1 and O_2, Fig. 167, separated by the distance apart of the two points of suspension, the value of λ^2 is given by the point of intersection; this is the only value of λ^2 consistent with both observations.

In Fig. 167, the line $O_1\,O_2$ represents the distance between the two alternative points of suspension, and the two curves representing possible values of λ^2 are found to intersect in the manner shown, giving the required value of λ^2. The plotting of these two curves may be effected over just the necessary portion

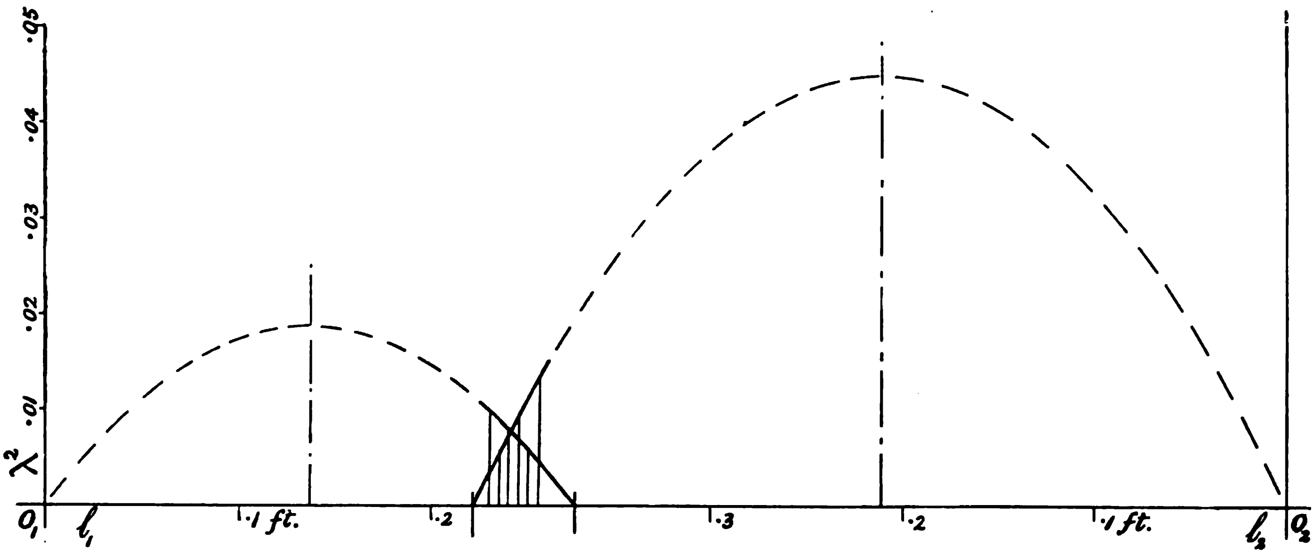


FIG. 167.

of the curves (where they overlap) as shown. In the example given the calculation is as follows :—

Suspension from origin O_1 gives $t_1 = \cdot 29$.

Now, $\lambda^2 = l\left(g\left(\frac{t}{\pi}\right)^2 - l\right)$, from which it is evident that λ falls to zero when $l_1 = o$ (*i.e.*, at origin), and when $l_1 = g\left(\frac{t_1}{\pi}\right)^2$ that is

$$l_1 = 32{\cdot}2\left(\frac{\cdot 29}{3{\cdot}14}\right)^2 = \cdot 274.$$

Suspension from O_2, $t = \cdot 36$

$$g\left(\frac{t}{\pi}\right)^2 = 32{\cdot}2\left(\frac{\cdot 36}{3{\cdot}14}\right) = \cdot 424.$$

These values give l_1 and l_2 respectively for $\lambda^2 = o$ laid off from O_1 and O_2. The intersection evidently lies between these points. We proceed to plot this portion of each curve as follows :—

l_1.		λ^2
·25	$\lambda^2 = \cdot 25 \times (\cdot 274 - \cdot 25)$	$= \cdot 0060$
·24	$\lambda^2 = \cdot 24 \times (\cdot 274 - \cdot 24)$	$= \cdot 0081$
·23	$\lambda^2 = \cdot 23 \times (\cdot 274 - \cdot 23)$	$= \cdot 0101$

And similarly for the curve of l_2. The ordinates representing these plottings are shown in the figure.

APPENDIX VII

THE GYROSCOPE

THE importance of the gyroscope in connection with the present subject is due to the fact that in certain actual cases, as in the flight of a projectile from a rifled gun, and in the flight of a boomerang, gyroscopic action plays an important part; and further, the gyroscope has actually been proposed as a means of securing stability in the flight of an aeronautical machine.

In view of these facts, and in view of the contingent possibility of the employment of gyroscopic action as a means of improving the stability of a flying machine, it has been thought desirable to include in the present work a discussion of the theory of the gyroscope, and an account of some of the more important of its present applications.

It is usual to treat the gyroscope and problems involving gyroscopic action exclusively by mathematical methods, in which the physical reality of the problem is partially obscured. One result of this is that, in spite of the unimpeachable nature of the said methods, mistakes are not infrequently made, even the direction of the gyroscopic torque is sometimes misstated.[1]

The author has for many years employed a direct method of

[1] In his "Dynamo Electric Machinery" (4th ed.), p. 388, Silvanus P. Thompson, discussing the gyroscopic couple, says:—". . . Then F = 30·6 lbs. on each bearing alternately acting up and down at each roll, if the axis of the dynamo lies athwart the ship." The actual direction of the gyroscopic couple is about an axis at right angles to the forced precession; in the case in point the resulting stresses on the bearings are horizontal, not vertical as stated.

dealing with the gyroscope which has many points in its favour. The account that follows forms a portion of a paper read before the Institution of Automobile Engineers.[1]

"The fundamental principle employed in the following exposition is termed the *principle of the conservation of angular momentum.* This is the rotational analogue of the conservation of (linear) momentum, which is corollary to the third law of motion—*When force acts on a body, the momentum generated in unit time is proportional to the force.* The rotational form of this law is—*When a couple or 'torque' acts on a body about a given axis the angular momentum generated per unit time about that axis is proportional to the magnitude of the torque.*

"In both cases the term 'body' may be construed as including not only a rigid body, or body in one piece, but a complex body of any kind whatever consisting of an indefinite number of parts associated amongst themselves by any known or unknown laws of attraction or repulsion. In other words, for *body* we may substitute the term *self-contained system*—such a system being one not associated with its surroundings in any way whatever except as regards the specified applied force, or torque, as the case may be.

"As it is important that the signification of the above principles should be made quite clear, an illustration will not be out of place. A well-known application of the principle of the conservation of momentum is the case of the gun and projectile: when a gun is mounted so that it is quite *free* to recoil, in fact, so that we may regard it as constituting with its charge and projectile a self-contained system, the firing of the charge has no influence on the motion of the mass centre of the system, so that when the shot and powder gases are projected forward the gun recoils backward, the minus momentum of the gun exactly neutralising the plus momentum of the charge. In order to reduce this problem to a nice simple form, it is usual to suppose that the

[1] "Some Problems Peculiar to the Design of the Automobile," Proc. Inst. A.E., read March, 1908.

powder gases are without mass, to imagine, in fact, that the projectile is discharged by a repulsive force instead of by gaseous pressure. As so presented the problem is reduced to simple arithmetic: thus, if the projectile weigh 1 oz. and the gun 100 ozs., and if the muzzle velocity of the shot be 2,000 ft./secs., the velocity of recoil is $\frac{1}{100} \times 2{,}000 = 20$ ft./secs.

"We may extend the above to deal with the spin of the projectile and the influence of the said spin on the motion of the gun. Thus, when the gun is fired, since there is no torque applied from without, the angular momentum of the self-contained system can undergo no change, and the angular momentum received by the bullet by virtue of the rifling will be exactly neutralised by the opposite angular momentum imparted to the gun. Thus, if the moment of inertia of the gun be 500 times that of the bullet, the angular velocity of the bullet will be 500 times that of the gun, the rotation being, of course, in opposite directions. It may be remarked that the question of the powder gases does not sensibly affect the rotational problem.

"Now in the foregoing illustrations the motions are confined to one line in the case of the linear problem, and about one axis in the analogous case of rotation, but the principles apply equally if the motions involve movements simultaneously along or about the other two co-ordinate axes.

"We will now apply the principle of the conservation of angular momentum to the study of the gyroscope. Taking the case of the ordinary lecture model (Fig. 168), in which the fly-wheel is mounted freely in gimbals, it may at once be noted that the mounting is such as to exercise no kind of rotational restraint on the spinning wheel; the latter is arranged, rotationally speaking, as a *self-contained system*. Thus, a movement of the stand which forms the mounting, although taking effect translationally on the gyroscope, has no effect rotationally, except so far as may be due to the friction of the gimbals, which is of the nature of a mechanical defect.

"If a torque or couple be applied to the axis of the wheel, or to

the hoop A, which in effect is the same, so long as the wheel is not in rotation, an angular velocity will be communicated about the axis of the applied torque in accordance with the law, the

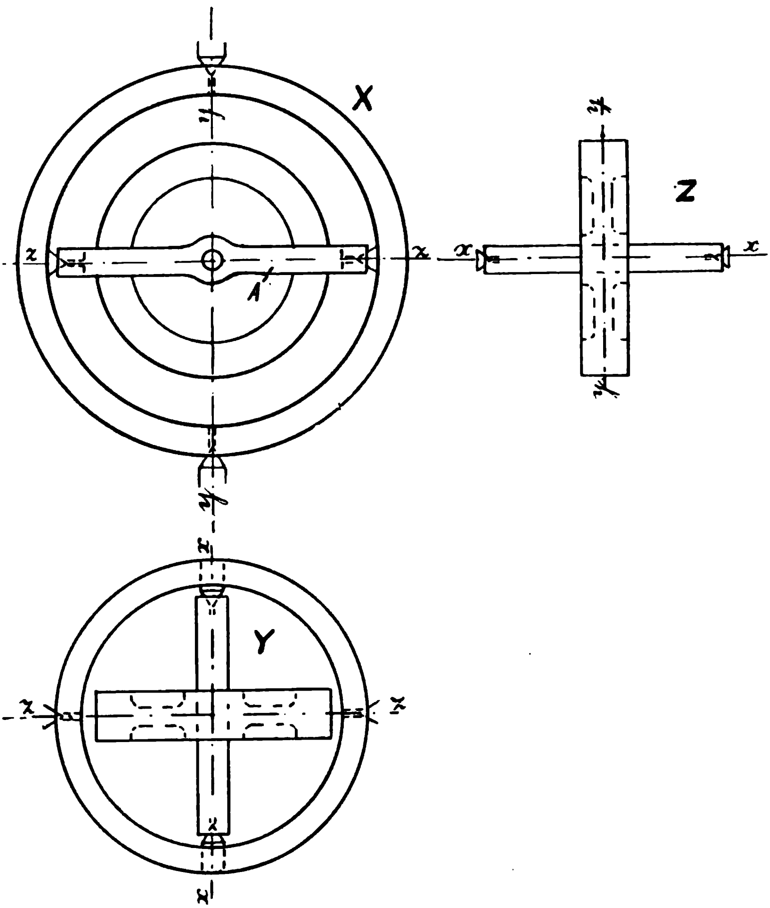


FIG. 168.

angular momentum **I** ω (where **I** is moment of inertia and ω is angular velocity), being equal to the applied torque multiplied by the time of its application; let us, however, suppose that the wheel be set in rapid rotation and examine the conditions that supervene.

" In Fig. 168, three co-ordinate diagrammatic projections of the gyroscope are given, which will be referred to as the X, Y, or Z view, according to the axis along which the view is supposed to be taken. A similar convention applies to the further diagrammatic Fig. 169.

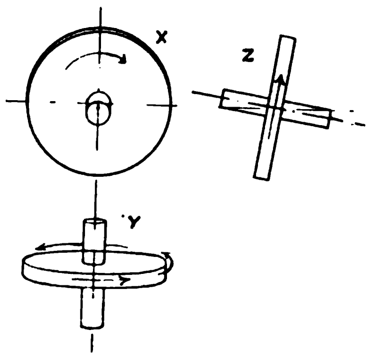


Fig. 169.

" Let us suppose now a clockwise torque about the axis of z, and let us assume provisionally that the effect of this torque is, as before, a movement in like direction about that axis (Fig. 169). Then in the Y view we have evidently an immediate development of angular momentum about the axis of y, *an axis about which no torque or couple exists;* but we know this is impossible, hence the provisional assumption is at fault, and the torque applied about the axis of z cannot give rise to motion about that axis.

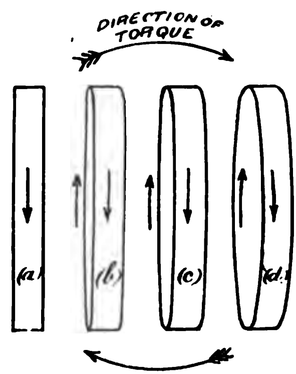


Fig. 170.

" We may term the angular momentum that makes its appearance about any axis owing to the movement of a rotating wheel, as in the above example, *angular momentum of aspect* owing to its being due to a change of *aspect* of the spinning wheel; thus, if in Fig. 170, (a) represents a spinning wheel in edge aspect, it will require the application of a clockwise torque to bring it into the successive positions (b), (c), (d), etc., this torque being, as indicated, about an axis at right angles to the plane of the paper.

"Now the motion supposed to take place in Fig. 169 corresponds

in every way to the requirements of a counter-clock torque applied about the axis of *y*; it is evident that if a torque be so applied, the angular momentum of the spinning wheel (in the motion depicted) makes its appearance in a manner that will fulfil the requirements of the principle of conservation. This reasoning may be extended to constitute a definite proof.

"Let a torque be applied about the axis of *y*, then since this torque has no component about the axis of *z*, and since there is no independent torque about that axis, there can be no change of angular momentum about that axis; consequently the motion of the gyroscope must be such as will not display momentum of

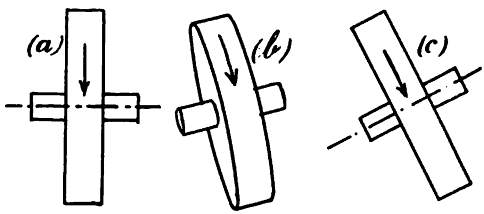


FIG. 171.

aspect *in Z view* (Fig. 169), and therefore it can only take place about either the axis of *x* or the axis of *z*. But the former is the axis of rotation, and change of momentum about this axis requires a torque about this axis which does not exist; therefore the only axis about which motion can take place is that of *z*, so that when a torque is applied about the axis of *y*, either there will be motion about the axis of *z*, or there will be no motion at all. But an unbalanced torque cannot act on a self-contained system without giving rise to angular momentum about the axis of its application, hence there must be motion, and it will be about the axis of *z*.

"We will now deal with the problem quantitatively. Let us first examine the question of angular momentum as due to aspect, with a view to expressing it in terms of the angular momentum of the wheel about its axis of rotation.

"In Fig. 172, let us represent the torque by which the angular momentum of the wheel may be imparted in unit time by the couple A B, and let each of the forces constituting the couple A B be resolved on to any two co-ordinate axes x and y into the two couples a_1 b_1 and a_2 b_2, this is merely a matter of resolving each of the two forces A and B by the usual parallelogram. Then the resolved couples are together in every respect the equivalent of the parent couple, and the angular momentum generated by

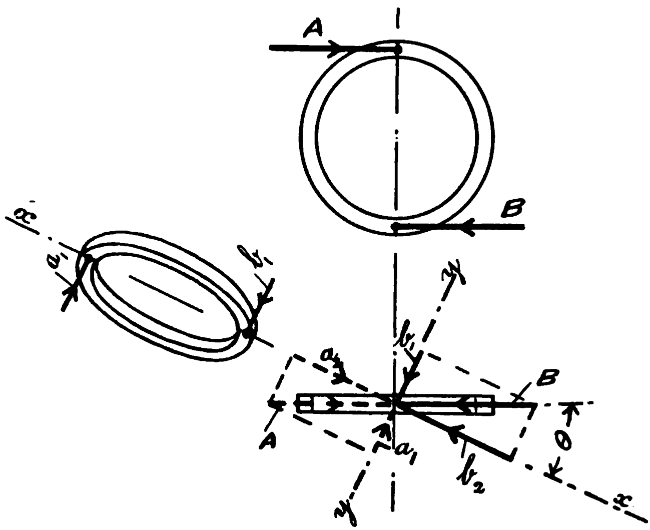


FIG. 172.

the couple a_1 b_1 will be to that generated by A B as the magnitudes of the individual forces a_1 b_1 are to the individual forces A B; therefore the angular momentum about the axis x is to the angular momentum about the axis of spin as the force a_1 is to the force A, or as the force b_1 is to the force B, that is, if θ be the angle made by the axis x to the plane of rotation, and if $\mathbf{I}$ be the moment of inertia and ω the angular velocity of spin of the wheel, so that $\mathbf{I}\,\omega$ is the angular momentum, we have the *angular momentum of aspect* $= \mathbf{I}\,\omega \sin\theta$.

"Let us represent the rate of change in the angle θ the symbol Ω, that is, $\Omega = \frac{d\theta}{dt}$, and is the angular velocity of precession.

"Now rate of communication of momentum due to change of aspect is $\frac{d\,(\mathrm{I}\,\omega\,sin\,\theta)}{dt}$, which, since $\mathrm{I}\,\omega$ is constant $= \mathrm{I}\,\omega\,\frac{d\,sin\,\theta}{dt}$ At the instant when $\theta =$ zero (that is, when the wheel is in edgewise aspect), this expression becomes $\mathrm{I}\,\omega\frac{d\theta}{dt}$ (θ being expressed in circular measure), which substituting for $\frac{d\theta}{dt}$ becomes $\mathrm{I}\,\omega\,\Omega$.

"But if we are employing absolute units, the gyroscopic torque, which we will denote by the symbol τ, is equal to the angular momentum communicated per second, that is,

$$\tau = \mathrm{I}\,\omega\,\Omega,$$

which is the well-known expression.

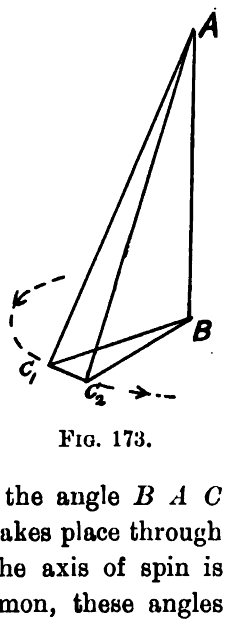


FIG. 173.

"The above deals immediately with the case of a precession about an axis at right angles to the axis of spin; it is easily extended, however, to include precession about an axis inclined at an angle. Thus in Fig. 173, let $A\,B$ represent the axis of precession, and let $A\,C_1\,A\,C_2$ represent two successive positions of the axis of spin; let the angle $B\,A\,C$ be denoted by a. Now while the precession takes place through the angle $C_1\,B\,C_2$, the angular motion of the axis of spin is denoted by $C_1\,A\,C_2$, and since $C_1\,C_2$ is common, these angles are in the relation $A\,C : C\,B$, that is, as 1 is to $sin\ a$, hence expression becomes:—

$$\tau = \mathrm{I}\,\omega\,\Omega\,sin\,a.$$

"There is one little difficulty still to discuss. In Fig. 169, the precessional motion about the axis of z exhibits direct angular momentum about that axis, and although this is constant after the motion is once initiated, it is at first

sight difficult to see how, under the conditions, it can arise. It is quite certain that if such angular momentum exists its initiation must involve a torque about the axis of z, which the conditions of free mounting definitely preclude. Hence we may boldly declare that the momentum that apparently must be there cannot exist. The precession of the wheel in Z view we cannot deny; such precession evidently involves angular momentum about z which is absent when the torque about y is removed and the precession ceases; and there is no torque at any time about the axis of z; the only solution is evidently that the system contains equal and opposite momentum about z somehow or other; and the only form in which such can exist is *momentum of aspect*. Here we have the solution: the commencement of the precession is marked by a slight yielding directly to the applied couple, and the termination of the precession is marked by the reverse effect.[1] Thus, in Fig. 171, (a) represents the wheel before the application of the torque; (b) represents the conditions during the application of the torque; and (c) after the torque is removed.

"We thus have some insight as to the mode of development of the precessional motion about the axis at right angles to the torque; there is an actual yielding when the torque is applied during a short period in which the precessional motion is generated, the extent of this yielding being just that necessary to give the equal and opposite momentum of aspect about the precessional axis. It is thus very little more than a matter of ordinary arithmetic to calculate the extent of the yielding that will take place under any stated conditions, a problem that as a matter of analytical dynamics can scarcely be regarded as simple."

[1] It is assumed that the couple is applied and withdrawn gradually, otherwise a vibration is set up.

APPENDIX VIIIA

THE RIFLED PROJECTILE

In the flight of a projectile fired from a rifled gun two influences are at work of an entirely different kind tending to modify the form of the trajectory[1]; these are gyroscopic action and the aerodynamic action dealt with in Vol. I. § 30, as an effect of discontinuity in the fluid motion on the flight of a ball in rotation.

Before the days of rifling the second of these influences alone took effect on the flight of the spherical bullets then in use, and as the accidental rotation that the ball acquired in the barrel of the gun was an unknown quantity, so was the resulting aerodynamic reaction, and the accuracy of the weapon was little better than that of the crossbow it superseded. About the middle of the 18th century Robins showed that a bend in the barrel would produce a rotation of the ball owing to the friction due to its centrifugal force, and that this rotation, after a comparatively short flight, would more than compensate for the direct effect of the bend; he further advocated *rifling* (a well-known expedient even then) as a remedy for the defects of the musket of that time.

When a spherical bullet is fired from a rifled gun the axis of rotation remains sensibly constant in direction, for the aerodynamic forces are initially symmetrical; the result of this is that at first the flight path is a simple trajectory, and thus the bullet is very soon travelling at an angle to its axis of

[1] In addition of course to the direct aerial resistance.

rotation and then the considerations of Vol. I. § 30, begin to have weight, and the ball moves laterally under the influence of the modified aerodynamic system. If the rifling is right-handed, *i.e.*, like a corkscrew, the wake current will be thrown to the right, and the reaction on the ball and its consequent drift are to the left. Conversely, if the rifling is left-handed the reaction on the ball and drift will be to the right.

In the case of elongate projectiles of the forms commonly employed, the conditions are different, and gyroscopic considerations require to be taken into account. Thus after a short distance has been traversed, owing to the curvature of the trajectory the direction of motion no longer coincides with the axis of rotation (Fig. 174), there is then a couple tending to turn the projectile into a broadside-on position,[1] and owing to the rotation this produces a precessional effect. It is evident that for an elongate projectile the end-on attitude is one of unstable equilibrium, just as in the case of a spinning top standing upon its peg; and that stability is given in both cases by the rotation. As in the case of the spinning top also, the effect of the gyroscopic precession is to make the axis of spin describe a cone about the axis of equilibrium, *i.e.*, about the axis of flight.

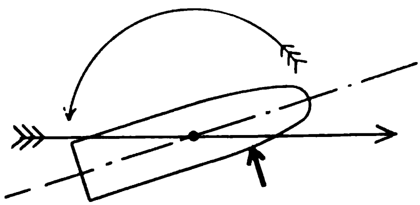


FIG. 174.

The condition of a steady state evidently requires that the projectile shall move in a spiral path, for the reaction of the air due to the inclination of the projectile to the line of flight causes a continual change in the direction of motion, and as this force is in a direction passing through the axis of precession, the equal

[1] This we know from aerodynamic considerations, the centre of lateral resistance is in front of the centre of figure, and so is in front of the centre of gravity.

and opposite inertia force will be the centrifugal force of a circular motion; the latter superposed on the motion of translation becomes a spiral. This condition is illustrated diagrammatically in Fig. 175, in which the plane of the paper contains the axis of the projectile for the time being; the motion will be of

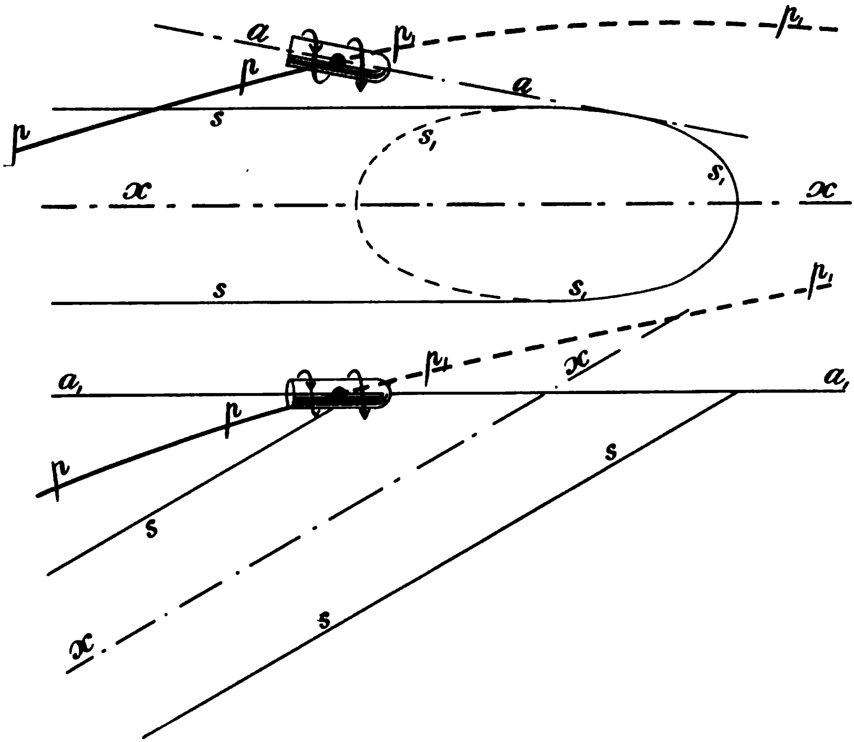


FIG. 175.

spiral form, but owing to the fact that the centrifugal force must be truly radial, the axis of rotation will not intersect the axis of flight, but will be tangent to a concentric cylinder in the manner shown.

In Fig. 175, x x is the axis of flight, and p p p_1 p_1 the spiral flight path. The axis of the projectile a a is directed inwards at an angle as shown is tangent to a cylindrical surface, s s s, whose

trace on the plane $a_1\ a_1$, containing the axis of the projectile, is $s_1\ s_1\ s_1$.

If there are no forces tending either to retard or hurry the precessional motion of the projectile in its spiral flight path, then that path will preserve its initial amplitude (except so far as the conditions may be affected by the falling off of the velocity owing to the resistance to flight). If forces exist tending to retard the precession, then as in the simple gyroscope the projectile will slowly yield to the applied torque and the amplitude of the spiral path will increase, in such a case the flight path would be "wild." If, on the contrary, there are forces tending to hurry the precession, the projectile will, as in the case of a common spinning top, move in opposition to the applied torque, and the amplitude of the spiral will diminish, the flight path settling down gradually to a closer and closer approximation to a smooth trajectory.[1]

[1] The damping of the amplitude of the spiral flight path is only a definite consequence of the diminution of the angle a by virtue of the law correlating the lateral pressure on the projectile and the angle a. If, in Fig. 200, a be the inclination of the axis $a\ a$ to the line of flight at any instant, and if the centre of lateral resistance for small angles be assumed constant in position, and situated a distance l front of the centre of gravity, then the torque will be proportional to the lateral force and the latter will be related to the angle a according to some law.

If for example we assume the $P_\beta \propto \sin \beta$ law (the law of the aeroplane for small angles), we have:—

$$F \propto \sin a \tag{1}$$

And the equation to the precession (App. VII.),

$$\tau = I\,\omega\,\Omega \sin a$$

$$F \propto \Omega \sin a \tag{2}$$

And writing r for the radius of the circular component of the / flight path, the equation giving the centrifugal force,

$$F = m\,r\,\Omega^2 \text{ or, } F \propto r\,\Omega^2 \tag{3}$$

where m is the mass of the projectile.

By (1) and (2),

$$\Omega \sin a \propto \sin a$$

or $\quad \Omega$ is constant

$\therefore$ by (3) $\quad F \propto r$

or by (1) $\quad r \propto \sin a$

Consequently as $\sin a$ diminishes, due to the hurrying of the precession, r diminishes in like ratio.

It is evident that since the want of alignment of the axis of spin and the axis of flight in the case of a spherical projectile is a thing of gradual growth, in the case of the elongate projectile the spiral of the flight path does not ordinarily acquire sensible magnitude, for the individual increments of change are infinitesimal; the whole effect is merely that the axis of the projectile follows the line of flight, the projectile flying point foremost, as is the case with a feathered arrow (Fig. 176), in contrast to the manner it is frequently depicted, as in Fig. 177. The spiral flight path only arises when some abrupt disturbance takes place, such as when at the time of firing the muzzle of the gun is situated in a cross-wind, or as when the bore at the muzzle end is damaged or irregular, when it is well known considerable disturbance of the trajectory results.

Since, in a well-conducted projectile, any initial "wildness"

We may similarly obtain a more general solution; thus for (1) we may write,

$$F \propto \sin^n a \qquad (4)$$

where according to the foregoing example $n = 1$, or according to the Newtonian law (*i.e.*, the law of the plane of extreme proportion in apteroid aspect), where $n = 2$. We have,

By (2) and (4),

$$\Omega \sin a \propto \sin^n a$$

or

$$\Omega \propto \sin^{n-1} a$$

and by (2) and (3)

$$r\,\Omega^2 \propto \Omega \sin a$$

or

$$\Omega \propto \frac{\sin a}{r}$$

$\therefore$

$$\frac{\sin a}{r} \propto \sin^{n-1} a$$

$\therefore$

$$r \propto \sin^{2-n} a$$

Thus in the case of the Newtonian law, $n = 2$, r becomes constant, and the diminution of the angle does not result in any change in the radius of the circular component of the spiral flight path.

In practice it may be taken as almost certain that the index n is less than 2 and probably greater than unity, consequently we may conclude that the reduction of $\sin a$ due to the hurrying of the precession has the effect of diminishing the value of r, although the said diminution may take place at a less rate than the simple ratio of the change in $\sin a$.

in the flight diminishes, so that the shooting may sometimes be relatively more consistent at 200 yards than at 100 yards, it is evident that there must be some influence at work tending to hurry the precession. The nature of this influence is in all probability that discussed in Vol. I. § 30, *i.e.*, it is due to the effect of "cut" or "side"; the projectile in its spiral flight and converging attitude (Fig. 175) experiences a

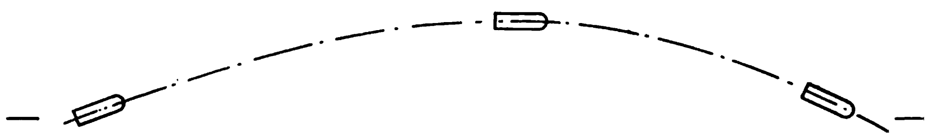


FIG. 176.

transverse reaction due to the want of symmetry of the lines of flow (comp. Fig. 22, Vol. I.); and this acting always tangentially to the cylinder containing the spiral flight path, tends to hurry the precessional motion. The effect so deduced is at least in the right direction, though in order to hurry the precession it would

FIG. 177.

appear not to be sufficient merely to chase the projectile sideways as a whole, but rather it is necessary to act upon it by a *torque*. This torque may conceivably arise as due to the streamlines acting more fully on the after body of the projectile than on the forward portion.

Apart from the explanation of the spiral flight path, a phenomenon that has frequently been observed[1] (but so far as

[1] The late Mr. R. H. Hausman informed the author that in bright sunshine it is frequently possible to actually *see* the bullet describing a corkscrew path, even at as high a velocity as 2,000 ft./sec., apart from evidence obtained by paper screens.

the author is aware not previously explained), the main result that has been deduced is the "tangent-to-trajectory" flight. Taking this as a basis, the author in 1904 succeeded in giving a quantitative solution to the "drift" problem, showing that the drift of an elongate projectile is almost entirely gyroscopic in origin.

The author's investigation on the "Drift of Projectiles" was contributed to *Technics* (No. 11), and the quantitative work related to the military ·303 Lee-Metford. By permission of Messrs. Geo. Newnes & Co., this article is here given *in extenso*.

The Drift of Projectiles.

"The cause of the deflection of a rifle bullet or projectile in its flight, to the right or left of the line of sight according to the direction of the rifling, is due to gyroscopic action. One of the conditions necessary to the proper flight of a rifle bullet is that its mass-centre (centre of gravity) shall not be too far forward; to be exact, it must be well behind the centre of wind pressure when the bullet has a slightly oblique motion. This is, in effect, how all bullets and projectiles for rifled guns are made. As a result, if the projectile were not spinning its equilibrium would be unstable, and it would tumble over in its flight.

"A parallel may be drawn between the rifle bullet and the ordinary spinning top, of which also the equilibrium is unstable in the absence of rotation. Just as the spinning top acquires stability by rotation, so does the projectile.

"This familiar parallel enables one to accept at once the well-known fact[1] that the axis of a projectile lies at all points of the flight in the direction of the trajectory; for, if we suppose this axis to maintain its original direction, so that after a time it becomes oblique to the direction of flight, we see that its condition

[1] The author has more recently come to the conclusion that this fact is not so well known as he at the time of writing supposed.

is that of a spinning top, resting permanently in an inclined position without precession, which is a condition we know to be impossible.

"If we suppose a bullet to be launched obliquely (as when the muzzle of the gun is in a strong wind) its subsequent career is of corkscrew form, owing to 'precession' such as shown by a

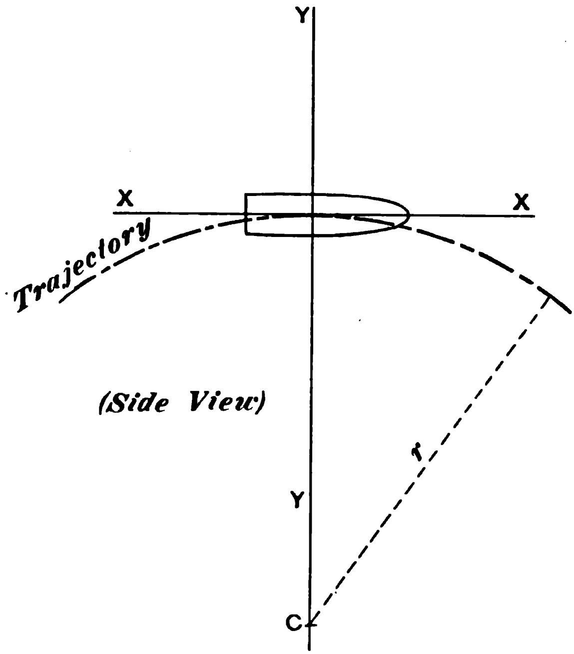


FIG. 178.

spinning top having an inclined axis; but if the change of condition is slow, such as when the course of the bullet is changed by gravity, the axis simply adapts itself to the new direction.

"I have so far dealt with the problem by analogy rather than by the more rigid method, in order to avoid becoming tedious and taking too much space; I propose now to take 'tangent-to-trajectory' flight of the projectile as an established hypothesis

(it is generally admitted as an experimental fact), and show how the drift follows as an immediate consequence.

"Fig. 178 represents a projectile and trajectory in side elevation.

" Fig. 179 gives same in plan.

"Fig. 180 shows the change of aspect in plan after a brief interval of time.

"Let r be the instantaneous radius of the trajectory, then

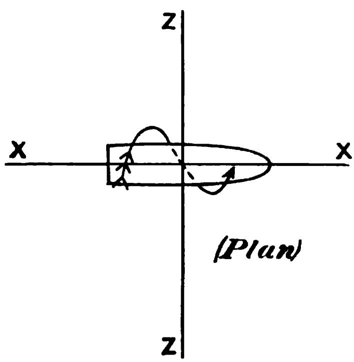


Fig. 179.

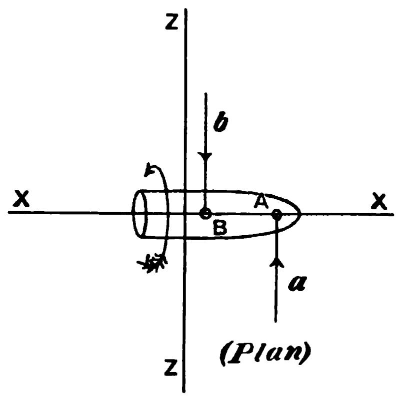


Fig. 180.

the projectile may be regarded as moving for the moment geometrically about the centre C (Fig. 178).

"Taking three co-ordinate axes as 'X' in the line of flight, 'Y' vertical, and 'Z' at right angles to both, we see that in plan Fig. 179, there is no angular momentum about the axis of 'Y'; but in plan Fig. 180, a small increment of time later, we find, due to the change of 'aspect,' the presence of angular momentum in a counter-clockwise direction when viewed from above (the rifling is taken as left-handed).

" Now, we know from first principles this signifies a 'torque' or 'couple' acting on the bullet in the direction of the angular momentum received; and we know that from the conditions of the problem, the elements of this couple can be resolved as a

lateral wind pressure acting at a point A (Fig. 181), and an equal and opposite inertia resistance at B (the centre of mass).

"This signifies that the condition of flight necessary to the axis 'following the trajectory' is that this axis is slightly oblique laterally, as indicated in Fig. 181. It is to this obliquity that the 'drift' is due.

"Of the two elements of the couple, *a* and *b*, it is seen that the force *a*, representing wind pressure, is the only one *acting from without*, and it is under the action of this force that the drift is produced: if we know the magnitude of the couple and the distance A—B, we can assign a definite value to the force *a*, and

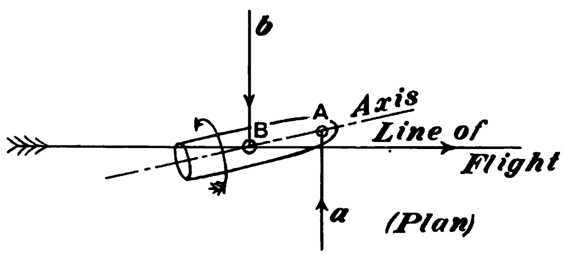


FIG. 181.

knowing the mass of the projectile, we can at once determine the acceleration of drift.

"Now, I know of no direct way by which the position of the point 'A' can be determined. This point may be described as the centre of lateral wind pressure for small angle; it might be found by experiments conducted on a carefully suspended model, but it is doubtful whether complete reliance could be placed on experiments at any speed that would be practicable. Probably the only way of arriving at a value for the distance A B is to deduce this from an actual measurement of the drift itself, thus inverting the problem in order to obtain the *constant* of any particular projectile.

"It is not actually certain that the position of point A is absolutely constant in respect to velocity, but this can probably

be relied on, provided that the range employed does not involve a critical velocity such as that of sound. In the absence of contrary data, it may be assumed as constant for our present purpose.

"Now, let l be the distance between A and B.

v be the velocity of the bullet.

r be the radius of the trajectory.

"Let Ω be the angular velocity of bullet about axis of Z (due to curvature of trajectory).

$I\omega$ the *angular momentum* of spin.

m the mass of the bullet.

f the lateral acceleration (*i.e.*, along the axis of Z) to which the drift is due.

"Then

$$r = \frac{v^2}{g} \text{ and } \Omega = \frac{v}{r} = v\,\frac{g}{v^2} = \frac{g}{v}$$

And *couple* $= I\omega\,\Omega = \dfrac{I\omega\, g}{v}$

But also *couple* $= f\,m\,l$

or,
$$f\,m\,l = \frac{I\omega\, g}{v}$$

$$\therefore \qquad f = \frac{I\omega\, g}{l\ m\ v} \text{ or } l = \frac{I\omega\, g}{f\,m\,v}$$

"In the above expression it will be seen that the quantities l and m are constants depending upon the bullet alone, the quantity $I\omega$ depends upon the moment of inertia I, the initial velocity, and the pitch of rifling. I estimate the value of $I\omega$ for the ·303 Lee-Metford at 2,000 ft./sec., as ·033 lb.-ft. ft./sec.[1]

[1] The moment of inertia I = ·000,0022 and ω (the angular velocity at 2,000 ft./sec.) = 15,000 radians/sec. giving $I\omega$ = ·033. In the original investigation an arithmetical error appears to have been made in the computation of the moment of inertia, the angular momentum being given as ·046 instead of ·033 as above. If the quantitative results had depended upon a prior knowledge of the value of l they would have been in error to a

"The value of g is taken as 32·2; the error introduced by neglecting the effective reduction in g caused by the 'soaring' of the bullet is discussed subsequently.

"If the range is sufficiently short, so that the velocity does not vary materially, then the value of f may be taken as constant, and the extent of drift bears a definite relation to the drop due to gravity; but in practice the velocity falls very rapidly, and the ratio of drift to drop increases with the range.

"Thus in Fig. 182, if the point 'O' be the axis of X and line $O\ P$ the projection of the path of the projectile on a vertical target, the curve will be of the form shown.

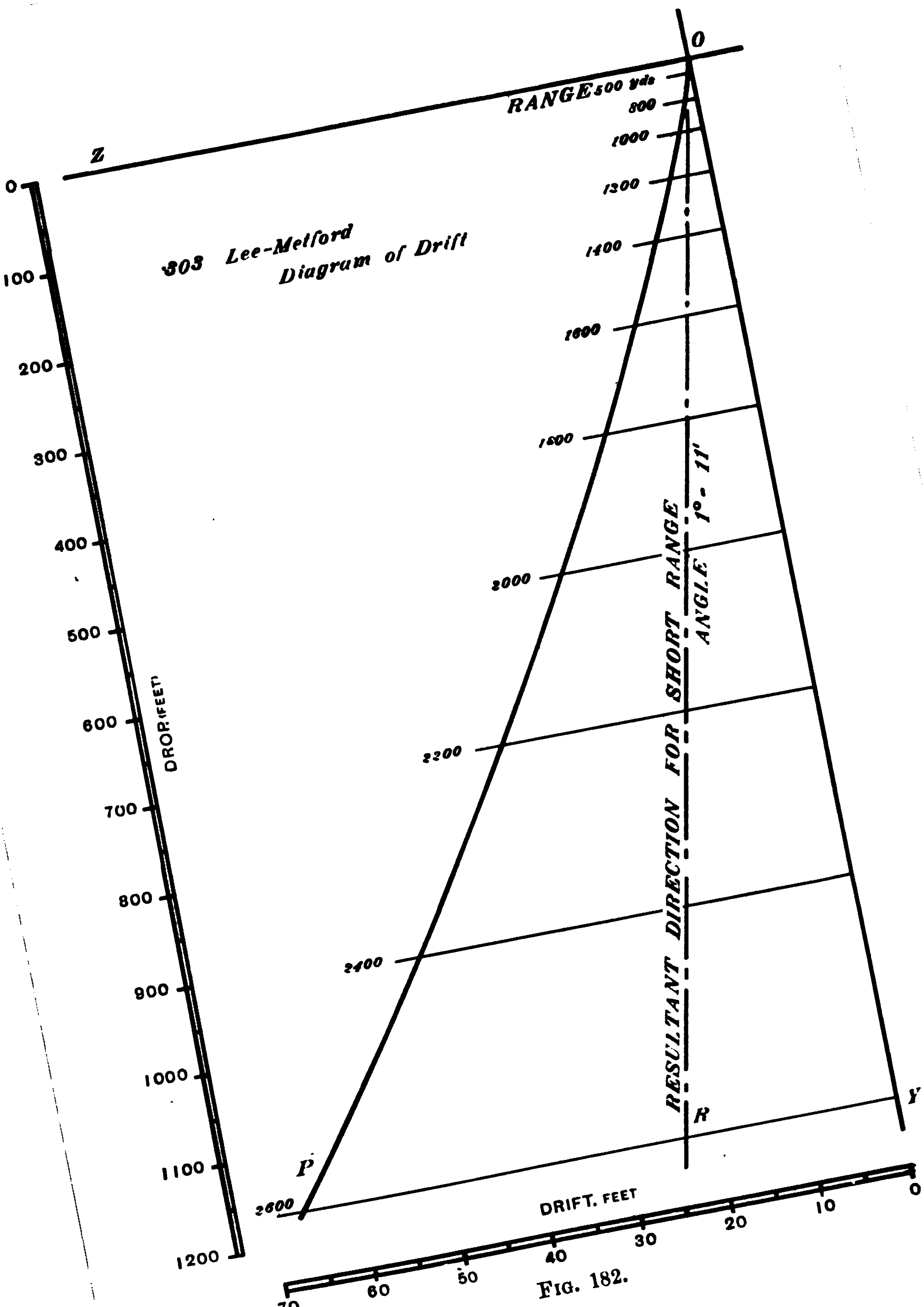
"The exact computation of drift necessitates plotting the velocity curve as in Fig. 183, where the abscissæ represent time. Curve v, v, v shows the velocity as determined by experiment or known data. Curve f, f, f gives acceleration of drift calculated from our formula. Curve w, w, w is the first integration of the curve f, f, f, giving the velocity of drift at successive intervals of time; and finally, in Fig. 184, curve h, h, h gives the further integration showing the actual drift displacement.

"I have taken the standard Lee-Metford rifle and service ammunition as the basis of calculation in the curves plotted in Figs. 182, 183 and 184, the former of which gives at a glance a comparison of drop to drift for ranges up to 2,600 yards, the ratio of scales being 10 : 1.

"The data as to velocity, plotted as a curve in Fig. 183, were supplied to me by Mr. R. H. Hausman, and, I understand, calculated from Mr. Bashforth's tables. The 'constant' on which, the *scale* of drift curves (Figs. 182 and 183) is based was determined by careful experiment at 1,000 yards range, the mean amount being found to be 30 inches. I am aware that this figure

---

like extent, but since the value of l was itself deduced from an actual measurement of the drift (30 inches at 1,000 yards), the drift values are correct, but the value of l originally given, *i.e.*, ·445 inch, is in error, the corrected value being ·31 inch or approximately $\frac{5}{16}$ths. The only effect of this is to throw Fig. 185 out of scale.



O
Z
RANGE 500 yds
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
·303 Lee-Metford
Diagram of Drift
DROP. (FEET)
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
RESULTANT DIRECTION FOR SHORT RANGE
ANGLE 1°- 11'
R
Y
P
DRIFT. FEET
70
60
50
40
30
20
10
0

FIG. 182.

differs from that frequently adopted, and that the exact determination of drift for some known range is at present wanting;

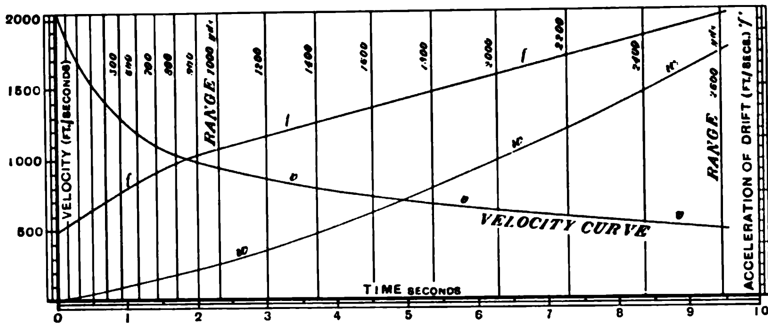


FIG. 183.

but pending further experimental determination (which should, if possible, be made at about 1,400 or 1,600 yards range) I think

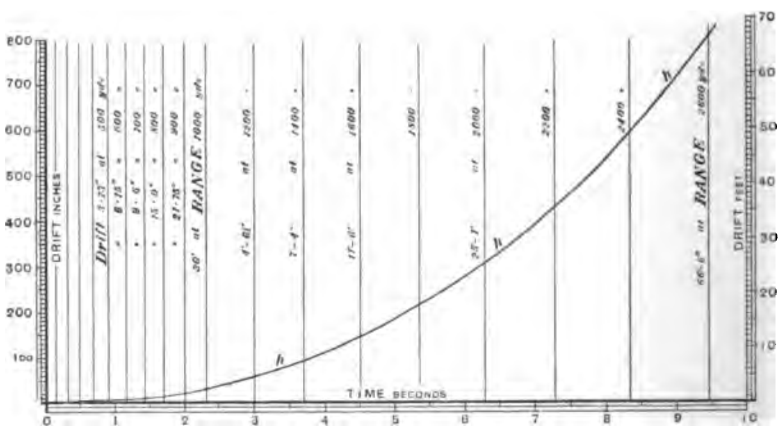


FIG. 184.

the value may be accepted as being certainly within 10 per cent. of the truth.

"The value of l for the Lee-Metford bullet on the above basis works out at 0·31 inch, that is to say, the effective centre of lateral pressure for evanescent angle is situated at about five-sixteenths from the nose of the bullet. It follows from our

equation that the amount of drift varies inversely as this quantity *l*, and consequently any new determination may affect the value here given; conversely, if the value 0·31 inch appears in any way improbable, it will for the present cast doubt either on the method of investigation, or on the accuracy of the experimental determination of the actual drift referred to above.

"It is of interest, therefore, to examine the question of the lateral wind pressure at small angle, with a view of testing the probability or otherwise of the figure we have obtained.

"Fig. 185 illustrates a supposed case of the resolution of forces required by the conditions of the problem. It will be seen that

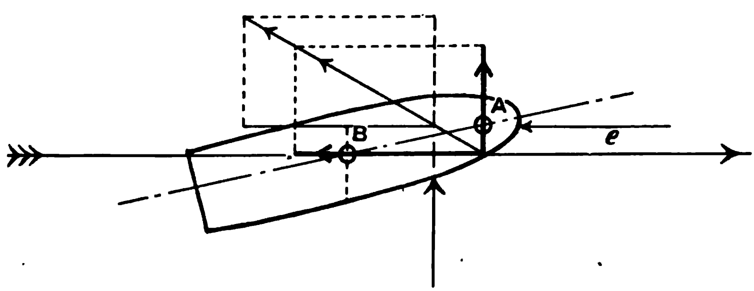


FIG. 185.

the pressures on the bullet must be analysed into a force in the axis of flight passing through the centre of gravity, and a force at right angles. From the construction shown it would appear to be possible that the point 'A' *might be* situated clear in front of the bullet, although this seems at first sight somewhat paradoxical; the actual interpretation is, of course, that the *real* couple *acting on* the bullet is in part due to the main resistance to flight not being aligned to the centre of gravity of the bullet as indicated by *e*, the resolution of forces shown being necessary to obtain the position 'A' as required by the assumption that the forces constituting the couple are transverse.

"It would thus appear that although at first sight the amount 0·31 inch would seem excessive, as placing the centre of lateral

pressure too far forward, closer examination puts quite a satisfactory interpretation on the matter.

"It is impossible in a brief article to deal fully with secondary effects, but attention may be called to one point in respect of which the above figures may require a correction.

"It is an experimental fact that rifled projectiles in flight 'soar' to a certain extent; that is to say, they do not drop as much as calculation would show, taking g as 32·2. One cause of soaring is without doubt the effect of 'cut,' owing to the axis being slightly oblique (sideways) to the line of flight. By 'cut' is meant the action which takes place when a golf ball soars, or a tennis or cricket ball swerves in the air. In the above numerical work the value of g in the equation $f\ \frac{\mathrm{I}\omega\ g}{l\ m\ v}$ has been taken as 32·2, whereas, though commencing at this value, it diminishes considerably as the range increases. The effect of error introduced from this cause will be to make the values given by the curve at long range too high. An error in the same direction, but of very small magnitude, will be introduced owing to the falling-off of the value of $\mathrm{I}\omega$ owing to the slowing down of the velocity of spin; it is believed that error from this cause is negligible.

"The experimental determination of drift is not so easy a matter as might be expected; a range of 1,000 or 1,200 yards is the least that is of real value, and almost perfectly still air is a *sine quâ non*. The Hon. T. F. Freemantle, in his 'Book of the Rifle,' confesses himself at a loss to judge between the different expert estimates, which vary from that of Mr. L. R. Tippens, who gives it as 4 feet at 1,000 yards, to the 'Musketry Regulations,' in which it is given as 11 inches for the same range.

"I venture to think that the drift diagram, Fig. 182, can be made of material assistance in bringing these widely different estimates into approximate harmony. Referring to the figure, it will be seen that the *true drift* is given by the departure of the

curve $O\ P$ from the axis of Y; but in order to exhibit this a match rifle, with spirit level and backsight set exactly at right angles, is necessary. If such a rifle, instead of having the level and sights set geometrically, were adjusted to shoot accurately at short range, say at 200 or 300 yards, the observed deflection at longer range will be an apparent drift given on the diagram by taking the resultant line $O\ R$ as datum.

"Now the apparent drift scaled from this diagram is about 1 foot at 1,000 yards and 2 feet at 1,200, but as the scale is very small I have calculated these quantities, and they come out as 11 inches at 1,000 yards and 23½ inches at 1,200 yards, and these figures are in remarkably close agreement with the 'Musketry Regulations,' in which the amounts are cited as 11 inches and 23 inches respectively.

"Before concluding, it is worthy of remark that spherical bullets fired from a rifled gun drift in the reverse direction to that indicated. This is owing to the points 'A' and 'B' being coincident, and the axis of rotation consequently retaining its initial direction; then as the bullet drops, the effect of 'cut' makes its appearance as a force acting laterally, and the direction of drift is negative. The negative drift sometimes reported with bullets of unusual shape or proportion is probably due to the same cause."

APPENDIX VIII B

THE BOOMERANG

THE boomerang originated as a weapon amongst the aborigines of Australia, both for use in war and for the pursuit of game. It must be looked upon from its origin as a *discovery* rather than a premeditated invention.

The Australian boomerang (Fig. 186) is commonly made from

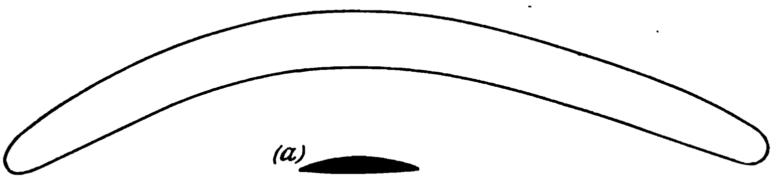


FIG. 186.

a naturally bent piece of hard wood, whittled down roughly to the form of section given at (*a*); the shape is that of two approximately straight limbs making an obtuse angle with one another, but the angle is sometimes a right angle.[1] The boomerang, when correctly thrown, has a spin imparted to it, the throw consisting in great part of a wrist motion, and the flight-path may be made to vary considerably, depending in part upon the form of the particular boomerang chosen, and in part upon the manner in which it is thrown. The most characteristic peculiarity of the flight is the well-known "return," though this feature is not invariably present.

[1] The form has been sometimes more accurately described as an approximate hyperbola.

It is believed that the boomerang was evolved by the natives from a wooden club or sword, which it became the fashion to throw at an adversary or quarry when otherwise out of reach; the bent form may in the first instance have been merely an accident, or possibly it was the result of a cunning endeavour

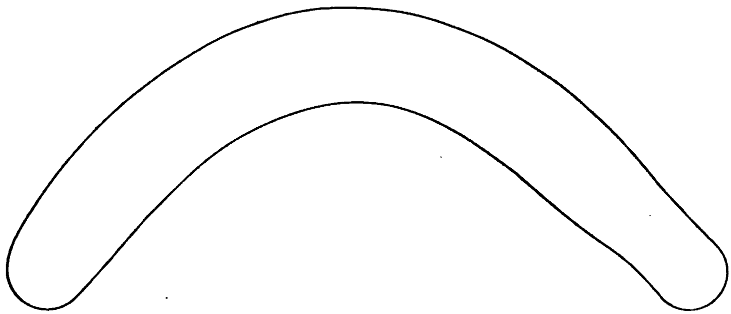


FIG. 187 (¼ full size).

to make a sword that would overreach the guard of an enemy; this explanation almost suggests itself when one handles a boomerang of the Australian type.

A convenient and handy size of boomerang termed the "Payne-Galway," Fig. 187, has been placed on the market,[1]

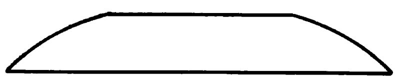


FIG. 188 (full size).

the section being as given in Fig. 188. This boomerang is of English ash, steam bent; it is of comparatively light weight and takes but little practice to throw successfully. The author has made boomerangs of sheet celluloid of three-limbed form (Figs. 189 and 190); these, when of the sizes and thicknesses shown, are suitable for indoor throwing or for use in a garden or other restricted area; the reason for and advantages of the three-limbed form will be explained later.

[1] Made by Messrs. Buchanan, of Pall Mall, London.

The theory of the flight of a boomerang has not been fully worked out. Probably the most advanced work that has been done on the subject is contained in a memoir by G. T. Walker,[1] in which an attempt is made to deal with the problem by mathematical analysis. It appears to the present author that this memoir, in spite of its unquestionable value, is not altogether sound in its initial premises; the results, however, speaking generally, are of the right kind.

In the discussion that follows no attempt is made to deal with the question quantitatively, but rather to elucidate the principles

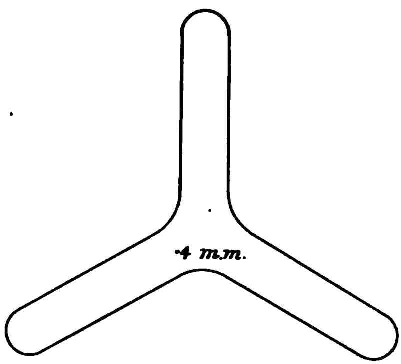


Fig. 189 (½ full size).

on which the peculiarities of the flight of the boomerang depend, regarding the latter as an example of the practical application of gyroscopic action for the maintenance of stability in flight.

Let us first examine the case of a circular flat disc, Fig. 191, projected edgewise, horizontally, with a rotary motion. Then the disc will begin to fall, and after a short time we may regard its weight as sustained by the aerodynamic reaction. But we know that the centre of pressure will then be in advance of the centre of gravity; consequently there will be a couple or torque tending to lift the leading edge, and this couple, counter-clock in Fig. 191, will give rise to angular momentum of like sense.

[1] Phil. Trans. cxc., 1897.

Owing to the spin of the disc this will be *angular momentum of aspect,* and the successive positions of the disc are as represented in the figure; as a secondary consequence the flight path is diverted from its original direction, owing to the lateral component of the pressure reaction. Thus if we throw a disc in the manner illustrated so that the direction of the spin viewed from

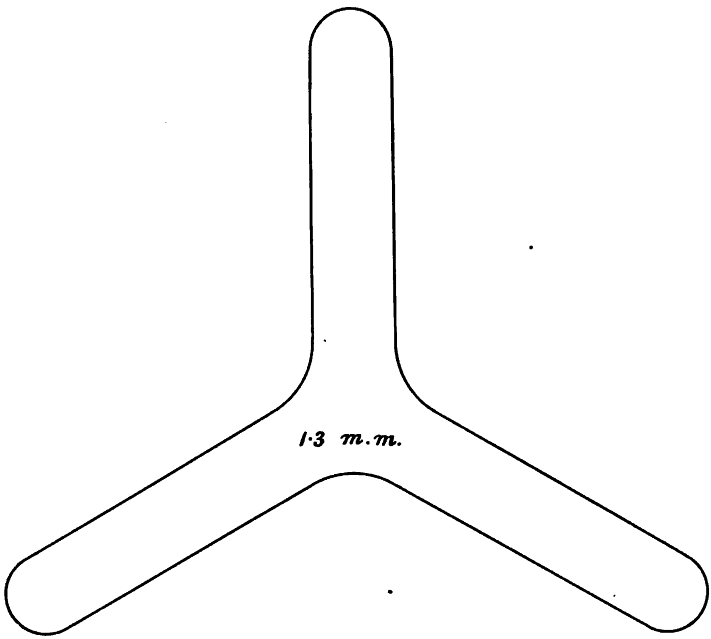


FIG. 190 ($\frac{1}{3}$ full size).

above is counter-clock, it will first tilt to the right and then undergo a change of *course* in like direction.

Let us now examine, under similar conditions, the behaviour of a boomerang. Instead of the simple disc, a boomerang, Fig. 192, is projected horizontally with a counter-clock spin about a vertical axis.

It is evident that the centre of pressure may still be in advance of the centre of gravity, but in addition to this the centre of pressure is displaced laterally. If we adopt the treatment of

§ 107 and suppose that each element of the area of the boomerang experiences a reaction proportional to the square of its velocity, we shall have the centre of pressure displaced as

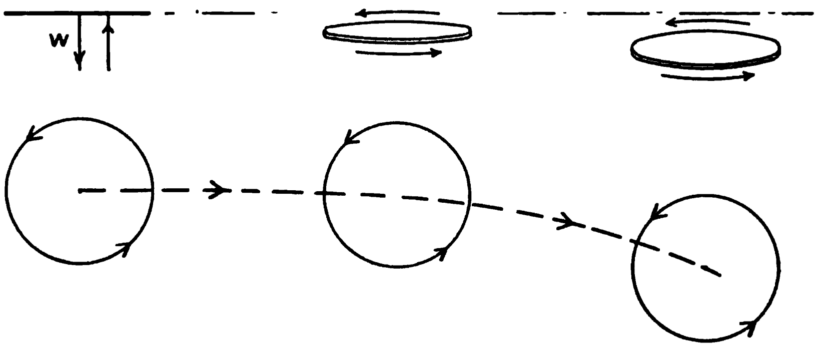


FIG. 191.

represented diagrammatically in Fig. 193 (*a*). In this figure it is evident that the motion of the boomerang viewed from above being counter-clock, the centre of pressure is displaced to the

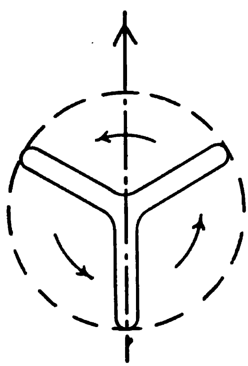


FIG. 192.

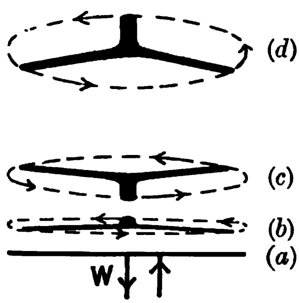


FIG. 193.

right, and the resulting torque, from the point of view of the thrower, is counter-clock. Consequently the angular momentum imparted to the boomerang about the axis of flight will also be counter-clock, and owing to the rotation it will be angular momentum of aspect; it is therefore the upper face of the

boomerang that will come into view as the result of the precession in the manner indicated in successive positions shown, (*b*), (*c*), and (*d*). As a consequence the path of the boomerang will take an upward trend, Figs. 194-5, and it will continue to rise until its kinetic energy is absorbed as potential, when it will

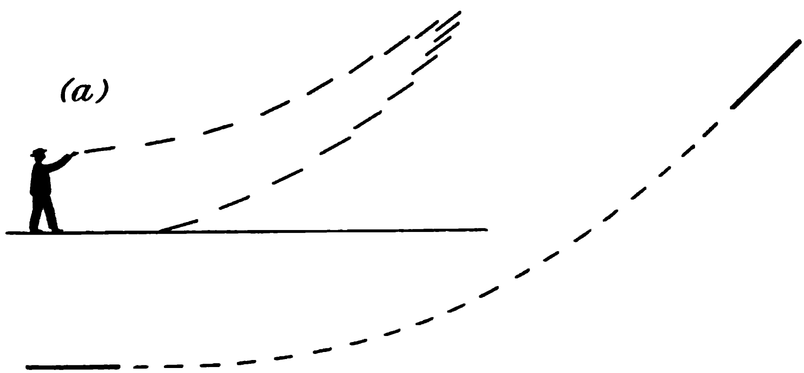


Fig. 194.

slide back along a similar path, and fall at the feet of the thrower, Fig. 194 (*a*).

When we take account of the forward displacement of the centre of pressure as well as its lateral displacement, we have

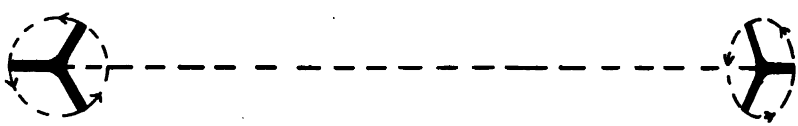


Fig. 195.

superposed on the motion as above depicted a further motion partaking of that of the previous example, the disc, and the flight path, instead of being in plan a straight line (Fig. 195), becomes a curve (Fig. 196).

If the plane containing the boomerang at the instant of projection is not horizontal, the flight path undergoes considerable modification. The form of trajectory given in Figs. 195-6 is not

good, because owing to the boomerang coming (translationally) to rest at the highest point of its flight path, its efficiency from an aerodynamic point of view becomes very poor, and the duration of the flight is correspondingly shortened. By initially inclining the plane of the boomerang when thrown as in Fig. 197, the flight path becomes (in plan) a loop of approximately circular

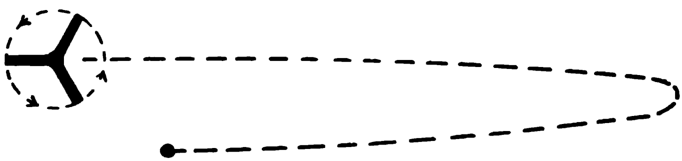


FIG. 196.

form, Fig. 198, and its velocity is nowhere unduly lowered. If the boomerang be supposed to lose none of its velocity, either rotational or translational, and neglecting the "disc effect," it would be possible to find conditions such that the flight path would be truly circular, the precession taking place about a vertical axis, as in the case of a spinning top, or more exactly a gyroscopic conical pendulum, Fig. 199. It is commonly the object of the thrower to approximate as closely as possible to such a circular path.[1]

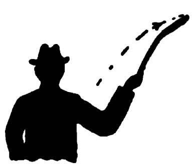


FIG. 197.

In making the comparison between a boomerang in flight and a gyroscope pendulum it is necessary to suppose the said pendulum loaded with a heavy counterpoise, in order to simulate the conditions as to the direction of torque and of precession; thus any influence acting to hurry the precession reduces the

[1] Experimenting in a large room with the boomerang shown in Fig. 189, the author has frequently obtained beautifully regular flight paths of converging spiral form, just as might be represented by the motion of the wheel of the gyroscopic pendulum with a torque applied to hurry the precession, and with a superposed motion of the whole instrument vertically downward, the latter being due in the case of the boomerang to the unavoidable dissipation of energy.

angle of inclination of the plane of rotation, and any influence acting to retard the precession has the opposite effect.

Now a displacement forward of the centre of pressure, as in the case of the disc or a simple aeroplane, manifestly tends to hurry the precession, for it is the equivalent of a force acting parallel to the pendulum rod but some little distance in advance of it in its motion;[1] hence the effect of such displacement is to reduce the inclination of the plane of rotation. This is actually found to take place when a boomerang is thrown; it becomes during its flight less and less inclined, that is to say, the axis of rotation becomes more and more nearly vertical, until after a complete loop has been traversed the boomerang will sometimes describe one or more simple oscillations of the kind already described with reference to Fig. 196. A diagram of a typical flight path is given in Fig. 198.

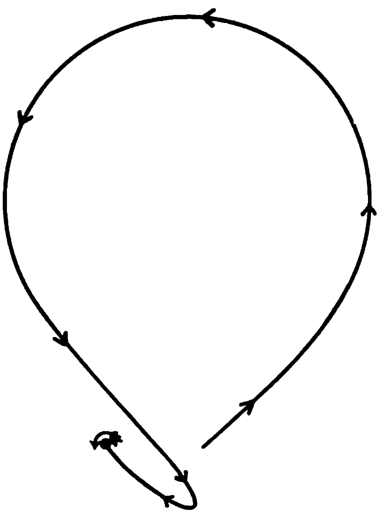


FIG. 198.

For the same reason, in order to obtain the longest possible flight, the initial angle should be very steep; in some cases, in fact, the boomerang is thrown in a nearly vertical plane.

The extent to which the disc effect is felt depends primarily upon the form of the boomerang; if the limbs are broad and the proportions are generally "chubby," the torque about the transverse axis will be relatively considerable. When, on the contrary, the limbs of the boomerang are thin and spider-like, the disc effect will not be so noticeable.

[1] As seen in plan it is evident that such a force exerts a turning effort about the vertical axis.

The dissipation of energy in the flight of the boomerang in all probability is not so very much different from that of a simple aerodone, and thus we may take the loss of energy as represented by the weight multiplied by γ times the distance traversed. The value of γ under the exceptional conditions in question we have

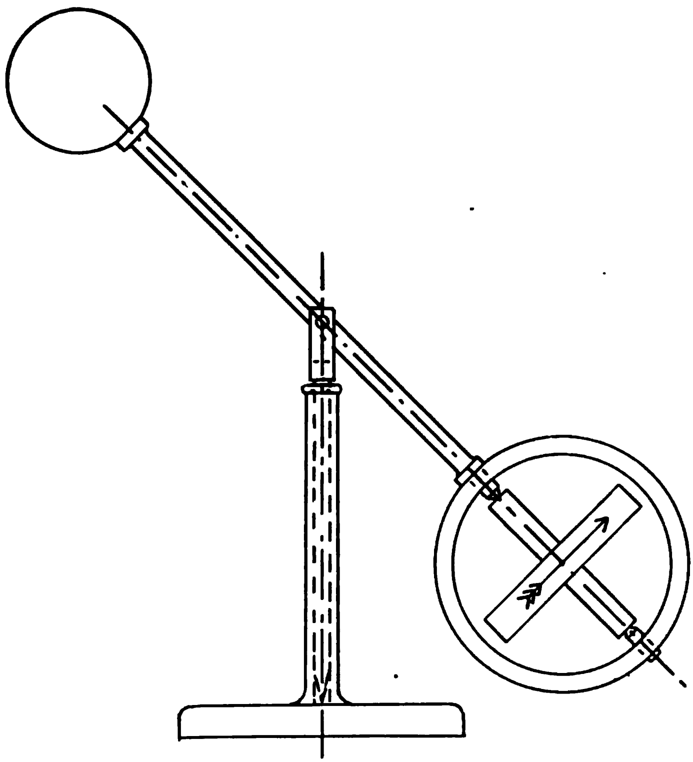


Fig. 199.

no means of calculating, but evidently if the boomerang is properly formed it will be of the same order of magnitude as the simple aerodone, or say, $= \cdot 2$.

Now, since the initial and final altitudes are approximately the same, we may take it that the whole of the energy expended is taken from that of initial velocity or initial rotation. Taking a

flight of 500 feet traversed, we have, if W be the weight, energy $= 100\, W$. Now, if this be taken wholly from the energy of translation, and if, as is sometimes the case, the velocity be approximately *nil* when the boomerang alights, the initial velocity will be that corresponding to a fall of 100 feet, or say 80 ft./sec., and the time of flight will be between six and seven seconds. As a matter of fact, the time for a flight of this length is usually about eight seconds, so that probably the velocity of projection is somewhat lower and some of the energy is derived from the rotation.

The author does not believe, in the case of a good boomerang, that much of the energy comes from the rotation, for after a successful flight the boomerang commonly comes to earth spinning briskly. In some cases it has even been observed that the speed of revolution is greater at the end of a flight than at the beginning, both as a matter of simple observation and when counted against a stop-watch.

The relative absorption in flight of the energy of translation and that of rotation depends upon the form of the boomerang. Thus if it be quite flat it is evident that both sources of energy will be utilised, whereas if it be of appropriate screw form the rotation may be maintained by the pressure reaction, when the energy of translation will alone be drawn upon. Again, if the said screw form be exaggerated the energy of rotation may actually be increased at the expense of that of translation. If, on the contrary, the screw form be of opposite "hand," *i.e.*, of corkscrewlike form, the energy of rotation may be drawn upon to any desired extent.

It is not always easy to appreciate from mere inspection of a boomerang whether it is, in effect, of right-handed or of left-handed screw form; thus the flat or under face may be markedly a right-hand screw and yet the whole form be in effect left-handed. The reason for this is to be found in the explanation given in Vol. I., § 30, of the *aerial tourbillion;* the flow round the convex leading edge of the boomerang ejects the dead water in that region and causes a suction by the centrifugal force of the adjacent

stream tending to maintain the rotation. This explanation in the present case is to some extent a conjecture, but the author believes that it will in due course receive confirmation.[1]

There is a further difficulty in judging the effective form of a boomerang, owing to its want of symmetry. If we suppose a two-limbed boomerang rotating truly about its principal axis, it is evident that there is nothing to balance the unsymmetrical distribution of the pressure reaction, and in practice, therefore, the boomerang will not spin exactly about its principal axis, but about an axis making a small angle with same, so that the resulting dynamic and aerodynamic couples[2] will be in equilibrium. It is for this reason that the author has recently adopted the three-legged form; such a form appears to be in every way equal to the Australian type, and has the advantage of being perfectly symmetrical so that it will spin true, and measurements of angles, etc., relatively to the containing plane may be relied upon.

The torque that gives rise to the gyroscopic precession is a variable depending upon the relation that exists between the velocity of rotation and that of translation, and upon the total pressure reaction. The latter is of necessity equal to the *apparent weight*, *i.e.*, the resultant of the weight and the centrifugal force of the boomerang at every instant; for any given value of the *apparent weight* the torque is proportional to the distance separating the centre of gravity and the centre of pressure, and hence depends upon the position of the latter. It

[1] Certain screw forms are very capricious in their behaviour. The aerial tourbillion described in Vol. I. (§ 30) may be looked upon as a windmill capable of behaving either as right or left handed. There is a certain form of screw that, mounted in a similar manner, will act as a windmill until a critical speed of revolution is reached, and then it will suddenly change its effective pitch and become a screw propeller eating its way up to windward until its velocity is reduced to a lower critical limit when the windmill conditions again supervene.

[2] The aerodynamic couple here referred to is not that to which the precessional motion is due, but one due to the want of symmetry of the boomerang; this couple is *mobile*, its axis rotates with the boomerang.

is evident that if the velocity of translation be zero, and that of rotation finite, the position of the centre of pressure will coincide with the centre of gravity. If, on the other hand, the velocity of rotation be small in comparison with that of translation, there will on the whole be but little lateral displacement of the centre of pressure, and in the limit, if we suppose the rotation to be extinguished, although the centre of pressure will be displaced *forward*, as in the simple aeroplane, there will be no lateral displacement, and the torque that produces the precession about the transverse axis will vanish.

Let us represent the boomerang by a straight narrow aeroplane as in Fig. 200, and let us suppose that the instantaneous

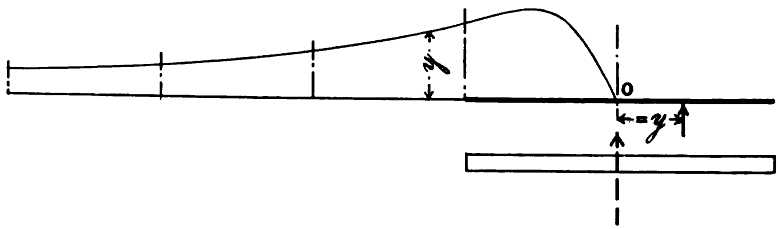


FIG. 200.

centre be moved from the geometric centre O away to infinity, so that in the first instance the motion is pure rotation and in the latter the motion is of pure translation. It is evident that, as above stated, for these extremes there is no displacement of the centre of pressure; on the basis of equal increments of K (comp. § 107), the pressure curve in the two cases is as represented respectively in Figs. 201 and 202. Now, for intermediate positions of the instantaneous centre of motion there will be displacement of the centre of pressure, as indicated in Fig. 200, in which the displacement of the centre of pressure is represented by the ordinates of a curve where abscissæ give the position of the instantaneous centre. As plotted, this curve is based on the V squared law and on the assumption of a uniform value of K per unit length. If the aeroplane is other than flat,

as when it is, in effect, of screw form, or if the distribution of K is not uniform, the form of the curve is different. In the former case, for example, the curve does not approach the line of zero torque asymptotically as the instantaneous centre runs to infinity, and the form of the curve is otherwise modified.

It is manifest that, from the considerations stated, the problem is far too complex at present for complete mathematical solution, that is for the flight path to be plotted from given data, etc., though it would appear possible to effect some approximation if the conditions are restricted artificially to a sufficient degree.

The author believes that under ordinary circumstances the

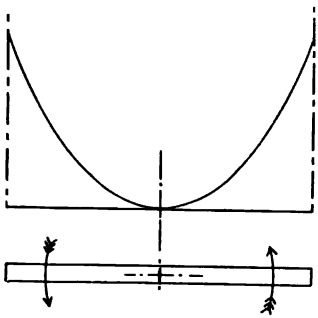


FIG. 201.

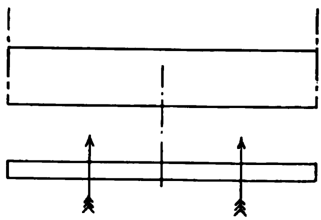


FIG. 202.

portion of the torque curve utilised is that in Fig. 200 in the region about and on the right-hand side of the maximum, so that the falling off of the velocity of translation, with that of rotation fully conserved, results on the whole in a diminution of the torque, and thus a reduction in the rate of precession, so that the flight path, instead of being a converging spiral, as would be the case on the basis of the gyroscopic pendulum, becomes of approximately circular form, so that the boomerang returns to its point of departure. This effect is probably helped by the fact that as the plane of rotation becomes more and more nearly horizontal the total pressure reaction becomes less and less, since the total pressure reaction may be computed by a simple resolution of forces.

Hence the torque is diminished from this cause as well as that stated.

It appears to the author that the above facts point to an opening for the quantitative treatment of the problem. Thus, for example, if we confine ourselves to the portion of the curve, Fig. 200, where the displacement of the centre of pressure is sensibly constant,[1] the torque that gives rise to the precession will, under changes of inclination, vary directly as the effective weight, that is inversely as *cos* a, where a is the angle made by the axis of rotation to the vertical (or by the plane of rotation to the horizontal). But by the equation to the gyroscope, App. VII.,

$$\tau = I\omega\,\Omega\,\sin a$$

hence we can correlate changes in the angle a and the rate of precession. Thus in the particular case where a is 45° the torque and *sin* a have the same relative rate of change, and at this critical value the rate of precession Ω is constant in respect of a.

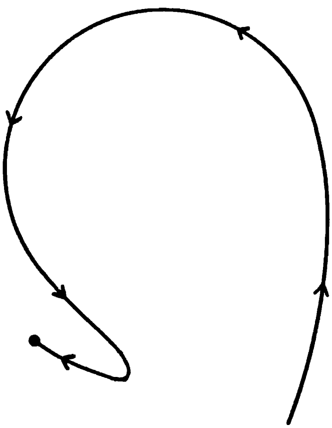


FIG. 203.

It is well known that the flight of a boomerang is greatly assisted by an appropriate wind. In still air considerable skill is required to make sure of a clean return, but flights of the form given in Fig. 203 are quite easy even to a novice. When such flights are made with a wind blowing from the "left front" a defective throw will become a good return. If, for example, a given throw in still air would give rise to a path of roughly cycloidal form, Fig. 204 (a), we know that with an appropriate superposed translation this becomes an approximate circle, Fig. 204 (b), and likewise for

[1] From an inspection of Fig. 200, and from the observed behaviour of a boomerang, it would appear that, for a considerable portion of the flight path at least, this assumption, as a first approximation, is justified.

other curves. A further effect of wind is that if opposed to the direction or projection it increases the energy of flight, so that the total time of flight may be greatly prolonged. A well-thrown boomerang in a high wind will return at a considerable height over the thrower's head with a velocity like that of driven grouse.

The whole question of wind effect, unless the weather is exceedingly boisterous, is a mere matter of relative motion, but even as such it possesses some subtleties. Thus, as remarked, the initial energy is enormously increased, in some cases being more

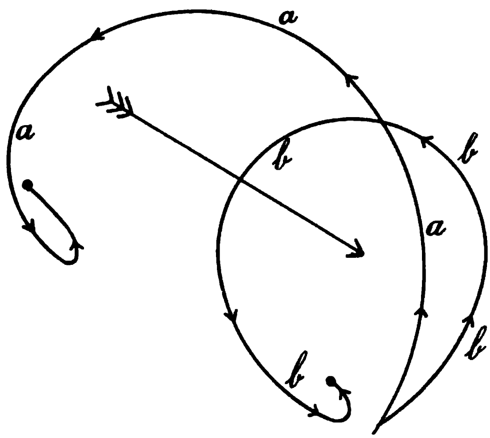


FIG. 204.

than doubled by the boomerang being thrown in the teeth of a gale. Also, the greater the head wind the more important it becomes to give an effective initial spin to the boomerang, for the wind adds to the velocity without adding to the rotation, and thus, unless the additional spin be given by the thrower, the behaviour of the boomerang in a high wind is not satisfactory. The author has found that in the case of boomerangs copied from Australian models, in which the angle is very open, and the principal axis consequently ill-defined, it is often easy to throw down wind (although it will not then return), but impossible to throw against even a moderate breeze, owing to the insufficiency of the initial spin.

APPENDIX VIIIc

THE SCHLICK MARINE GYROSCOPE

THE Schlick marine gyroscope is a recent invention for preventing or minimising the rolling of ships at sea This apparatus, illustrated diagrammatically in Fig. 205, consists of a fly-wheel A, whose bearings B, B, are carried by the frame C, which in turn is mounted on gimbals or trunnions D, D, arranged transversely to the vessel, the axis of rotation under normal conditions being vertical. When the vessel attempts to roll in a sea-way the gyroscope rocks or precesses on its gimbals, and by its precession it counteracts the turning moment, which would otherwise give the vessel angular momentum about a longitudinal axis—in other words, would produce a rolling motion.

The action of the gyroscope is illustrated in the series of diagrams given in Fig. 206. The lettered diagrams (*a*), (*b*), (*c*), etc., represent the vessel in the various phases of a passing wave, the direction of the torque due to the slope of the wave being clockwise when the water is higher on the left-hand side and *vice versâ*. The angular momentum communicated by this torque appears in the *momentum of aspect* (comp. Appendix VII.) of the gyroscope, instead of appearing in the rolling of the vessel, as would under ordinary conditions be the case. Thus, referring to Fig. 206 (*a*), we have a condition of maximum clockwise torque, and the gyroscope is precessing in such a manner as to exhibit clockwise momentum as at (*b*), the maximum condition being reached when, as at (*c*), the applied torque has fallen to zero. The wave surface then begins to slope

in the opposite direction, and the torque becomes counter-clock; the result of this is a precession of the gyroscope in the opposite sense and a diminution of the clockwise *momentum of aspect*, until, as shown at (*d*), nearly the whole of the clockwise momentum is given up, following which the momentum of aspect

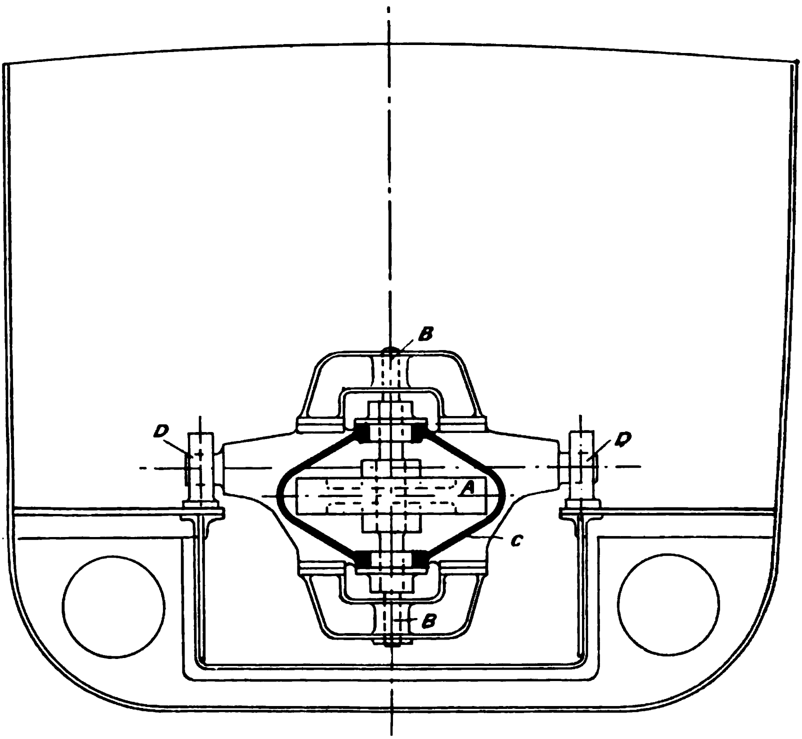


FIG. 205.

becomes counter-clock, (*e*), and continues to increase until the counter-clock torque falls to zero, when the precession, having reached its maximum value, ceases also, (*f*). The applied torque then again becomes clockwise, and the condition of the gyroscope returns to that shown in diagram (*a*) and the whole cycle is repeated.

Trials of this device have shown it to be highly efficient, and it

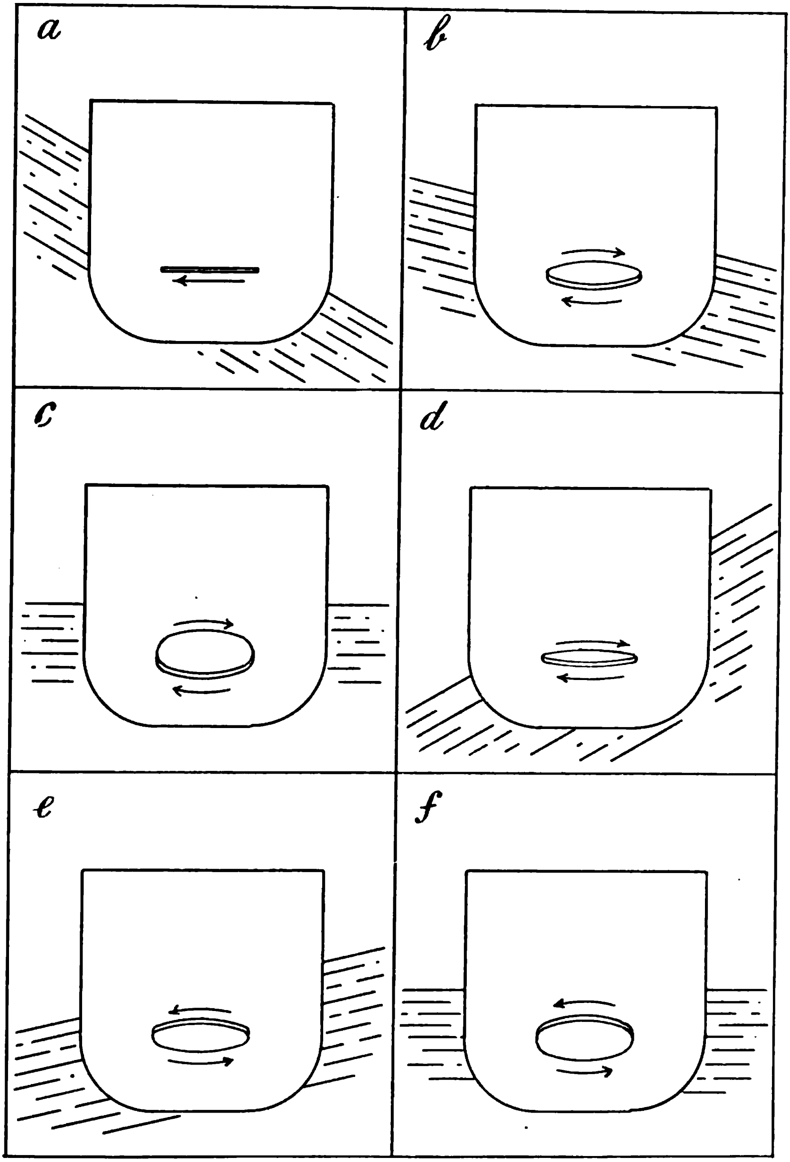


Fig. 206.

will doubtless come into more general use. An account by Sir William White has been published in *Engineering*, Vol. LXXXIII., p. 448.

In order to ensure the mean position of the gyroscope being that shown, *i.e.*, axis vertical, its mass centre is arranged slightly below the gimbal centres.[1]

It has been suggested by some that the force necessary to hold a vessel steady in a sea-way must be such as to cause an undue strain on the vessel, but such is by no means necessarily the case. Where no gyroscope is fitted and the rolling of the vessel is largely due to synchrony, the stresses and strains may sometimes be very much greater than would be the case were the vessel prevented from rolling altogether by brute force, whether supplied in the form of a gyroscope or otherwise. Of course, under all conditions the gyroscope should be fitted in the immediate neighbourhood of a transverse bulkhead, and where a vessel is of great size it would be obviously expedient to employ more than one gyroscope distributed along the length of the vessel in approximate proportion to the displacement.

[1] The above exposition, although fundamental and sufficient for the present purpose, is not altogether complete; complications arise under certain circumstances, as for example when the gyroscope is set in motion in a vessel already rolling. (Reference should be made, for more complete detail, to the article by Sir William White already cited.)

APPENDIX VIIID

THE GYROSCOPE FOR DIRECTION MAINTENANCE—THE WHITEHEAD TORPEDO

THE gyroscope in the Whitehead torpedo differs as to its mode of employment from the preceding examples; its function here is purely directive, it is not in any way concerned with the maintenance of equilibrium.

In brief, the gyroscope is mounted in a similar manner to a lecture model, on double gimbals, so that it is unaffected by the rotational movements of its surroundings; the torpedo may change its course upward or downward, or to the right or left, without the axis of rotation of the gyroscope undergoing any change of direction. The outer gimbal of the mounting is arranged to actuate a small rotary valve which controls the admission of air to a pneumatic cylinder; thus the gyroscope is utilised as if it were an accurately suspended mass of great moment of inertia.

In order to obtain a clear idea of the action of the above combination it is convenient to regard the rotary valve as being rigidly attached to one of the outer ring trunnions of the gyroscope, and so as being fixed in space, whilst the valve case is fixed to the torpedo and shares with the latter any change of course; then, every time the torpedo from any cause tries to depart from the ordained course, it moves the valve case round relatively to the valve (the latter being held stationary by the gyroscope), and so admits air under 250 lbs. pressure to the

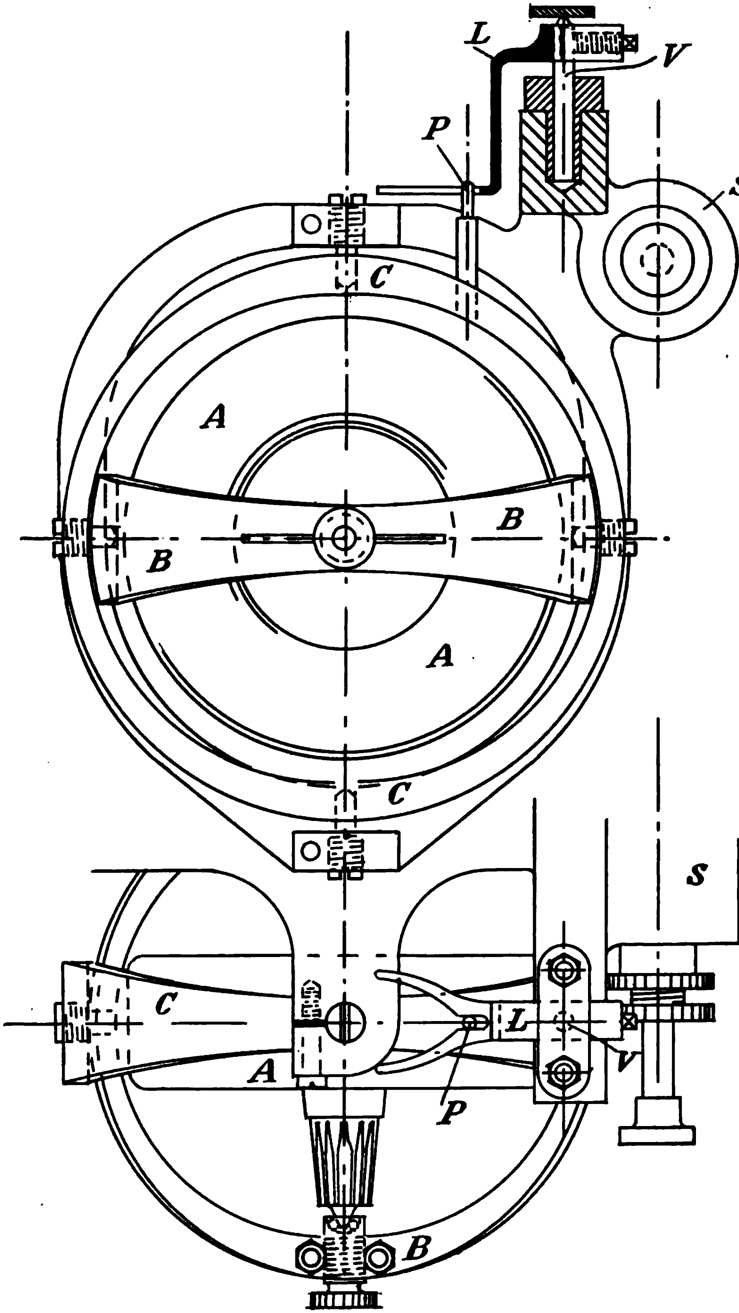


Fig. 207.

steering cylinder and almost immediately rectifies the error of direction. As a matter of fact, the course of a torpedo controlled by a gyroscope is not a straight line, but rather a just perceptible zig-zag whose mean direction is the course set by the gyroscope.

Referring to Fig. 207, in which the more essential parts are shown dissected from their surroundings, and to the photograph, Fig. 208, the gyroscope wheel A is mounted on ball-bearings in the ring B, which is pivoted in the outer ring C, which in turn is mounted on fixed centres in the usual manner. The rotary valve is not actually mounted on the outer ring trunnion as described, but is operated by a pin P projecting upward from the outer ring arranged to engage in a slot in the lever L fixed to the valve V, the end of this lever being so formed that the valve is capable of being moved through a certain definite range and no more, any further motion of the torpedo away from its prescribed course being without effect. The air is controlled to and from the steering piston, contained in the cylinder S, by passages in the cylindrical valve V; this valve is of small size, and but little force is required to move it one way or the other.

The effect of the little resistance that the valve offers is to cause a precession of the gyroscope about the trunnion axis of the inner ring B, without any appreciable yielding of the outer ring C; consequently the course is not in the least affected by valve-friction. The course could only become involved if the precession were to continue until it could go no further, as when the ring B reaches the limit of its motion, or, if no stops exist, until the axis of spin becomes coincident with the mounting of the ring C. Under the conditions of service such a contingency never arises, the total "flight" of a torpedo occupies but a few minutes, and as it has at the worst but little "bias" one way or the other, the reactions the gyroscope experiences are sometimes in one direction and sometimes in the other; consequently the precessional movements in great part correct each other, and

FIG. 208.

there is no continued precession such as would be necessary to put an end to the efficiency of the apparatus.

The gyroscope is put in action automatically immediately after launching, the initial spin being given by a toothed segment *D* (Fig. 208), actuated by a spring, and initially in gear with the pinion teeth on the axle of the fly-wheel. The friction on the ball-bearings is so small that the initial spin imparted in this way lasts for upwards of 20 minutes.

APPENDIX VIIIE

OTHER APPLICATIONS OF THE GYROSCOPE

OTHER applications of the gyroscope exist that are of interest from the present point of view. Thus in the Brennan mono-rail the equilibrium is maintained by a gyroscope combination, in which the *precession* is hastened by the action of a small roller mounted on an extension of the spindle of the revolving wheel. The equilibrium thus has some analogy to that of a spinning top, in which the precession is hastened by the frictional contact with the ground.[1]

In 1891 Sir Hiram Maxim proposed to secure the longitudinal stability of his aerodrome, or flying machine, by means of a gyroscope arranged to actuate horizontal bow and stern rudders, through the medium of a steam or pneumatic relay, in a manner similar to the maintenance of *course* in the Whitehead torpedo. As this machine never got beyond the captive stage, it may be presumed that the arrangement in question was not carried into effect.[2]

The Tower "steady platform"[3] is another device in which a gyroscope is arranged to give constancy of direction through the medium of a relay. In this device the gyroscope is arranged to run as a spinning top with its axis vertical. Water under pressure passes into a conduit system in the rotating wheel

[1] A description of the Brennan apparatus has appeared in *Engineering*, Vol. LXXXIII., pp. 623 and 794.

[2] Patent specification 19228 of 1891.

[3] See description in *The Engineer*, Vol. LXVII., p. 324 (1889).

through a ball-and-socket joint at the lower end of the spindle and escapes through apertures arranged tangentially at the periphery and through an axial hole at the upper end of the spindle; the former acts by recoil, after the manner of the well-known "Hero" engine, to maintain the velocity of rotation, and the latter acts in a very ingenious manner to operate one or another of a series of hydraulic cylinders by which the "platform" (on which may be mounted a searchlight or a light gun) is held in a horizontal plane. It is well known that water, escaping by an orifice, is capable of generating a pressure little short of that by which it is impelled. Thus in close proximity with the axial nozzle four apertures are arranged, each communicating with one of four hydraulic cylinders arranged to act on the platform. The latter is mounted on trunnions to swivel universally and carries the gyroscope mounting, so that if by the rolling or pitching of the vessel the platform begins to move rotationally in any direction, the jet of water issuing from the axial nozzle passes into one or other of the orifices, and there generates pressure in whichever of the hydraulic cylinders is in a position to correct the motion of the platform, and so maintain its constancy of direction. The platform is thus made to copy at all times the position of the gyroscope, and so partakes of its steadiness. The above form of relay is a singularly beautiful contrivance, inasmuch as it acts without any contact, frictional or otherwise, so that the gyroscope is unconstrained, and is not influenced by the mechanism on which it operates.

The weight of the gyroscope wheel is borne in most part by the water pressure at the socket joint, which serves as a "foot-step" bearing. Any remaining friction is utilised to retard the precession and so to bring the wheel into its position of static stable equilibrium; the centre of gravity of the rotating mass is arranged slightly below the point of support.

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BRADBURY AGNEW, & CO. LD., LONDON AND TONBRIDGE.